Journal of the Korean Physical Society, Vol. 71, No. 7, October 2017, pp. 396∼403

Non-Static Spherically Symmetric Exact Solution of the Einstein-Maxwell Field Equations

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(Received 25 May 2017, in final form 23 June 2017)

We present a class of exact spherically symmetric and non-static solutions of Einstein-Maxwell's field equations. We have assumed isotropic pressure distribution and have taken ansatz on two of the gravitational potentials. The solutions admit negative pressure. We show that the solutions satisfy physical boundary conditions associated with the Einstein-Maxwell exact solutions. Therefore, these solutions can model physical systems such as moving dark energy stars.

PACS numbers: 04.20.Jb, 04.40.Nr
Keywords: Non-static, Einstein-Maxwell equations, Exact solutions
DOI: 10.3938/jkps.71.396

I. INTRODUCTION

Astrophysical systems such as stellar interiors can be modeled by solutions of Einstein–Maxwell's field equations. Many attempts have been made to solve this set of differential equations both for static and non–static conditions. Several solutions of Einstein's field equations are found in [1]. The first solution of the Einstein–Maxwell equations was obtained by H. Reissner and G. Nordström [2]. D. Lovelock [3] obtained a solution of the sourceless Einstein–Maxwell's equations for a static massless charged particle. J. Hajj–Boutros and J. Safeila [4] obtained a general plane–symmetric solution for non–static charged dust. A. Melfo and H. Rago [5] obtained a solution for a charged anisotropic fluid sphere under the assumption of a conformally flat interior metric. M. K. Bashar *et al.* [6] discussed these equations for static, spherical distribution of matter in the form of a charged perfect fluid and found a class of their analytic solutions. Brendan S. Guilfoyle [7] discussed static solutions of the electro–gravitational field equations exhibiting a functional relationship between the electric and gravitational potentials. J. Carminati and C. B. G. McIntosh [8] obtained exact solutions of the Einstein–Maxwell equations for the non–static metric of the form

$$ds^2 = e^{2h(t)}dt^2 - e^{2A(t)}(dx^2 + dy^2) - e^{2B(t)}dz^2. \quad (1)$$

M. Čermak [9] integrated these equations for stationary cylindrical space–times by restricting the range of free parameters involved. A class of asymptotically flat static solutions of Einstein–Maxwell's equations was obtained by S. M. Abramyan and Ts. I. Gutsunaev [10]. K. D. Krori and T. Chaudhury [11] developed a technique to solve the Einstein–Maxwell equations for conformally flat space–times for both static and non–static cases.

In the last few years, models of stars with charge and different equations of state have attained attention. M. Humi and J. Mansour [12] obtained solutions of the Einstein–Maxwell equations both for spherically and plane symmetric space–times under the assumption that the equation of state has the form $p = n\rho$, where p is the pressure, ρ is mass density and $n \in [0, 1]$. Assuming a similar kind of the equation of state, Z. PengFei and Z. DongPei [13] obtained a solution for static charged spherically symmetric space–times in a higher dimension. K. Komathiraj and S. D. Maharaj [14] obtained two classes of solutions for a static charged sphere representing quark matter, by assuming an equation of state $p = \frac{1}{3}(\rho - 4B)$ (B is a constant) and the first component of the metric tensor $e^{2\nu} = A^2(a + \sqrt{cr^2})^2$ or $e^{2\nu} = A^2(a + cr^2)^4$ (where A, a and c are constants). V. Varela *et al.* [15] assumed the form of second component of the metric tensor to be $e^{\lambda} = \frac{1+ar^2}{1+(a-b)r^2}$ and the electric field intensity of the form $E^2 = \frac{k(3+ar^2)}{(1+ar^2)^2}$ (where k, a and b are constants) for charged anisotropic matter, both with linear and nonlinear equations of state. S. Thirukkanesh and S. D. Maharaj [16] have assumed a similar kind of λ and E^2 and a linear equation of state is considered for anisotropic matter. S. D. Maharaj and S. Thirukkanesh [17] have taken assumptions on λ and E^2 and have obtained a linear relation between p and ρ. T. Feroze [18] obtained two classes of exact solutions of static spherically symmetric anisotropic perfect fluid distribution with linear equation of state and a particular form of gravitational potential. T. Feroze and A.

