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Gravitational Wave from Cosmic Inflation in a Gravity with Two Small Four-derivative Corrections

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We investigate a model of inflationary cosmology where the minimally coupled scalar field theory is modified by additional correction terms. Among the most general ten correction terms remarked by Weinberg in context of effective field theory, we consider only two terms, $f_1(\phi)R^2$ and $f_2(\phi)R^{ab}R_{ab}$, following the work by Noh and Hwang where f_1 and f_2 are constant. The fourth order differential equations for the background universe and the tensor-type perturbation are derived out of this model. We show that these equations can be reduced to second order equations, supposing that f_n are small. From these approximated equations, we find that the propagation speed of gravitational wave is slightly less than the speed of light due to f_2 term, and that the evolution of the tensor-type perturbation is conserved in the large scale limit.

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I. INTRODUCTION

For a generic description of the very early universe governed by high energy physics where effects of quantum gravity can occur, Weinberg [1] suggested the most general corrections with four spacetime derivatives, ΔL,

$$\begin{aligned}\Delta L &= \sqrt{-g}\,\big[f_1(\phi)(\phi^{;c}\phi_{,c})^2 + f_2(\phi)\phi^{;c}\phi_{,c}\Box\phi \\ &\quad + f_3(\phi)(\Box\phi)^2 - f_4(\phi)R^{ab}\phi_{,a}\phi_{,b} \\ &\quad - f_5(\phi)R\phi^{;c}\phi_{,c} - f_6(\phi)R\Box\phi \\ &\quad + f_7(\phi)R^2 + f_8(\phi)R^{ab}R_{ab} + f_9(\phi)C^{abcd}C_{abcd}\big] \\ &\quad + f_{10}(\phi)\eta^{abcd}C_{ab}{}^{ef}C_{cdef}. \qquad (1)\end{aligned}$$

Here, R_{ab} is the Ricci tensor, R is the Ricci scalar, η^{abcd} is a totally antisymmetric Levi-Civita tensor density, and C_{abcd} is the Weyl tensor. This ΔL is added to the standard Lagrangian of the minimally coupled scalar field (MSF), L_0, describing the universe filled with scalar field [2–4], given by

$$L_0 = \sqrt{-g}\Big[\frac{1}{16\pi G}R - \frac{1}{2}\phi^{;c}\phi_{,c} - V(\phi)\Big], \qquad (2)$$

where G is Newton's constant and $V(\phi)$ is a potential as a function of single scalar field ϕ.

These correction terms with just four spacetime derivatives have been previously discussed by Elizalde *et al.* [5,6] in a different context. R^2 or $R^{ab}R_{ab}$ terms were studied by DeWitt (1967) searching for quantum theory of gravity and by Birrell and Davis studying on quantum fields in curved space [7,8]. In earlier times, Weyl, Pauli, and Eddington suggested a simpler version of the additional term(s) [9–11]. Especially, the term proportional to R^2, in a pure gravity theory without scalar field, has been discussed by Starobinsky [12], a special example of general $f(R)$ gravity [13] which substitutes the standard Einstein-Hilbert action. An inflation model based on Starobinsky gravity as well as non-minimally coupled scalar field theory [14–17] well explains the observational results pictured in the n_s(spectral index)-r(tensor-to-scalar ratio) plane, and these are preferred among other inflationary models by Planck Collaboration [18] who measures the cosmic microwave background (CMB) anisotropy. In addition to inflation, dark energy related scenarios are well accommodated by theories of modified gravity and scalar field [19,20]. Weinberg in his 2008 paper derived the tensor mode equation for only f_{10} correction [1]. Noh and Hwang considered f_7 and f_8 as constants without other correction terms and aimed at the explanation of cosmological gravitational wave [21]. Here, we mainly generalize this theory such that f_7 and f_8 are small corrections as functions of a scalar field.

