Journal of the Korean Physical Society, Vol. 75, No. 1, July 2019, pp. 54∼62

Excitation and Formation Conditions of Monotonic Shock Waves in Magnetized Plasmas with Superthermality Distributed Electrons

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(Received 6 November 2018; revised 15 February 2019; accepted 7 March 2019)

Nonlinear excitation and the properties of ion-acoustic shock waves (IASWs) in a magnetoplasma model composed of viscous ions with two-temperature superthermality distributed electrons are studied by employing the well-known reductive perturbation analysis to obtain a nonlinear Zakharov-Kuznetsov-Burgers equation (ZKBE), which admits the excitation of nonlinear IASWs in superthermal plasmas. Applying the tanh method, we discuss the solutions of the ZKBE. The asymptotic behavior and the stability of the analytical shock wave solution are studied. In general, nonlinear ion-acoustic disturbances are found analytically to exhibit only monotonic shock structures in the proposed model. For different situations, the effects of the dispersion and the dissipation coefficients on the profiles of the shock structures are discussed. The findings here demonstrate that the effective features of nonlinear IASWs depend strongly on the dispersion and the dissipation coefficients, which include physical parameters such as the superthermality of cold electrons, the cold superthermal electron-to-ion number density ratio, the ion kinematic viscosity and the ion cyclotron frequency. The current work may be helpful for an advanced comprehension of the physical nature of shock waves in astrophysical plasma situations.

PACS numbers: 52.27.−h, 52.35.Tc, 52.25.Xz, 52.35.Fp
Keywords: Monotonic shock waves, Superthermality distributed electrons, Zakharov-Kuznetsov-Burgers equation, Magnetized plasmas
DOI: 10.3938/jkps.75.54

I. INTRODUCTION

In the last few decades, numerous attempts have been made to examine the effective features of nonlinear waves for different models of plasmas in astrophysical environments [1–5]. In particular, the examinations of superthermal plasmas have drawn the attention of most investigators due to their existence in astrophysical plasma situations [6–8]. For instance, superthermal particles are found naturally in different astrophysical situations, such as those involving Earth, Saturn, Mercury and Uranus [9–11]. Rapid plasma particles are well known to be described by using a high-energy tail in the velocity distribution of superthermal plasmas. In addition to the collisions between waves and particles, an external force acts in astrophysical situations to generate superthermal plasma particles, such as strong radiation in the interplanetary medium and solar wind. One can say that superthermal particles seem to be associated with most astrophysical environments, and in the presence of strong shock waves, the particle acceleration is most effective. For example, the solar wind accelerates through a bow shock when the magnetic pressure becomes comparable with the momentum flux in the solar wind at $r\sim 10$ Earth radii [12]. Because of the presence of superthermal particles, the distribution function deviates from a simple Maxwellian to a non-Maxwellian distribution (*i.e.*, superthermal distribution). This kind of a superthermal distribution, is generally known as a non-Maxwellian/kappa (*i.e.*, κ-) distribution. Actually, the existence of particles with kappa distributions in a plasma model modifies the model's [13–15]. For example, El-Shamy [15] discussed the fundamental characteristics of positron periodic travelling waves due to the presence of superthermal electrons as well as positrons. El-Shamy [15] demonstrated that the superthermality of hot positrons plays a pivotal role in the nonlinear propagation of positron periodic travelling waves. On the one hand, a low value of the superthermal parameter κ represents a distribution with a large component of "superthermal particles". On the other hand, at a very large value of κ, the velocity distribution func-

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tion approaches a Maxwellian distribution. In addition, the effective thermal velocity is determined as $\theta = \{(\kappa - 3/2)/\kappa\}^{1/2}(2k_BT/m)^{1/2}$, where $T(m)$ is the kinetic temperature (mass) of superthermal particles, in which case one physically consider $\kappa > 3/2$.

Voyager plasma science (PLS) [16,17] encounters with Saturn are well known to have provided a powerful way to examine the interactions of acoustic waves in the environment of Saturn. Furthermore, brief bursts of ion-acoustic waves in the outer region of the magnetosphere are considered as one of the main observations of the Voyager 1 encounter with Saturn [16, 17]. Recently, two-distinct kappa distributions of the electron population in Saturn's magnetosphere were recorded by the Cassini plasma spectrometer (CAPS) [18], with independent low values of the superthermal parameters, κ_h for hot electrons and κ_c for cold electrons [19]. Moreover, many authors have already investigated theoretically the properties of solitons in plasmas with a Maxwellian distribution/non-Maxwellian distribution of hot and cold electrons [20–23]. A nonlinear medium famously admits soliton solutions. This happens due to the balance between the nonlinearity and the dispersion of the medium. For example, Saini *et al.* [23] derived the Zakharov-Kuznetsov (ZK) equation by balancing the nonlinearity of the medium with the dispersion of the medium. They found that the stability region of ion acoustic (IA) solitary waves is increased by decreasing the superthermality of electrons. In addition, Panwar *et al.* [24] investigated the oblique propagation of ion acoustic cnoidal waves in a magnetized plasma consisting of cold ions and two-temperature superthermal electrons. They demonstrated that the superthermality of cold electrons increases (decreases) the amplitude of compressive (rarefractive) ion acoustic cnoidal waves. Recently, the propagation of shock-like solutions in superthermal plasmas has received considerable attention and has been extensively investigated. In general, shock waves are created due to the balance between nonlinearity (causing wave steepening) and dissipation (caused by viscosity, collisions, wave particle interactions) of the nonlinear dispersive medium. The dissipative effect is one of the effects that play crucial roles in the excitation of the shock waves [25]. Furthermore, when a medium has both dispersive and dissipative effects, then the excitation of small amplitude shock waves can be appropriately described by using the KortewegdeVriesBurgers (KdVB) equation/the Zakharov-Kuznetsov-Burgers equation (ZKBE). The Burger term in the ZKBE arises by considering the kinematic viscosity among the superthermal plasma constituents. The domination of dissipation is well known to lead to a monotonic shock wave while the shock structures are oscillatory shock waves when the dispersion term dominates over the dissipative term in a plasma medium [26–28].