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A. Siddiqui [19], and S. D. Maharaj and P. M. Takisa [20] assumed a quadratic equation of state and particular forms of gravitational potential and electric field intensity. N. Pant *et al.* [21], N. Pradhan and N. Pant [22], M. H. Murad [23], and N. Pant *et al.* [24] have discussed and developed the models of charged strange stars by assuming particular forms of one of the gravitational potentials and the electric field intensity and have considered a usual linear equation of state. Some other notable solutions are presented in [25–27].

A. H. Abbasi and S. Gharanfoli [28] and A. H. Abbasi [29] have obtained non–static spherically symmetric solutions of Einstein's vacuum field equations with a cosmological constant. M. Sharif and T. Iqbal [30] have investigated solutions of Einstein's field equations for the non–static spherically symmetric perfect fluid case using different equations of state. D. Shee, *et al.* [31] have proposed a model for relativistic compact star with anisotropy and analytically obtained exact spherically symmetric solutions describing the interior of the dense star admitting non–static conformal symmetry.

The main objectives of different cosmological models include the description of different phases of the Universe. It may concern the time evolution of the acceleration field of the Universe. It is now well known that the Universe is dominated by the so–called dark energy but the nature of this dark energy is still unknown. It is also believed that the dark energy has large negative pressure that leads to accelerated expansion of the Universe. Due to this fact much importance is given to the study of dark energy models by many authors. The simplest example of dark energy is a cosmological constant, introduced by Einstein in 1917 [32]. A. Cappi [33] has discussed different cosmological models with the equation of state of the form $\omega = P/\rho c^2$ (where P is pressure, ρ is mass density, and c is speed of light) and has discussed different models for $\omega = -1, -1 < \omega < 0, \omega < -1$. B. Saha [34] has solved the Einstein field equations for a system of Bianchi type–I gravitational field and a binary mixture of perfect fluid and dark energy given by a cosmological constant. Some other dark energy solutions include [35–37]. It is to be noticed that these solutions are obtained for static space–time structure and in view of [38, 39] the configurations of stars may not be static. Keeping this fact in mind we aim to obtain solution of Einstein–Maxwell's field equations for non–static space–time geometry that also represent a dark energy model.

In this paper, we find a class of exact solutions of the Einstein–Maxwell field equations for non–static spherically symmetric conditions. The pressure distribution is assumed to be isotropic and ansatz are taken on the first and the third metric components. The solutions admit negative pressure. In the following Section II, we discuss the Einstein–Maxwell field equations. In Section III, we present a new class of solutions of the field equations. In Section IV, we present the analysis for our solution to be physically acceptable. In Section V, we present a brief conclusion and identify the types of physical systems that this solution can model.

II. THE EINSTEIN–MAXWELL FIELD EQUATIONS

The general form of Einstein–Maxwell's field equations is given as

$$T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{2}$$

$$(\sqrt{-g}(F^{\mu\nu}))_{,\nu} = \sqrt{-g} j^{\mu}, \tag{3}$$

where $(\mu, \nu = 0, 1, 2, 3)$, $T_{\mu\nu}$ is the stress energy tensor, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, g is the determinant of metric tensor, $F^{\mu\nu}$ is the electromagnetic tensor and j^{μ} is the current density. If the trace of the stress energy tensor, T, is non–zero, then the electromagnetic field is a field with a source and is sourceless otherwise.

A general non–static spherically symmetric space–time has a metric of the form

$$ds^2 = -e^{\nu(t,r)} dt^2 + e^{\lambda(t,r)} dr^2 + \mu^2(t, r) d\Omega^2, \tag{4}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. In spherically symmetric space–times, only radial components of electric and magnetic fields survive. Therefore, non–zero components of the electromagnetic tensor are $F_{01} = E = -F_{10}$ and $F_{23} = -B = F_{32}$, where E is the electric field intensity and B is the strength of the magnetic field. We omit the magnetic field from our calculations for simplification. The electromagnetic tensor and the electromagnetic part of the stress energy tensor are related by the expression

$$T_{\mu\nu} = F^{\alpha}_{\mu} F_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}. \tag{5}$$