In Sec. II, we derive gravitational field equations and scalar field equation of motion. In Sec. III, we apply the standard cosmological metric to the equations derived in section II. In Sec. IV, we use a perturbative approximation and obtain solutions under the condition of large scale limit; these are our main results. In Sec. V, we briefly discuss our results. We take the convention of

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Hawking and Ellis [22] and the notation of Hwang and Noh [23]. Here, $c \equiv 1 \equiv \hbar$.

II. EINSTEIN EQUATIONS AND EQUATION OF MOTION WITH TWO CORRECTION TERMS

The action considered here is

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}R - \frac{1}{2}\phi^{;c}\phi_{,c} - V(\phi) + f_1(\phi)R^2 + f_2(\phi)R^{ab}R_{ab}\Big], \tag{3}$$

where $f_1(\phi)$ and $f_2(\phi)$ are the dimensionless functions corresponding to f_7 and f_8 respectively in Eq. (1). Varying the action (Eq. (3)) with respect to the metric and the scalar field [4, 24–26] yields the gravitational field equations (GFE) and equation of motion (EOM):

$$R_{ab} - \frac{1}{2}g_{ab}R - 8\pi G(T^{(f_1)}_{ab} + T^{(f_2)}_{ab}) = 8\pi G T^{(MSF)}_{ab}, \tag{4}$$

where

$$T^{(MSF)}_{ab} = \phi_{,a}\phi_{,b} - \Big(\frac{1}{2}\phi^{;c}\phi_{,c} + V\Big)g_{ab}\,, \tag{5}$$

$$T^{(f_1)}_{ab} \equiv 2f_1\left(\frac{1}{2}R^2 g_{ab} - 2RR_{ab} - 2g_{ab}\Box R + 2R_{;ab}\right) -8f_{1,c}R^{;c}g_{ab} + 8f_{1,(a}R_{,b)} + 4f_{1;ab}R - 4\Box f_1 R g_{ab}\ , \tag{6}$$

$$\begin{aligned}T^{(f_2)}_{ab} \equiv{}& f_2 g_{ab}R^{cd}R_{cd} - 2g_{ab}(f_2R^{cd})_{;cd} + 4(f_2R_{(a}{}^{c})_{;b)c} - 2\Box(f_2R_{ab}) - 4f_2R_a{}^cR_{bc}\\ ={}& 2f_2\Big(\frac{1}{2}R^{cd}R_{cd}g_{ab} + R_{;ab} - 2R^{cd}R_{acbd} - \frac{1}{2}g_{ab}\Box R - \Box R_{ab}\Big)\\ &+ 2(-g_{ab}f_{2,c}R^{;c} - 2f_{2,c}R_{ab;d}g^{cd} + 2f_{2,c}R^c_{(a;b)} + f_{2,(a}R_{,b)})\\ &+ 2(-f_{2;cd}R^{cd}g_{ab} - \Box f_2R_{ab} + 2f_{2;c(a}R^c_{b)}),\end{aligned} \tag{7}$$

and

$$\Box\phi = V_{,\phi} - f_{1,\phi}R^2 - f_{2,\phi}R^{ab}R_{ab}\ . \tag{8}$$

Here, semicolons denote covariant derivatives, symmetrization of a tensor is defined as $T_{(ab)} \equiv \frac{1}{2}(T_{ab}+T_{ba})$, d'Alembertian of ϕ is written as $\Box\phi \equiv g^{ab}\phi_{,a;b}$, $V_{,\phi} \equiv \frac{\partial V}{\partial\phi}$, and $\dot\phi \equiv \frac{\partial\phi}{\partial t}$. In Eq. (7), the Bianchi identities [26] have been used in order to specify each component of the energy-momentum tensor conveniently. If f_1 and f_2 are constants, then the GFE are in agreement with the previous results by Noh and Hwang [21].