The nonlinear behavior of shock structures in superthermal plasmas has been investigated by many authors [29–32]. For instance, Alam *et al.* [29, 30] studied the basic features of planar and nonplanar dust-ion-acoustic (DIA) shock waves, respectively, in a dusty plasma that contained inertial ions, superthermal hot and cold electrons, and stationary negative dust grains. They found that the height and the steepness of a cylindrical shock wave are larger than those of a planar shock wave, but smaller than those of a spherical shock wave. Bains *et al.* [31] examined, in a one-dimensional plasma model, the oblique shock waves in two-electron-temperature superthermally magnetized plasmas. They stated that the shock amplitude and the velocity could be increased by increasing the ion kinematic viscosity. For numerical investigations, they [23,24,29–31] chose the numerical parameters of Saturn's magnetosphere. Furthermore, in all the mentioned works [23, 24, 29–31], the investigators reported that the obtained results were helpful in understanding the fundamental features of electrostatic waves in Saturn's magnetosphere. In astrophysical plasmas, such as Saturn's magnetosphere, the fact that cannot be ignored is that the one-dimensional propagation of shock waves may not only be a real situation in a plasma, but that the three-dimensional propagation of ion-acoustic shock waves (IASWs) may also be more realistic in superthermal magnetoplasmas. Therefore, our target is to examine, in a three-dimensional plasma system, the nonlinear excitation of IASWs in magnetized plasmas with non-Maxwellian hot and cold electrons. In addition, of interest is to obtaining the formation conditions of oscillatory and monotonic IASWs in this study. Moreover, we will pay attention to the effects of two-temperature κ-distributed electrons, the ion kinematic viscosity, an external magnetic field and obliqueness on the nature (the strength and the steepness) of nonlinear IASWs in superthermal plasmas.

The work is organized in the following manner: The basic equations of the proposed fluid plasma model are provided in Sec. II. Furthermore, the ZKBE is derived using the reductive perturbation analysis. In Sec. III, the analytical solution and the formation conditions of oscillatory or monotonic IASWs are discussed in two cases: asymptotic behavior and the stability of a small perturbation around the analytical solution. Moreover, a brief summary of the numerical investigations is given. In Sec. IV, shock wave solutions in two limiting cases are discussed. Conclusions are finally presented in Sec. V.

II. GOVERNING EQUATIONS

We consider a three-component collisionless magnetized plasma containing inertial viscous positive ions and two types of superthermal electrons with temperatures T_c and T_h. Therefore, at the equilibrium condition, $Z_iN_i^{(0)} = N_c^{(0)} + N_h^{(0)}$, where $N_i^{(0)}$, $N_c^{(0)}$, and $N_h^{(0)}$ are, respectively, the unperturbed number densities of ions and cold and hot superthermal electrons, and $Z_i = 1$ is a singly ionized plasma. The electron inertia is neglected if one considers the phase speed of the IASWs (*i.e.*, v_{Ph}) lies in the range $v_{thi} \ll v_{Ph} \ll v_{thce,thhe}$, where v_{thi} is the ion thermal velocity, and v_{thce} (v_{thhe}) is the cold (hot) electron thermal velocity. The set of normalized dynamic equations for nonlinear IASWs in a superthermal plasma can be given as follows [14,23,24,29–31]:

$$\frac{\partial N_i}{\partial T} + \grave{\nabla}\cdot(N_i\vec{V}_i) = 0, \tag{1}$$

$$\frac{\partial \vec{V}_i}{\partial T} + (\vec{V}_i\cdot\grave{\nabla})\vec{V}_i + \grave{\nabla}\varphi - \Omega(\vec{V}_i\times\hat{e}_z) = \eta_i\grave{\nabla}^2\vec{V}_i, \tag{2}$$

$$\grave{\nabla}^2\varphi = \mu_c N_c + \mu_h N_h - N_i, \tag{3}$$

$$N_c = \Big(1 - \frac{\sigma_c\varphi}{\kappa_c - 3/2}\Big)^{(-\kappa_c+1/2)}, \tag{4}$$

$$N_h = \Big(1 - \frac{\sigma_h\varphi}{\kappa_h - 3/2}\Big)^{(-\kappa_h+1/2)}, \tag{5}$$

where N_i is the ion number density, the physical quantities N_c and N_h are the number densities of cold and hot superthermal electrons, respectively, and $\vec{V}_i$ and φ are, respectively, the velocity of inertial viscous ions and the electrostatic wave potential. In addition, $\grave{\nabla} = (\partial/\partial X, \partial/\partial Y, \partial/\partial Z)$, where X, Y, and Z are the space coordinates. Here, T is the time variable. η_i is the ion kinematic viscosity. These physical quantities are scaled by $N_i \to N_i/N_i^{(0)}$, $N_c \to N_c/N_c^{(0)}$, $N_h \to N_h/N_h^{(0)}$, $V_{i(x,y,z)} \to V_{i(x,y,z)}/C_s$, $\varphi \to e\varphi/K_BT_{eff}$, $T \to T\omega_{pi}$, $\grave{\nabla} \to \grave{\nabla}\lambda_{eff}$, and $\eta_i \to \eta_i/\omega_{pi}\lambda_{eff}^2$, where C_s ($=\sqrt{K_BT_{eff}/m_i}$) is the ion acoustic speed, ω_{pi} ($=\sqrt{4\pi e^2N_i^{(0)}/m_i}$) is the ion plasma frequency, and λ_{eff} ($=\sqrt{K_BT_{eff}/4\pi e^2N_i^{(0)}}$) is the Debye radius. Moreover, the ion cyclotron frequency is Ω ($=(eB_0/m_ic)/\omega_{pi}$), and the effective temperature is T_{eff} ($=T_c/(\mu_c+\mu_h\sigma)$). The superthermality of cold (hot) electrons is κ_c (κ_h), $\sigma = T_c/T_h$, $\sigma_c = T_c/T_{eff}$, $\sigma_h = T_h/T_{eff}$, $\mu_c = \frac{N_c^{(0)}}{N_i^{(0)}}$, and $\mu_h = \frac{N_h^{(0)}}{N_i^{(0)}}$, where $\mu_c + \mu_h = 1$.