Using above expression and adding the matter part we obtain the stress energy tensor for the metric (4) as

$$T_{\mu\nu} = diag\left(\rho e^{\nu} + \frac{1}{2} e^{-\lambda} E^2, p_r e^{\lambda} - \frac{1}{2} e^{-\nu} E^2, (p_t + \frac{1}{2} e^{-(\nu+\lambda)} E^2)\mu^2, (p_t + \frac{1}{2} e^{-(\nu+\lambda)} E^2)\mu^2 \sin^2\theta\right), \tag{6}$$

where ρ is the mass density, E is the electric field intensity, p_r is the radial pressure and p_t is the tangential pressure (a matter distribution is isotropic if $p_r = p_t$ and is anisotropic otherwise). For the metric (4) and the stress energy tensor given by Eq. (6), the Einstein–Maxwell field eqs. (2) and (3) take the form

$$e^{-\lambda}\left(\frac{\lambda'\mu'}{\mu}-2\frac{\mu''}{\mu}-\frac{\mu'^2}{\mu^2}\right)+e^{-\nu}\left(\frac{\dot{\lambda}\dot{\mu}}{\mu}+\frac{\dot{\mu}^2}{\mu^2}\right)+\frac{1}{\mu^2} = \rho+\frac{1}{2}e^{-(\nu+\lambda)}E^2, \tag{7}$$

$$e^{-\lambda}\left(\frac{\nu'\mu'}{\mu}+\frac{\mu'^2}{\mu^2}\right)+e^{-\nu}\left(\frac{\dot{\nu}\dot{\mu}}{\mu}-2\frac{\ddot{\mu}}{\mu}-\frac{\dot{\mu}^2}{\mu^2}\right)-\frac{1}{\mu^2} = p_r-\frac{1}{2}e^{-(\nu+\lambda)}E^2, \tag{8}$$

$$\begin{aligned}&\frac{1}{4}e^{-\lambda}\left(4\frac{\mu''}{\mu}-2\frac{\lambda'\mu'}{\mu}+2\frac{\nu'\mu'}{\mu}-\nu'\lambda'+2\nu''+\nu'^2\right)\\ +\ &\frac{1}{4}e^{-\nu}\left(-4\frac{\ddot{\mu}}{\mu}+2\frac{\dot{\nu}\dot{\mu}}{\mu}-2\frac{\dot{\lambda}\dot{\mu}}{\mu}+\dot{\nu}\dot{\lambda}-2\ddot{\lambda}-\dot{\lambda}^2\right)=p_t+\frac{1}{2}e^{-(\nu+\lambda)}E^2,\end{aligned} \tag{9}$$

$$-2\dot{\mu'}+\nu'\dot{\mu}+\dot{\lambda}\mu' = 0, \tag{10}$$

$$j_0 = \frac{1}{\mu^2}e^{-\lambda}(E\mu^2)', \tag{11}$$

$$j_1 = \frac{1}{\mu^2}e^{-\nu}(E\mu^2)^{\boldsymbol{\cdot}}. \tag{12}$$

Here '. 'and ', 'are derivatives with respect to t and r respectively. The trace, T, of the stress energy tensor is

$$\begin{aligned}T = &-\frac{2}{\mu^2}+e^{-\lambda}\left(-2\frac{\lambda'\mu'}{\mu}+4\frac{\mu''}{\mu}+2\frac{\mu'^2}{\mu^2}+2\frac{\nu'\mu'}{\mu}-\frac{1}{2}\nu'\lambda'+\nu''+\frac{1}{2}\nu'^2\right)\\ &+e^{-\nu}\left(-2\frac{\dot{\lambda}\dot{\mu}}{\mu}-4\frac{\ddot{\mu}}{\mu}-2\frac{\dot{\mu}^2}{\mu^2}+2\frac{\dot{\nu}\dot{\mu}}{\mu}+\frac{1}{2}\dot{\nu}\dot{\lambda}-\ddot{\lambda}-\frac{1}{2}\dot{\lambda}^2\right).\end{aligned} \tag{13}$$