III. EVOLUTION OF BACKGROUND UNIVERSE AND GRAVITATIONAL WAVE

We assume a homogenous, isotropic, and spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric [26] for the description of the background universe and consider tensor-type linear perturbation:

$$ds^2 = a^2\big[-d\eta^2 + (\delta_{\alpha\beta} + 2C_{\alpha\beta})dx^\alpha dx^\beta\big]. \tag{9}$$

Here, $a(t)$ is the cosmic scale factor, $x^0 \equiv \eta$, and $dt \equiv ad\eta$. According to the notation of Hwang and Noh [23] who have formulated cosmological linear perturbation theory in various generalized gravity including scalar- and tensor-type perturbation, $C^{(t)}_{\alpha\beta}$ should be used instead of $C_{\alpha\beta}$ to indicate the tensor mode. However, the superscript (t) is omitted in this paper, since we deal with only gravitational wave. $C_{\alpha\beta}(\mathbf{x}, t)$ is tracefree and transverse with respect to the flat three-dimensional metric $\delta_{\alpha\beta}$, $C^\alpha_\alpha \equiv 0 \equiv C^\alpha_{\beta,\alpha}$. $C_{\alpha\beta}$ is also invariant under a gauge transformation [3,23,27–30]. Useful quantities calculated from the metric (Eq. (9)), are listed in the appendices of Noh and Hwang [21]. They include $G^a_b, \Box R$, *etc.* By substituting the metric (Eq. (9)) into GFE (Eq. (4)) and EOM (Eq. (8)), we obtain

$$\begin{aligned}&8\pi G T^{0(MSF)}_0\\ &= -3H^2 - 96\pi G\big[(3f_1 + f_2)(2H\ddot H - \dot H^2 + 6H^2\dot H) + \dot f_1 HR + \dot f_2(3H^3 + 2H\dot H)\big],\end{aligned} \tag{10}$$

$$T^{0(MSF)}_\alpha = T^{\alpha(MSF)}_0 = 0\,, \tag{11}$$

$$\begin{aligned}&8\pi G T^{\alpha(MSF)}_\beta\\ &= -(2\dot H + 3H^2)\delta^\alpha_\beta + D^\alpha_\beta\\ &- 8\pi G\Big\{4(3f_1 + f_2)\delta^\alpha_\beta(2\dddot H + 12H\ddot H + 9\dot H^2 + 18H^2\dot H)\\ &+ 8\dot f_1\delta^\alpha_\beta(\dot R + HR) + 4\ddot f_1R\delta^\alpha_\beta - 4f_1(RD^\alpha_\beta + \dot R\dot C^\alpha_\beta)\\ &- 4\dot f_1R\dot C^\alpha_\beta + 2\dot f_2\delta^\alpha_\beta(8\ddot H + 36H\dot H + 12H^3)\\ &+ 4\ddot f_2\delta^\alpha_\beta(2\dot H + 3H^2)\\ &+ 2f_2[\ddot D^\alpha_\beta + 3H\dot D^\alpha_\beta - 6(\dot H + H^2)D^\alpha_\beta - \frac{\Delta}{a^2}D^\alpha_\beta\\ &- 6(\ddot H + 2H\dot H)\dot C^\alpha_\beta - 4\dot H\frac{\Delta}{a^2}C^\alpha_\beta]\\ &+ 2\dot f_2[2\dot D^\alpha_\beta + 3HD^\alpha_\beta - 6(\dot H + H^2)\dot C^\alpha_\beta] + 2\ddot f_2 D^\alpha_\beta\Big\},\end{aligned} \tag{12}$$

and

$$\ddot\phi + 3H\dot\phi + V_{,\phi} - 36f_{1,\phi}\big(\dot H^2 + 4\dot HH^2 + 4H^4\big) -12f_{2,\phi}\big(\dot H^2 + 3\dot HH^2 + 3H^4\big) = 0\,, \tag{13}$$

where the Hubble parameter, $H \equiv \dot a/a$, the Ricci scalar, $R = 6(\dot H + 2H^2)$, and

$$D^\alpha_\beta \equiv \ddot C^\alpha_\beta + 3H\dot C^\alpha_\beta - \frac{\Delta}{a^2}C^\alpha_\beta\,. \tag{14}$$

Putting f_1 and f_2 to be constant and removing the ϕ-dependent terms, we get the results which agree with those of Noh and Hwang [21]. Therefore, their remarks on the qualitative sameness of the background contribution from R^2 and $R^{ab}R_{ab}$ theories also hold in this case.