To examine the propagation of nonlinear IASWs in superthermal plasmas, we apply a reductive perturbation analysis. Then, we can expand the physical perturbed quantities N_i, $V_{i(x,y,z)}$, and φ about their equilibrium values in a power series of ε as [33]

$$\begin{aligned}\Psi &= \Psi^{(0)} + \sum_{n=1}^{\infty}\varepsilon^n\Psi^{(n)}, \text{ and}\\ V_{i(x,y)} &= \varepsilon^{3/2}V_{i(x,y)}^{(1)} + \varepsilon^2V_{i(x,y)}^{(2)} + \cdots,\end{aligned} \tag{6}$$

where

$$\Psi = [N_i, V_{iz}, \varphi] \text{ and } \Psi^{(0)} = [1, 0, 0]. \tag{7}$$

Now, we present the stretched coordinates as follows [34]:

$$\begin{aligned}X &= \varepsilon^{1/2}x,\\ Y &= \varepsilon^{1/2}y,\\ Z &= \varepsilon^{1/2}(z-\lambda t),\\ T &= \varepsilon^{3/2}t.\end{aligned} \tag{8}$$

Let us assume a weak damping due to the cold ion kinematic viscosity; then, $\eta_i = \varepsilon^{1/2}\eta_0$ [35]. This scaling hypothesis, with a view to analytical convenience, relates the various mechanisms involved in the dynamics. Furthermore, this assumption reflects the fact that the dissipative effect is considered to be small and finite, where ε is the strength of the nonlinearity. Furthermore, λ denotes the linear phase velocity. Now, putting Eqs. (6)–(8) into Eqs. (1)–(5) and collecting the nonzero order of ε, we obtain the following relations:

$$N_i^{(1)} = \frac{1}{\lambda^2}\varphi^{(1)}, \tag{9}$$

$$V_{ix}^{(1)} = -\frac{1}{\Omega}\frac{\partial\varphi^{(1)}}{\partial y}, \tag{10}$$

$$V_{iy}^{(1)} = \frac{1}{\Omega}\frac{\partial\varphi^{(1)}}{\partial x}, \tag{11}$$

$$V_{iz}^{(1)} = \frac{1}{\lambda}\varphi^{(1)}. \tag{12}$$

Here, $\lambda\Big(=\frac{1}{\sqrt{\mu_c\sigma_c\frac{(\kappa_c-1/2)}{(\kappa_c-3/2)} + (1-\mu_c)\sigma_h\frac{(\kappa_h-1/2)}{(\kappa_h-3/2)}}}\Big)$ is the linear phase velocity of the shock wave. Now combining Eqs. (9)–(12), we have the following evolution equation:

$$\begin{aligned}\frac{\partial\varphi}{\partial t} + A\varphi\frac{\partial\varphi}{\partial z} + B\frac{\partial^3\varphi}{\partial z^3} + C\frac{\partial}{\partial z}\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\Big)\varphi\\ -D\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\Big)\varphi = 0,\end{aligned} \tag{13}$$

where A is the nonlinearity coefficient, B and C are the dispersion coefficients, and D is the dissipation coefficient. Equation (13) is the ZKBE, which governs the nonlinear behavior of IASWs. For simplicity, we consider $\varphi = \varphi^{(1)}$:

$$\begin{aligned}A &= B\Bigg[\frac{3}{\lambda^4} - 2\Bigg(\mu_c\sigma_c\Big(\frac{(2\kappa_c-1)(2\kappa_c+1)}{2(2\kappa_c-3)^2}\Big)\\ &\quad + \mu_h\sigma_h\Big(\frac{(2\kappa_h-1)(2\kappa_h+1)}{2(2\kappa_h-3)^2}\Big)\Bigg)\Bigg],\\ B &= \frac{\lambda^3}{2}, \quad C = B\Big(1+\frac{1}{\Omega^2}\Big), \quad D = \frac{\eta_0}{2}.\end{aligned} \tag{14}$$

III. ANALYTICAL SOLUTIONS AND THE CONDITIONS OF OSCILLATORY AND MONOTONIC IASWS

Using $\varphi(x,y,z,t) = \phi(\zeta)$, with the transformation $\zeta\big(=\frac{1}{\gamma}(\ell_x x + \ell_y y + \ell_z z - ut)\big)$, normalized the constant speed of the reference frame u to the ion acoustic speed C_s, using $\frac{1}{\gamma}$ as the IASWs width and $\ell_x^2 + \ell_y^2 + \ell_z^2 = 1$, where ℓ_x, ℓ_y, and ℓ_z are, respectively, the directional cosines of the wave vector along the x-, y-, and z-axes, and assuming the boundary conditions $\phi(\zeta) \to 0$, $\phi'(\zeta) \to 0$, $\phi''(\zeta) \to 0$, and $\phi'''(\zeta) \to 0$ as $\zeta \to \pm\infty$, Eq. (13) becomes

$$-u\frac{d\phi}{d\zeta} + A\ell_z\phi\frac{d\phi}{d\zeta} + \frac{\ell_z}{\gamma^2}\big((B-C)\ell_z^2 + C\big)\frac{d^3\phi}{d\zeta^3} - \frac{D}{\gamma}\frac{d^2\phi}{d\zeta^2} = 0. \tag{15}$$

Now, let us discuss the possibility of the existence of oscillatory or monotonic IASWs. Integrating Eq. (15) and applying the above-mentioned boundary conditions, we obtain

$$\frac{d^2\phi}{d\zeta^2} = \frac{u\gamma^2}{\ell_z((B-C)\ell_z^2 + C)}\phi - \frac{A\gamma^2}{2((B-C)\ell_z^2 + C)}\phi^2 + \frac{\gamma D}{\ell_z((B-C)\ell_z^2 + C)}\frac{d\phi}{d\zeta}. \tag{16}$$