III. SOLUTION OF THE FIELD EQUATIONS

We consider an isotropic fluid distribution *i.e.* $p_r = p_t = p$ and take ansatz on metric coefficients. A. Qadir and M. Ziad [40], while classifying spherically symmetric space–times, concluded that for a non–static solution μ can only be some function of $t \pm r$. Further, the coefficients of the metric should be continuous and non–sigular [14]. Keeping these conditions in view, we consider μ to be of the form

$$\mu = a_0\left(1+\left(\frac{t+r}{a}\right)^2\right), \tag{14}$$

where a and a_0 are constants, and ν to be

$$\nu = -\frac{1}{1+\left(\frac{t+r}{a}\right)^2}, \tag{15}$$

Inserting these values in Eq. (10), we obtain

$$\lambda = \frac{1}{1+\left(\frac{t+r}{a}\right)^2}+\ln\left(\frac{t+r}{a}\right)^2. \tag{16}$$

Using values of λ, ν and μ in Eqs. (7) - (9), we get

$$-4e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left(\frac{1/a^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}+\frac{1/a^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^3}\right)+4e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left(\frac{1/a^2}{1+\left(\frac{t+r}{a}\right)^2}+\frac{\frac{1}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}-\frac{\frac{1}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^3}\right)$$
$$+\frac{1}{a_0{}^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}=\rho+\frac{E^2}{2\left(\frac{t+r}{a}\right)^2}, \tag{17}$$

$$4e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left(\frac{1/a^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}+\frac{1/a^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^3}\right)-4e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left(\frac{1/a^2}{1+\left(\frac{t+r}{a}\right)^2}+\frac{\frac{1}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}-\frac{\frac{1}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^3}\right)$$
$$-\frac{1}{a_0{}^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}=p-\frac{E^2}{2\left(\frac{t+r}{a}\right)^2}, \tag{18}$$

$$2e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left(\frac{1/a^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^4}\right)+e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left(-\frac{4/a^2}{1+\left(\frac{t+r}{a}\right)^2}+\frac{4/a^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}-\frac{\frac{2}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^4}\right)=p+\frac{E^2}{2\left(\frac{t+r}{a}\right)^2}. \tag{19}$$

From Eqs. (17) and (18), we have

$$\rho+p=0, \tag{20}$$

which is the equation of state. Solving Eqs. (18) and (19), the pressure and the electric field intensity are given as

$$\begin{aligned}p \;=\; & e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left(\frac{2/a^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}+\frac{2/a^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^3}+\frac{1/a^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^4}\right)\\
&+e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left(-\frac{4/a^2}{1+\left(\frac{t+r}{a}\right)^2}+\frac{2/a^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}-\frac{\frac{2}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}+\frac{\frac{2}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^3}-\frac{\frac{1}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^4}\right)\\
&-\frac{1}{2a_0{}^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^2},\end{aligned} \tag{21}$$

and

$$\begin{aligned}E^2 \;=\; & e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left(-\frac{\frac{4}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}-\frac{\frac{4}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^3}+\frac{\frac{2}{a^2}\left(\frac{t+r}{a}\right)^2}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^4}\right)\\
&+e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left(\frac{\frac{4}{a^2}\left(\frac{t+r}{a}\right)^2}{1+\left(\frac{t+r}{a}\right)^2}-\frac{\frac{4}{a^2}\left(\frac{t+r}{a}\right)^4}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^3}-\frac{\frac{2}{a^2}\left(\frac{t+r}{a}\right)^4}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^4}\right)\\
&+\frac{\left(\frac{t+r}{a}\right)^2}{a_0{}^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}.\end{aligned} \tag{22}$$

Using values of λ, μ, ν and E^2 in Eqs. (11) and (12), the components of the current density are obtained as