We can split the energy momentum tensor into the background part (function of only time) and the small perturbed part (function of both time and space) in the cosmological linear perturbation theory based on the typical FLRW model [3,4,28], $T^a_b(\mathbf{x},t) = \overline{T^a_b}(t) + \delta T^a_b(\mathbf{x},t)$. The background parts are easily read off from the Eqs. (10) and (12):

$$\begin{aligned} &H^2 + 32\pi G\Big[(3f_1+f_2)(2H\ddot{H} - \dot{H}^2 + 6H^2\dot{H}) \\ &\quad + 6\dot{f}_1(2H^3 + H\dot{H}) + \dot{f}_2(3H^3 + 2H\dot{H})\Big] \\ &\quad = -\frac{8\pi G}{3}T^{0(MSF)}_0 = \frac{8\pi G}{3}\mu^{(MSF)} = \frac{8\pi G}{3}\Big(\frac{\dot{\phi}^2}{2} + V\Big), \\ &\dot{H} + 16\pi G\Big[2(3f_1+f_2)(\dddot{H} + 3H\ddot{H} + 6\dot{H}^2) \\ &\quad +6\dot{f}_1(2\ddot{H} + 7H\dot{H} - 2H^3) + \dot{f}_2(4\ddot{H} + 12H\dot{H} - 3H^3) \\ &\quad +6\ddot{f}_1(\dot{H} + 2H^2) + \ddot{f}_2(2\dot{H} + 3H^2)\Big] \\ &\quad = 4\pi G\Big(T^{0(MSF)}_0 - \frac{1}{3}\overline{T^{\alpha}_{\alpha}}^{(MSF)}\Big) = -4\pi G\dot{\phi}^2. \end{aligned} \tag{15}$$

The second equation can also be checked by diffentiating the first one and by using the EOM (Eq. (13)). The perturbed part of Eq. (12) is

$$\begin{aligned} &D^\alpha_\beta + 8\pi G\Big\{4f_1(RD^\alpha_\beta + \dot{R}\dot{C}^\alpha_\beta) + 4\dot{f}_1R\dot{C}^\alpha_\beta \\ &-2f_2[\ddot{D}^\alpha_\beta + 3H\dot{D}^\alpha_\beta - 6(\dot{H}+H^2)D^\alpha_\beta - \frac{\Delta}{a^2}D^\alpha_\beta \\ &-6(\ddot{H} + 2H\dot{H})\dot{C}^\alpha_\beta - 4\dot{H}\frac{\Delta}{a^2}C^\alpha_\beta] \\ &-2\dot{f}_2[2\dot{D}^\alpha_\beta + 3HD^\alpha_\beta - 6(\dot{H}+H^2)\dot{C}^\alpha_\beta] - 2\ddot{f}_2D^\alpha_\beta\Big\} = 0\ . \end{aligned} \tag{16}$$

Equation (16) is a fourth order differential equation for $C^\alpha_\beta(\mathbf{x},t)$. Thus, it is theoretically hard to deal with because more initial conditions are required for numerical analysis and these equations allow unnecessary unphysical solutions. With this concern for the problems of higher-derivative theories, the research on a perturbative method for reducing the order of derivatives has been done by Simon *et al.* [31–34].

IV. SECOND ORDER DIFFERENTIONAL EQUATIONS AFTER FEEDBACK

Considering the quantum corrections are small and neglecting f^2 terms allow the order reduction of the differential Eqs. (15) and (16):