Clearly, Eq. (16) represents the equation of motion for a unit mass particle in a force field, where ϕ and ζ are a generalized coordinate and the time, respectively [36]. In general, the structure of IASWs is well known depend strongly on the physical parameters in the dissipation coefficient. To find the critical value of the dissipation coefficient corresponding to oscillatory and monotonic IASWs, one can examine the asymptotic behavior of the solution of Eq. (16). By linearizing Eq. (16) and considering $\phi = \phi_0 + \widetilde{\phi}$, where $\phi_0 = 2u/A\ell_z$ and $\phi_0 \gg \widetilde{\phi}$, we have [37],

$$\frac{d^2\widetilde{\phi}}{d\zeta^2} - \frac{\gamma D}{\ell_z((B-C)\ell_z^2 + C)}\frac{d\widetilde{\phi}}{d\zeta} + \frac{u\gamma^2}{\ell_z((B-C)\ell_z^2 + C)}\widetilde{\phi} = 0. \tag{17}$$

The solution of Eq. (17) is directly proportional to $\exp(m\zeta)$, where $m = \frac{\gamma D}{2\ell_z((B-C)\ell_z^2+C)} \pm \sqrt{\left(\frac{\gamma D}{2\ell_z((B-C)\ell_z^2+C)}\right)^2 - \frac{u\gamma^2}{\ell_z((B-C)\ell_z^2+C)}}$.

Accordingly, monotonic IASWs will be created if $D > \sqrt{4u\ell_z((B-C)\ell_z^2 + C)}$ whereas oscillatory IASWs will be produced if $D < \sqrt{4u\ell_z((B-C)\ell_z^2 + C)}$.

Now, when the tanh method is employed, the analytical solution of ZKBE is given as [38],

$$\phi(\zeta) = a_0 + a_1\tanh(\zeta) + a_2(\tanh(\zeta))^2, \tag{18}$$

where $a_0 = \frac{u}{A\ell_z} + \frac{12((B-C)\ell_z^2+C)}{\gamma^2 A}$, $a_1 = -\frac{6D^2}{25A\ell_z^2((B-C)\ell_z^2+C)}$, $a_2 = -\frac{12((B-C)\ell_z^2+C)}{\gamma^2 A}$, the shock wave width $\frac{1}{\gamma} = \frac{D}{10\ell_z((B-C)\ell_z^2+C)}$, and the constant speed of the reference frame $u = \frac{6D^2}{25\ell_z((B-C)\ell_z^2+C)}$. Therefore, Eq. (18) can be written as,

$$\phi(\zeta) = \phi_{\max}\Big(1 + \frac{1}{2}\text{sech}^2(\zeta) - \tanh(\zeta)\Big), \tag{19}$$

where $\phi_{\max}(= 6D^2/25A\ell_z^2((B-C)\ell_z^2 + C))$ represents the amplitude of the IASWs. In the present work, assuring that the condition $D > \sqrt{4u\ell_z((B-C)\ell_z^2 + C)}$ is satisfied, regardless of the numerical values of B, C and ℓ_z, is essential. Therefore, the suggested model exhibits only monotonic IASWs.

Now, examining the stability of a small perturbation ($\widetilde{\phi}_1$) around the analytical solution given in Eq. (19) in the form $\Phi = \phi(\zeta) + \varepsilon\widetilde{\phi}_1$, where $\varepsilon \ll 1$, is important. Putting Φ in Eq. (16) and then linearizing with respect to $\widetilde{\phi}_1$, one can obtain the differential equation for the perturbation $\widetilde{\phi}_1$ [35] as,

$$\frac{d^2\widetilde{\phi}_1}{d\zeta^2} = \frac{\gamma D}{\ell_z((B-C)\ell_z^2 + C)}\frac{d\widetilde{\phi}_1}{d\zeta} + \frac{u\gamma^2}{\ell_z((B-C)\ell_z^2 + C)}\widetilde{\phi}_1 - \frac{A\gamma^2}{2((B-C)\ell_z^2 + C)}\phi\widetilde{\phi}_1. \tag{20}$$

Equation (20) has a solution proportional to $\exp(p\zeta)$, where the parameter p is given by [35],

$$p_{1,2} = \frac{\gamma}{2\ell_z((B-C)\ell_z^2 + C)}\left(D \pm \sqrt{D^2 - 2\ell_z((B-C)\ell_z^2 + C)(\ell_z A\phi - 2u)}\right). \tag{21}$$

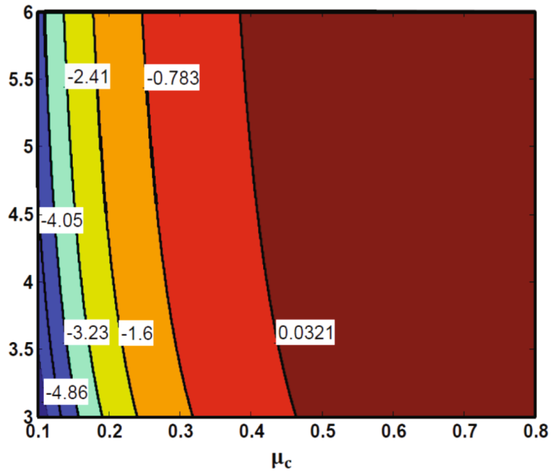

Fig. 1. (Color online) Variation of coefficient A in the μ_c-κ_c plane, where the numbers on the contour lines indicate the values of the coefficient A for $\kappa_h = 3$ and $\sigma = 0.01$.