$$
\begin{aligned}
j_0 =& \frac{1}{2}\frac{e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}}{\left(\frac{t+r}{a}\right)^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^2} \\
&\times\Bigg[e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left\{-\frac{4}{a^2}\left(\frac{t+r}{a}\right)^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^2-\frac{4}{a^2}\left(\frac{t+r}{a}\right)^2\left(1+\left(\frac{t+r}{a}\right)^2\right)+\frac{2}{a^2}\left(\frac{t+r}{a}\right)^2\right\} \\
&+e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left\{\frac{4}{a^2}\left(\frac{t+r}{a}\right)^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^3-\frac{4}{a^2}\left(\frac{t+r}{a}\right)^4\left(1+\left(\frac{t+r}{a}\right)^2\right)-\frac{2}{a^2}\left(\frac{t+r}{a}\right)^4\right\}+\left(\frac{t+r}{a}\right)^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^2\Bigg]^{-\frac{1}{2}} \\
&\times\Bigg[e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\Bigg\{\frac{4}{a^3}\left(\frac{t+r}{a}\right)-\frac{16}{a^3}\left(\frac{t+r}{a}\right)^3-\frac{8}{a^3}\left(\frac{t+r}{a}\right)\left(1+\left(\frac{t+r}{a}\right)^2\right)\left(2+\left(\frac{t+r}{a}\right)^2\right)-\frac{16}{a^3}\left(\frac{t+r}{a}\right)^3\left(1+\left(\frac{t+r}{a}\right)^2\right)-\frac{\frac{8}{a^3}\left(\frac{t+r}{a}\right)^3}{1+\left(\frac{t+r}{a}\right)^2}+\frac{\frac{4}{a^3}\left(\frac{t+r}{a}\right)^3}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}\Bigg\} \\
&+e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\Bigg\{-\frac{32}{a^3}\left(\frac{t+r}{a}\right)^3\left(1+\left(\frac{t+r}{a}\right)^2\right)+\frac{8}{a^3}\left(\frac{t+r}{a}\right)\left(1+\left(\frac{t+r}{a}\right)^2\right)^3+\frac{24}{a^3}\left(\frac{t+r}{a}\right)^3\left(1+\left(\frac{t+r}{a}\right)^2\right)^2+\frac{\frac{8}{a^3}\left(\frac{t+r}{a}\right)^5}{1+\left(\frac{t+r}{a}\right)^2}+\frac{\frac{4}{a^3}\left(\frac{t+r}{a}\right)^5}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}\Bigg\} \\
&+\frac{2}{a}\left(\frac{t+r}{a}\right)\left(1+\left(\frac{t+r}{a}\right)^2\right)\left(1+3\left(\frac{t+r}{a}\right)^2\right)\Bigg]\,,
\end{aligned}
$$

and

$$
\begin{aligned}
j_1 =& \frac{1}{2}\frac{e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2} \\
&\times\Bigg[e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left\{-\frac{4}{a^2}\left(\frac{t+r}{a}\right)^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^2-\frac{4}{a^2}\left(\frac{t+r}{a}\right)^2\left(1+\left(\frac{t+r}{a}\right)^2\right)+\frac{2}{a^2}\left(\frac{t+r}{a}\right)^2\right\} \\
&+e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\left\{\frac{4}{a^2}\left(\frac{t+r}{a}\right)^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^3-\frac{4}{a^2}\left(\frac{t+r}{a}\right)^4\left(1+\left(\frac{t+r}{a}\right)^2\right)-\frac{2}{a^2}\left(\frac{t+r}{a}\right)^4\right\}+\left(\frac{t+r}{a}\right)^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^2\Bigg]^{-\frac{1}{2}} \\
&\times\Bigg[e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\Bigg\{\frac{4}{a^3}\left(\frac{t+r}{a}\right)-\frac{16}{a^3}\left(\frac{t+r}{a}\right)^3-\frac{8}{a^3}\left(\frac{t+r}{a}\right)\left(1+\left(\frac{t+r}{a}\right)^2\right)\left(2+\left(\frac{t+r}{a}\right)^2\right)-\frac{16}{a^3}\left(\frac{t+r}{a}\right)^3\left(1+\left(\frac{t+r}{a}\right)^2\right)-\frac{\frac{8}{a^3}\left(\frac{t+r}{a}\right)^3}{1+\left(\frac{t+r}{a}\right)^2}+\frac{\frac{4}{a^3}\left(\frac{t+r}{a}\right)^3}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}\Bigg\} \\
&+e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}\Bigg\{-\frac{32}{a^3}\left(\frac{t+r}{a}\right)^3\left(1+\left(\frac{t+r}{a}\right)^2\right)+\frac{8}{a^3}\left(\frac{t+r}{a}\right)\left(1+\left(\frac{t+r}{a}\right)^2\right)^3+\frac{24}{a^3}\left(\frac{t+r}{a}\right)^3\left(1+\left(\frac{t+r}{a}\right)^2\right)^2+\frac{\frac{8}{a^3}\left(\frac{t+r}{a}\right)^5}{1+\left(\frac{t+r}{a}\right)^2}+\frac{\frac{4}{a^3}\left(\frac{t+r}{a}\right)^5}{\left(1+\left(\frac{t+r}{a}\right)^2\right)^2}\Bigg\} \\
&+\frac{2}{a}\left(\frac{t+r}{a}\right)\left(1+\left(\frac{t+r}{a}\right)^2\right)\left(1+3\left(\frac{t+r}{a}\right)^2\right)\Bigg]\,.
\end{aligned}
\tag{23}
$$