$$\begin{aligned} &H^2 = 8\pi G\Big\{\frac{1}{3}\mu^{(MSF)} \\ &+8\pi G(3f_1+f_2)\big[8\pi G\big(\mu^{(MSF)} + p^{(MSF)}\big)^2 + 4H\dot{p}^{(MSF)}\big] \\ &+32\pi GH\big[\dot{f}_1\big(3p^{(MSF)} - \mu^{(MSF)}\big) + \dot{f}_2p^{(MSF)}\big]\Big\} \\ &= 8\pi G\Big\{\frac{1}{3}\Big(\frac{\dot{\phi}^2}{2} + V\Big) \\ &-64\pi G(3f_1+f_2)\Big[4\pi G\dot{\phi}^2\Big(\frac{\dot{\phi}^2}{4} + V\Big) + H\dot{\phi}V_{,\phi}\Big] \\ &+32\pi GH\Big[\dot{f}_1(\dot{\phi}^2 - 4V) + \dot{f}_2\Big(\frac{\dot{\phi}^2}{2} - V\Big)\Big]\Big\} \end{aligned} \tag{17}$$

and

$$\begin{aligned} &D^\alpha_\beta + 32\pi G\Big\{f_1\dot{R}\dot{C}^\alpha_\beta + \dot{f}_1R\dot{C}^\alpha_\beta \\ &\quad + f_2[3(\ddot{H} + 2H\dot{H})\dot{C}^\alpha_\beta + 2\dot{H}\frac{\Delta}{a^2}C^\alpha_\beta] \\ &\quad + 3\dot{f}_2(\dot{H} + H^2)\dot{C}^\alpha_\beta\Big\} = 0\ . \end{aligned} \tag{18}$$

A much simplified second order differential equation (Eq. (18)) for C^α_β is obtained by a feedback method: inserting $D^\alpha_\beta = \mathcal{O}(f^1_n)$ from Eq. (16) into the big curly brackets in Eq. (16) itself and neglecting very small $\mathcal{O}(f^2_n)$ terms. Likewise, using Eq. (15) and Eq. (13), we derived a modified Friedmann Eq. (17) in which the curly brackets may be regarded as $\frac{1}{3}$ of the effective energy density in this model.

Meanwhile, it is allowed to add a term of f^2_n-order, $96\pi Gf_2(\dot{H} + H^2)D^\alpha_\beta$, to Eq. (18) and to recover the f_1 gravity terms before the feedback:

$$\begin{aligned} &D^\alpha_\beta + 32\pi G\Big\{\big(f_1R\big)^{\cdot}\dot{C}^\alpha_\beta + f_1RD^\alpha_\beta + 3\big[f_2(\dot{H} + H^2)\big]^{\cdot}\dot{C}^\alpha_\beta \\ &\qquad +3f_2(\dot{H} + H^2)D^\alpha_\beta + 2f_2\dot{H}\frac{\Delta}{a^2}C^\alpha_\beta\Big\} \\ &\qquad = FD^\alpha_\beta + \dot{F}\dot{C}^\alpha_\beta + 64\pi Gf_2\dot{H}\frac{\Delta}{a^2}C^\alpha_\beta = 0\ , \end{aligned} \tag{19}$$

where

$$F \equiv 1 + 32\pi G[f_1R + 3f_2(\dot{H} + H^2)]\ . \tag{20}$$

Dividing Eq. (19) by F and using the definition of D^α_β in Eq. (14) lead to an equation for the tensor mode in the compact form:

$$\begin{aligned} &\frac{1}{a^3F}\big(a^3F\dot{C}^\alpha_\beta\big)^{\cdot} - \big(1 - 64\pi Gf_2\dot{H}\big)\frac{\Delta}{a^2}C^\alpha_\beta \\ &= \frac{1}{a^2z}\Big[v^{\alpha\prime\prime}_\beta - \Big(\frac{z''}{z} + c^2_T\Delta\Big)v^\alpha_\beta\Big] = 0\ , \end{aligned} \tag{21}$$

$$v^\alpha_\beta \equiv zC^\alpha_\beta\ , \quad z \equiv a\sqrt{F}\ , \tag{22}$$

and

$$c^2_T \equiv 1 - 64\pi Gf_2\dot{H}\ . \tag{23}$$

Here, $' \equiv \frac{\partial}{\partial\eta}$. Equation (21) is often called Mukhanov-Sasaki equation [2,30,35]. If c_T is the gravitational wave propagation speed, then it is affected not by the general function $f_1(\phi)$, but by the small f_2 correction term depending on time. Moreover, c_T should be less than the speed of light, thus the constraint that $f_2\dot{H} > 0$ is required.