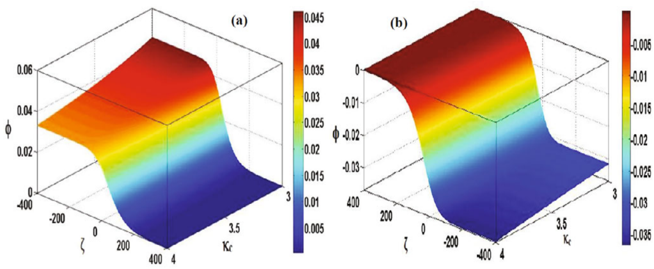

Fig. 2. (Color online) Three-dimensional profiles of the shock wave ϕ for various values of κ_c and $\kappa_h = 3.2$, $\sigma = 0.01$, $\Omega = 0.15$, $\eta_0 = 0.5$, and $\ell_z = 0.5$: (a) $\mu_c = 0.7$ (compressive IASWs) and (b) $\mu_c = 0.3$ (rarefactive IASWs).

Now, let us distinguish three different cases. Clearly, a small perturbation around the solution exhibits an exponentially behavior, which increases (decreases) if p is positive (negative). Furthermore, it has an oscillatory behavior if p is imaginary. Therefore, the small perturbation has an oscillatory perturbation if the following condition is met:

$$D^2 - 2\ell_z((B-C)\ell_z^2 + C)(\ell_z A\phi - 2u) < 0. \qquad (22)$$

Putting Eq. (19) into Eq. (22), the condition becomes

$$4\{1 + 12[1 + \tanh(\zeta)]^2\} < 0. \qquad (23)$$

Clearly, Eq. (23) can never be achieved, regardless of the value of ζ. Interestingly, the latter expression shows that oscillatory perturbations are not allowed, regardless of the plasma features. Consequently, the two roots of Eq. (21) must be real. One can obtain the analytic solution of Eq. (20) in the form

$$\widetilde{\phi}_1 = G_1 \exp(p_1\zeta) + G_2 \exp(p_2\zeta), \qquad (24)$$

where G_1 and G_2 are integration constants. Evidently that the sum of the two roots $p_1 + p_2 > 0$. Demonstrating from the sum of the two roots that at least one of the two roots p_1 or p_2 must be positive, which means that a small perturbation around the analytical solution given in Eq. (24) will be exponentially growing in ζ and will then disrupt the shock wave propagation is straightforward. Therefore, one can say that no perturbations of the shock wave structure take place in this region. Accordingly, the analytical solution provides only a strictly monotonic shock structure, which is unstable to external perturbations.

We are interested here in examining the behavior of monotonic IASWs. Figures 1–8 present the dependences of monotonic IASWs numerically on the physical parameters κ_c, κ_h, η_0, μ_c, σ, Ω, and ℓ_z. Using the numerical data of Saturn's magnetosphere is instructive [19,23,24, 29–31]. The density of cold superthermal electrons is $\sim$0.1 to 10 cm^{-3}, and the density of hot superthermal electrons is $\sim$0.1 to 1 cm^{-3}. Furthermore, the suprathermal electron temperature is observed to increase from $\sim$100 eV to 10 keV. Therefore, we can utilize σ ($\sim$0.005 to 0.5), μ_c ($\sim$0.2 to 0.7), κ_c ($\sim$3 to 5), κ_h ($\sim$2 to 3.5), Ω ($\sim$0.1 to 0.6), and ℓ_z ($\sim$0.2 to 0.8). Now let's start by studying the polarity of the monotonic IASWs. Evidently the signs of A and B have pivotal roles in determining the polarity of the IASWs. Clearly, B is always positive; hence, the monotonic IASWs have positive polarity (*i.e.*, compressive monotonic IASWs) if $A > 0$ and negative polarity (*i.e.*, rarefactive monotonic IASWs) if $A < 0$.

Figure 1 shows the effects of κ_c and μ_c on the nonlinear coefficient A. The contour plot demonstrates that the polarity of the IASWs depends on the numerical values of A, and in particular the numerical values of μ_c. Obviously a critical value of μ_c exists, below which rarefactive monotonic IASWs are formed. Figure 2(a) (2(b)) illustrates the variations of the compressive (rarefactive) monotonic IASWs profiles with the superthermality of cold electrons κ_c. Clearly the amplitude of the compressive (rarefactive) monotonic IASWs decreases (increases) with increasing of the superthermality of cold electrons κ_c. This is because, by increasing κ_c, the superthermality of the plasma system decreases, which leads to an increase (a decrease) in the magnitude of the nonlinear coefficient A (see Fig. 1). From the analytical solution of the shock wave structure (Eq. (19)), the amplitude of shock wave is seen to be basically controlled by the nonlinear coefficient; thus, as A increases (decrease), the shock wave potential decreases (increases). In contrast, the width of the compressive (rarefactive) monotonic IASWs increases (decreases) with increasing of κ_c. That is to say, the superthermality of cold electrons κ_c has powerful effects on the structures of the monotonic IASWs. The results are shown in Fig. 3(a) (3(b)), showing the changes in the profiles of the compressive (rarefactive) monotonic IASWs with the superthermality of hot electrons κ_h. The amplitude and the steepness of

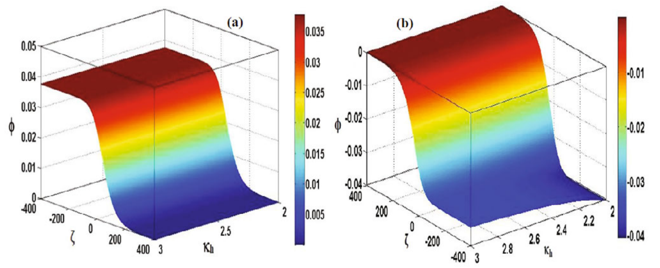

Fig. 3. (Color online) Three-dimensional profiles of the shock wave ϕ for various values of κ_h and $\kappa_c = 3.5$, $\sigma = 0.01$, $\Omega = 0.15$, $\eta_0 = 0.5$, and $\ell_z = 0.5$: (a) $\mu_c = 0.7$ (compressive IASWs) and (b) $\mu_c = 0.3$ (rarefactive IASWs).