The metric of our solution is

$$
ds^2=-e^{-\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}dt^2+\left(\frac{t+r}{a}\right)^2e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}}dr^2+{a_0}^2\left(1+\left(\frac{t+r}{a}\right)^2\right)^2d\Omega^2. \tag{24}
$$

IV. PHYSICAL ANALYSIS OF THE SOLUTION

In the previous section, we obtained a class of exact solutions of the Einstein–Maxwell field equations for charged isotropic non–static spherically symmetric conditions. The analysis below shows our solution to be physically acceptable. Here we use the physical constraints for the Einstein–Maxwell exact solutions that are identified in N. Pant *et al.* [41,42], Y. K. Gupta and S. K. Maurya [43] and T. Feroze *et al.* [44].

• There is no singularity in the solution other than $t=-r$ as is evident from the metric (24). Also, it is a coordinate singularity as the curvature invariants given in Appendix, are defined at $t=-r$.

• The square of the electric field intensity, E^2, is continuous, bounded and a positive function of both r and t, for all values of the parameters a and a_0 and is shown in

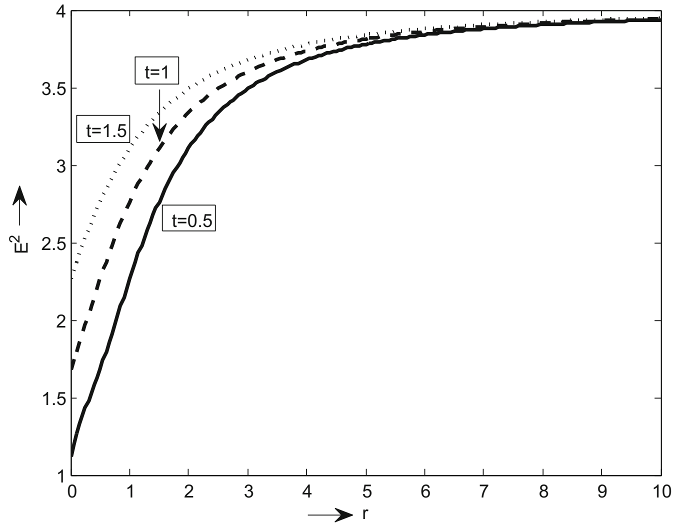

Fig. 1. The square of electric field intensity, E^2, is shown with respect to r for $t = 0.5, 1$ and 1.5. For all cases $a = a_0 = 1$.

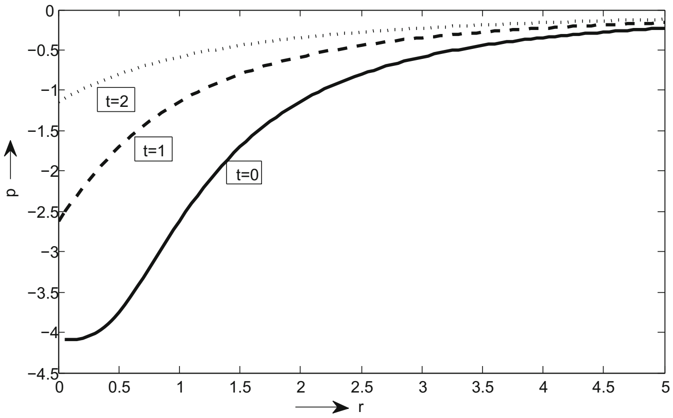

Fig. 2. The pressure, p, is shown with respect to r for $t = 0, 1$ and 2. For all cases $a = a_0 = 1$.