In the large scale limit, a general integral form solution is obtained:

$$C^\alpha_\beta(\mathbf{x}, t) = c^\alpha_\beta(\mathbf{x}) + d^\alpha_\beta(\mathbf{x}) \int^t \frac{dt}{a^3 F} \ , \tag{24}$$

where $c^\alpha_\beta(\mathbf{x})$ and $d^\alpha_\beta(\mathbf{x})$ are the time-independent integration constants. Ignoring the decaying transient d-solution in an expanding universe, we note that the evolution of tensor type perturbation in the large scale limit is described by the conserved quantity $c^\alpha_\beta(\mathbf{x})$.

V. DISCUSSIONS

We have derived complicated fourth order differential equations of the gravitational wave as well as the background evolution in the inflationary universe implemented with the additional two modified gravity theories including a scalar field. Reducing the order by the perturbative approximation yields the more tractable equation and its solutions in the large scale limit. With model-dependent variables F, z, or c_T [36–38] , the form of Eq. (21) is maintained in various generalized gravity theories such as a model motivated by string theory. Those variables have been tabulated in Ref. 23 . If f_1 and f_2 are constants, Einstein gravity and Starobinsky gravity correspond to a limit of $F = 1$ and $F = 1 + 32\pi G f_1 R$ respectively . It would be more appropriate to call Eq. (21) Field-Shepley [39] equation if the priority were concerned.

According to Weinberg [1] , if the field equations derived from the MSF Lagrangian (Eq. (2)) are used in the correction Lagrangian (Eq. (1)) and ϕ and $V(\phi)$ are suitably redefined, then Eq. (1) can be simplified to have only three terms, f_1, f_9, and f_{10} . In other words, the ten terms in Eq. (1) are not independent to one another if the perturbative method at the action level and the redefinition approach are applied. We suggest an interpretation of the logic behind his argument that is simpler than our approach to the full Lagrangian as follows. Assuming that ΔL (Eq. (1)) is much smaller than L_0 (Eq. (2)), Einstein's equation (we set $8\pi G \equiv 1$ in this section only)

$$R_{ab} = \phi_{,a}\phi_{,b} + g_{ab}V \tag{25}$$

derived from L_0 (Eq. (2)) and its trace equation

$$R = 2(X + 2V) \tag{26}$$

with a convenient definition $X \equiv \frac{1}{2} g^{ab}\phi_{,a}\phi_{,b}$ can be put into ΔL (Eq. (1)). Assuming that $f_8 = -4f_7$,

$$f_7 R^2 + f_8 R^{ab} R_{ab} = -12 f_7 X^2 \equiv 4 f_1 X^2. \tag{27}$$

Thus, the f_1-gravity form [40] is obtained from the seventh and eighth terms in ΔL (Eq. (1)) with the above-mentioned assumptions. Our approach in a different context results in a modified propagation speed of gravitational wave that is measurable in principle. We selected and considered only two terms, f_7 and f_8 in the correction Lagrangian (Eq. (1)) and directly analyzed the action without any redefinitions and simplification, while we and Weinberg share the same assumption that the correction Lagrangian is small. We used the approximation at the wave equation (Eq. (16)), while he did the approximation at the action level. Comparison between two methods may be another issue.

There are several future investigations about this research. Firstly, quantizing Eq. (21) from the action level is straightforward by following the known prescriptions [23,30]. The unitarity shall be considered during quantization of the theories here to preserve the inner product of quantum states; however, the unitarity-violating term is encountered in a study of quantum cosmology [41] . Indeed, quantizing gravity is an abstruse issue for the very early universe. More fundamentally, various generalized gravity theories with higher-derivative expansion are motivated by string theory [42–45] . Secondly, if the Riemann-tensor-squared Lagrangian is studied, then the tensor mode equations in this paper will be able to transform into Weinberg's counterpart [1]. Thirdly, a heavy numerical analysis may allow a comparison of the exact equations and the approximate equations.

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