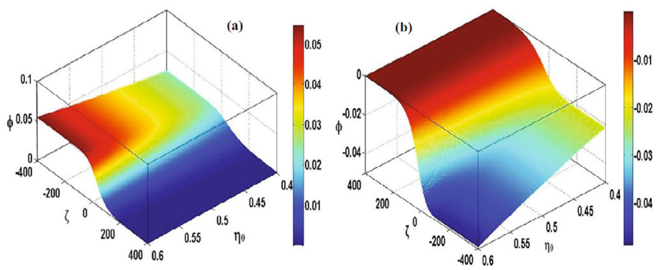

Fig. 4. (Color online) Three-dimensional profiles of the shock wave ϕ for various values of η_0 and $\kappa_c = 3.5$, $\kappa_h = 3.2$, $\sigma = 0.01$, $\Omega = 0.15$, and $\ell_z = 0.5$: (a) $\mu_c = 0.7$ (compressive IASWs) and (b) $\mu_c = 0.3$ (rarefactive IASWs).

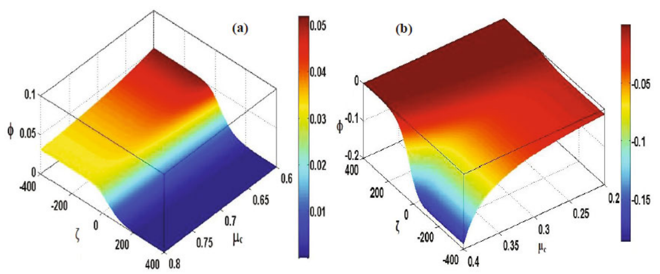

Fig. 5. (Color online) Three-dimensional profiles of the shock wave ϕ for various values of μ_c and $\kappa_c = 3.5$, $\kappa_h = 3$, $\sigma = 0.01$, $\Omega = 0.15$, $\eta_0 = 0.5$, and $\ell_z = 0.5$: (a) compressive IASWs and (b) rarefactive IASWs.

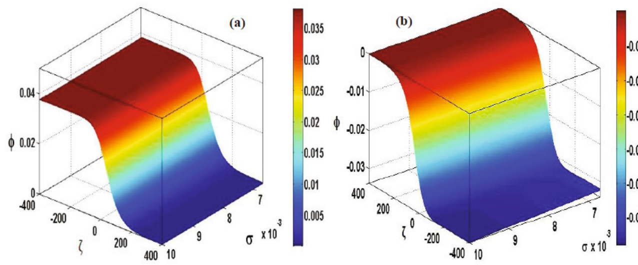

Fig. 6. (Color online) Three-dimensional profiles of the shock wave ϕ for various values of σ and $\kappa_c = 3.5$, $\kappa_h = 3$, $\Omega = 0.15$, $\eta_0 = 0.5$, and $\ell_z = 0.5$: (a) $\mu_c = 0.7$ (compressive IASWs) and (b) $\mu_c = 0.3$ (rarefactive IASWs).

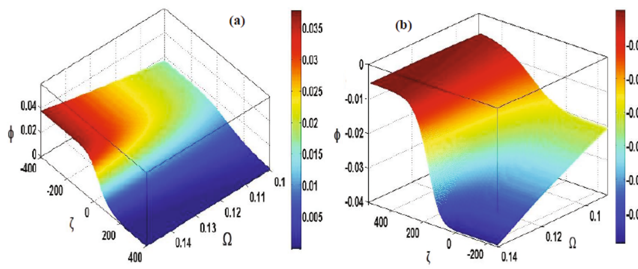

Fig. 7. (Color online) Three-dimensional profiles of the shock wave ϕ for various values of Ω and $\kappa_c = 3.5$, $\kappa_h = 3$, $\sigma = 0.01$, $\eta_0 = 0.5$, and $\ell_z = 0.5$: (a) $\mu_c = 0.7$ (compressive IASWs) and (b) $\mu_c = 0.3$ (rarefactive IASWs).

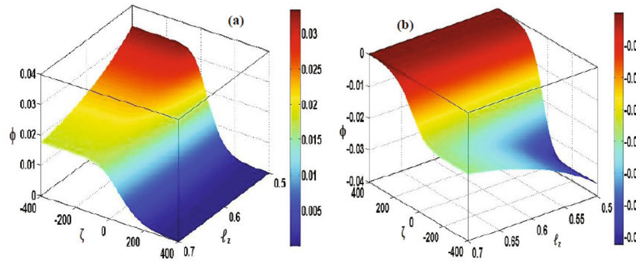

Fig. 8. (Color online) Three-dimensional profile of the shock wave ϕ for various values of ℓ_z and $\kappa_c = 3.5$, $\kappa_h = 3.2$, $\sigma = 0.01$, $\eta_0 = 0.5$, and $\Omega = 0.15$: (a) $\mu_c = 0.7$ (compressive IASWs) and (b) $\mu_c = 0.3$ (rarefactive IASWs).

the compressive (rarefactive) monotonic IASWs decrease slightly (increase) due to the decrease of the superthermality of hot electrons κ_h.