Fig. (1).
• The mass density, ρ, is positive for all values of the parameters satisfying the relation $11a_0^2 + a^2 \geq 0$, and this relation is true for any arbitrary values of a and a_0. Thus, ρ is a positive and decreasing function of both r and t. It is shown in Fig. (5).
• The pressure, p, is required to be positive, continuous, bounded and a smooth function, but in our case it is a negative function of both r and t and its value asymptotically approaches zero. It is shown in Fig. (2).
• The weak energy condition *i.e.*, $\rho \geq 0$, $\rho + p \geq 0$ and the dominant energy condition *i.e.*, $\rho \geq |p|$ are also satisfied.
• The causality condition, *i.e.*, $0 < \frac{dp}{d\rho} \leq 1$ is not satisfied as pressure is negative and $\frac{dp}{d\rho} = -1$.
• The non–zero components of the electric current den-

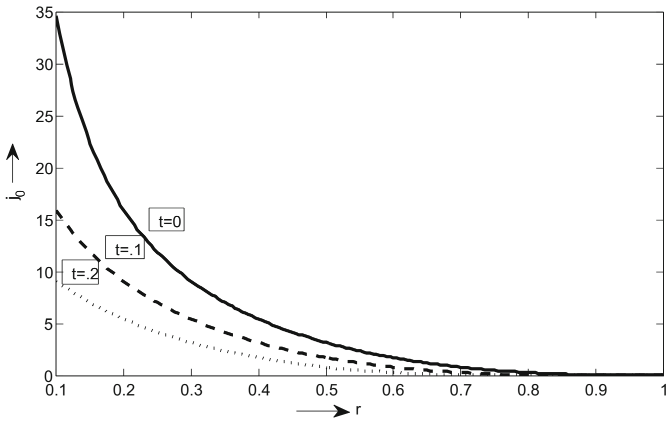

Fig. 3. The component, j_0, of the current density is shown with respect to r for $t = 0, 0.1$ and 0.2. For all cases $a = a_0 = 1$.

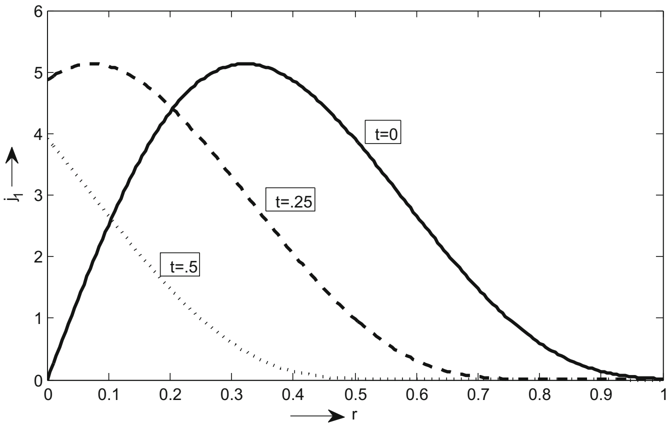

Fig. 4. The component, j_1, of the current density is shown with respect to r for $t = 0, 0.25$ and 0.5. For all cases $a = a_0 = 1$.

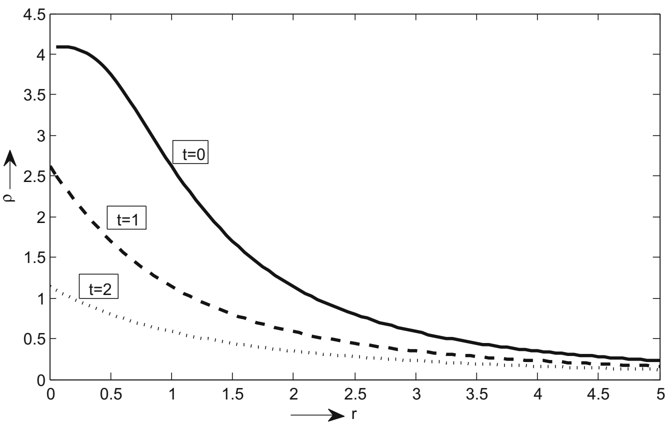

Fig. 5. The mass density, ρ, is shown with respect to r for $t = 0, 1$ and 2. For all cases $a = a_0 = 1$.

sity j_0 and j_1 are each continuous and decreasing functions of both r and t and are shown in Figs. (3) and (4).