Figure 4(a) (4(b)) exhibits the influence of the ion kinematic viscosity η_0 on the compressive (rarefactive) monotonic IASWs. Obviously, both the strength and the steepness of the IASWs increase as η_0 increases. The existence of η_0 among the plasma constituents is well known to give rise to the Burgers term D in the ZKBE. Therefore, the strength and the steepness of the nonlinear monotonic IASWs are enhanced. Figure 5(a) (5(b)) demonstrates the impact of the cold superthermal electron-to-ion number density ratio μ_c on the profiles of the compressive (rarefactive) monotonic IASWs. For compressive monotonic IASWs, the amplitude and the width of the compressive monotonic IASWs are found to decrease and increase, respectively, due to the enhancement of μ_c. On the contrary, for rarefactive monotonic IASWs, the amplitude and the width of the rarefactive monotonic IASWs are found to increase deeply and decrease sharply, respectively, because of the growing of the cold superthermal electron-to-ion number density ratio μ_c. The effect of the cold superthermal electron-to-hot superthermal electron temperature ratio σ on the structures of monotonic IASWs is significant. The strength and the steepness are seen to increase (decrease) slightly by decreasing the values of σ as shown in Fig. 6(a) (6(b)). Figure 7(a) (7(b)) represents the relation be-

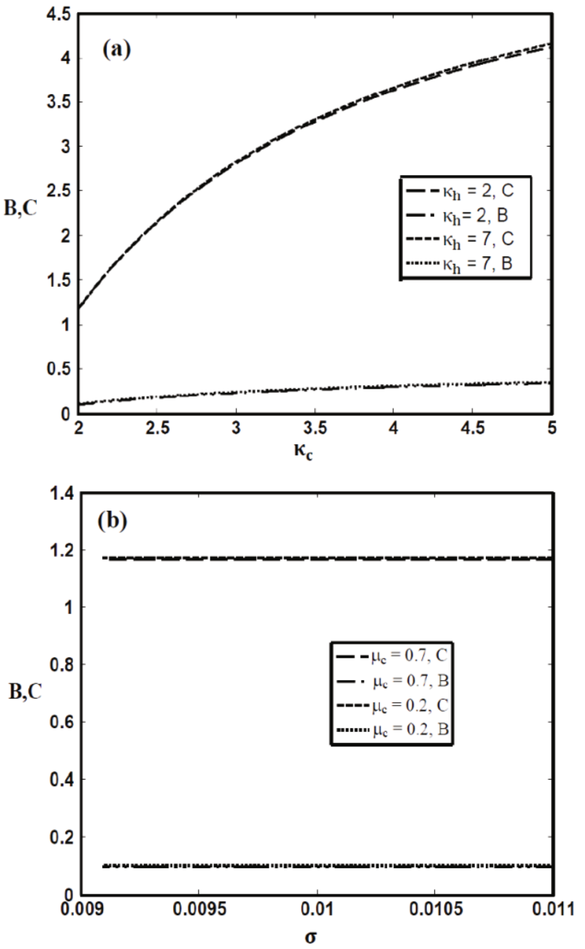

Fig. 9. (a) Variation of the dispersion coefficients B and C with κ_c for different values of κ_h. (b) Variation of the dispersion coefficients B and C with σ for different values of μ_c.

tween the nonlinear compressive (rarefactive) monotonic IASWs and the ion cyclotron frequency Ω. Clearly, the amplitude and the steepness of the compressive (rarefactive) monotonic IASWs are increased by increasing of Ω. Finally, let us pay attention to the effects of the direction cosines ℓ_z on the compressive (rarefactive) monotonic IASWs, as displayed in Fig. 8(a) (8(b)). A reduction in ℓ_z is seen to make the amplitude and the width of monotonic IASWs larger and narrower, respectively. Clearly both Ω and ℓ_z have strong effects on the structures of monotonic IASWs. Physically, the enhancement of monotonic IASWs means that the potential drop across the monotonic IASWs increases. Accordingly, more particles will be accelerated. Interestingly, comparing our results with the results of Bains *et al.* [31] turns out to be very valuable. The results of the current investigation for a one-dimensional plasma model without the formation conditions of monotonic/oscillatory IASWs when the asymptotic behavior and the stability of a small perturbation around the analytical solution are neglected and without the shock wave solution in limiting cases agree with the results of Bains *et al.* [31].

IV. SHOCK WAVE SOLUTIONS IN LIMITING CASES

Evidently, the nonlinearity coefficient A and the dispersion coefficients B and C depend on the superthermal parameters (*i.e.*, κ_c and κ_h), the cold superthermal electron-to-ion number density ratio μ_c and the cold superthermal electron-to-hot superthermal electron temperature ratio σ. In addition, C depends on the magnetic field strength Ω while the dissipation coefficient D depends only on the ion viscosity η_0 (see Eq. (14)). Now, the influences of the mentioned physical parameters on the shock wave solutions in limiting cases remain to be examined.

To proceed, we should mention that the features of the solutions to Eq. (19) depend mainly on the interplay of the plasma dispersion (*i.e.*, B and C) and dissipation (*i.e.*, D). Thus, the effects of κ_c, κ_h, μ_c, and σ on the dispersion coefficients B and C must be discussed. In Fig. 9(a), obviously the superthermality of cold electrons κ_c has a strong effect on the dispersion coefficients B and C while the superthermality of hot electrons κ_h has a very weak impact on B and C. In Fig. 9(b), clearly both μ_c and σ have no influence on the dispersion coefficients B and C. Therefore, we can say that the superthermality of cold electrons κ_c and the ion viscosity η_0 have strong roles in determining the dispersion coefficients B and C, and the dissipation coefficient D, respectively. Based on the two physical parameters κ_c and η_0, let us examine the shock wave solutions in two distinct limiting cases of the present model. First, by considering the case of strongly superthermal plasmas (*i.e.*, κ_c in the vicinity of 3/2; see Fig. 9(a)), the dispersion coefficients B and C are negligible because the ion viscosity η_0 is assumed to be greater than unity. In this situation, upon neglecting the dispersion coefficients B and C, Eq. (15) becomes

$$-u\frac{d\phi}{d\zeta} + A\ell_z\phi\frac{d\phi}{d\zeta} - \frac{D}{\gamma}\frac{d^2\phi}{d\zeta^2} = 0. \qquad (25)$$

The solution of Eq. (25) is represented by

$$\phi(\zeta) = \phi_0(1 - \tanh(\zeta)). \qquad (26)$$

Equation (26) is a monotonic IASW solution for which the amplitude, the speed and the width can be written as ϕ_0 $(= u/(A\ell_z))$, u and $\frac{1}{\gamma}$ $(= 2D/u)$, respectively.

At this point, examining the propagation characteristics of monotonic IASWs numerically based on the solution in Eq. (26) would be instructive. Figure 10(a) (10(b)) describes the variations of the compressive (rarefactive) monotonic IASWs with the cold superthermal electron-to-ion number density ratio μ_c. As seen from the figure, with increasing μ_c, the shock potential and

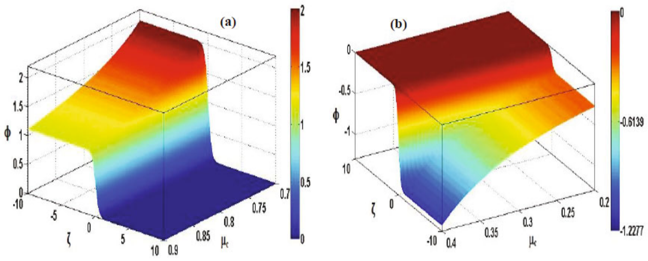

Fig. 10. (Color online) Three-dimensional profiles of the shock wave ϕ for various values of μ_c and $\kappa_c = 2$, $\kappa_h = 2$, $\sigma = 0.01$, $\Omega = 0.15$, $\eta_0 = 1.1$, and $\ell_z = 0.9$: (a) compressive IASWs and (b) rarefactive IASWs.

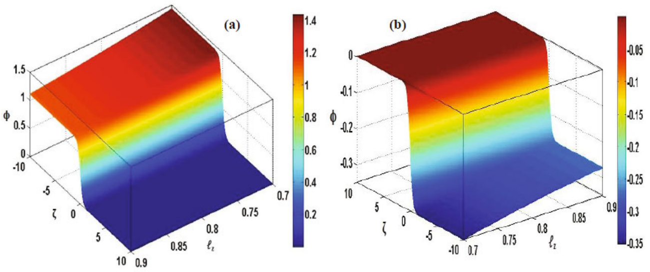

Fig. 11. (Color online) Three-dimensional profiles of the shock wave ϕ for various values of ℓ_z and $\kappa_c = 2$, $\kappa_h = 2$, $\sigma = 0.01$, $\eta_0 = 1.1$, and $\Omega = 0.15$: (a) $\mu_c = 0.9$ (compressive IASWs) and (b) $\mu_c = 0.2$ (rarefactive IASWs).

the steepness decrease (increase). In Fig. 11(a) (11(b)), the behavior of the compressive (rarefactive) monotonic IASW profile is studied for different chosen values of the direction cosines ℓ_z. Here, we can observe in the two modes (*i.e.*, compressive and rarefactive modes) that the strength and the steepness of the IASWs grow with decreasing direction cosines ℓ_z.

Now, comparing our limiting case with those of Alam *et al.* [29] and Sultana *et al.* [35] would be interesting. The results of Alam *et al.* [29] without dust grains agree with the present results in a one-dimensional plasma system for only one component of superthermal electrons and without an external magnetic field. On the other hand, the results of the present study, for a one-dimensional plasma model with one component of superthermal electron and without an external magnetic field agree with those of an earlier investigation by Sultana *et al.* [35].

We next consider the case for an increase in the superthermality of cold electrons κ_c (*i.e.*, weakly superthermal plasmas) and ion viscosity $\eta_0 \ll 1$. In this situation, the dispersion term dominates over the dissipative term; hence, we can neglect the dissipative term. Therefore, the nonlinear ZKBE will lead to a natural convergence to the well-known nonlinear Zakharov-Kuznetsov equation, which generates ion-acoustic solitons. This case has been investigated by Saini *et al.* [23], so it will not be considered here.

V. CONCLUSION

Using the proposed model, we have examined the characteristics of nonlinear IASWs in a collisionless magnetized plasma containing inertial viscous ions and two superthermal electrons. For a three-dimensional superthermal magnetoplasma model, the ZKBE is deduced employing a reductive perturbation analysis. In addition, using the tanh method, we have discussed the analytical solutions and the formation conditions of nonlinear oscillatory/monotonic IASWs. Moreover, the most significant findings are as follows:

1. The current model supports the finite amplitude of IASWs, whose fundamental features (*i.e.*, strength, steepness, *etc.*) depend on inertial viscous positive ions and two superthermal electrons (see Figs. 1–4).
2. The superthermality of cold electrons κ_c, the ion kinematic viscosity η_0, the cold superthermal electron-to-ion number density ratio μ_c, the ion cyclotron frequency Ω, and the direction cosines ℓ_z play powerful roles in determining the nature of monotonic IASWs (see Figs. 2, 4, 5, and 7).
3. The present model admits compressive and rarefactive IASWs. Furthermore, the existence of positive and negative shock potentials strongly depends on the cold superthermal electron-to-ion number density ratio μ_c (see Fig. 1).
4. In the general situation, a strictly monotonic shock wave solution is analytically obtained.
5. Analytically, several types of nonlinear ion-acoustic disturbances were demonstrated to exist depending on the relation between the dispersive terms (*i.e.*, B and C) and the dissipative term D.
6. The superthermality of cold electrons κ_c and the ion viscosity η_0 are observed to have strong effects on the dispersion coefficients B and C and the dissipation coefficient D, respectively.
7. In a strongly superthermal plasmas situation (*i.e.*, κ_c in the vicinity of 3/2) and ion viscosity η_0 greater than unity, the dissipation coefficient D dominates the dispersion coefficients B and C. In this case, due to the Burgers equation, positive and negative potential shock structures can propagate (see Figs. 9–11).
8. When the dissipation is weak (*i.e.*, $D \to 0$) and the dispersion strong, the nonlinear ion-acoustic disturbances exhibit only ion-acoustic solitons.

Finally, we believe that the current study may be helpful for a broad and advanced understanding of the effective features of IASWs in astrophysical plasma situations with two superthermal electrons, such as Saturn's magnetosphere.

ACKNOWLEDGMENTS

The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University for funding this work through the research groups program under grant number KKUR.G.P.1/38/39. The authors also thank the editor and his staff for their kind cooperation.

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