V. CONCLUSION

In this paper, we have obtained a class of non–static spherically symmetric solutions of the Einstein–Maxwell equations. We have assumed the pressure distribution to be isotropic. The mass density, ρ, is a positive decreasing function of both r and t. The square of the electric field intensity, E^2, is a positive–definite and bounded function of both r and t. The pressure, p, turns out to be a negative function of both r and t and asymptotically approaches to zero. Here ρ, p and E^2 are all symmetric with respect to t and r.

The solution obtained may be thought to represent a moving dark energy compact object. The causality condition is not satisfied by the solution, as is expected for a dark energy object with negative pressure. All other physical conditions are shown to be satisfied.

There are numerous static solutions of the Einstein–Maxwell equations to model dark energy objects. However, there is hardly any literature for non–static case. Keeping in view that the configuration of such objects may not be static [38, 39], we have made a successful attempt to find a class of non–static solutions of the Einstein–Maxwell equations representing dark energy objects.

APPENDIX

The curvature invariants corresponding to the metric (4) are given by

$$
\begin{aligned}
R_1 &= R \\
&= \frac{1}{a_0{}^2 a^8 e^{\frac{-1}{1+\left(\frac{t+r}{a}\right)^2}}\left[1+\left(\frac{t+r}{a}\right)^2\right]^4}\Bigg[2a^8 e^{\frac{-1}{1+\left(\frac{t+r}{a}\right)^2}} \\
&- 20a^6 a_0{}^2 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} + 8a^6 a_0{}^2 + 4a^8\left(\frac{t+r}{a}\right)^2 e^{\frac{-1}{1+\left(\frac{t+r}{a}\right)^2}} \\
&- 24a^6 a_0{}^2\left(\frac{t+r}{a}\right)^2 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} + 36a^6 a_0{}^2\left(\frac{t+r}{a}\right)^2 \\
&+ 2a^8\left(\frac{t+r}{a}\right)^4 e^{\frac{-1}{1+\left(\frac{t+r}{a}\right)^2}} - 8a^6 a_0{}^2\left(\frac{t+r}{a}\right)^4 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 48a^6 a_0{}^2\left(\frac{t+r}{a}\right)^4 + 24a^6 a_0{}^2\left(\frac{t+r}{a}\right)^6\Bigg]
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
R_2 &= R^{ab}_{cd}R^{cd}_{ab} \\
&= \frac{16}{a^{20}a_0{}^4\left[1+\left(\frac{t+r}{a}\right)^2\right]^8}\Bigg[4a^{20}\left(\frac{t+r}{a}\right)^8 + 128a^{18}a_0{}^2\left(\frac{t+r}{a}\right)^8 e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 24a^{20}\left(\frac{t+r}{a}\right)^4 + 16a^{20}\left(\frac{t+r}{a}\right)^2 + 4a^{20} + 6a^{16}a_0{}^4 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 67a^{16}a_0{}^4 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} - 128a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{10} - 516a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^8 \\
&- 780a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^6 - 532a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^4 - 150a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^2 - 8a^{16}a_0{}^4 \\
&- 32a^{18}a_0{}^2 e^{\frac{-1}{1+\left(\frac{t+r}{a}\right)^2}} + 32a^{18}a_0{}^2\left(\frac{t+r}{a}\right)^{10} e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}} + 16a^{20}\left(\frac{t+r}{a}\right)^6 \\
&+ 192a^{18}a_0{}^2\left(\frac{t+r}{a}\right)^6 e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}} + 128a^{18}a_0{}^2\left(\frac{t+r}{a}\right)^4 e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}} \\
&- 12a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^6 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} + 32a^{18}a_0{}^2\left(\frac{t+r}{a}\right)^2 e^{\frac{1}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 4a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^2 + 2a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^4 + 2a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^8 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 4a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^6 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} + 2a^{16}a_0{}^4 + 2a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^4 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&- 32a^{18}a_0{}^2\left(\frac{t+r}{a}\right)^8 e^{\frac{-1}{1+\left(\frac{t+r}{a}\right)^2}} - 128a^{18}a_0{}^2\left(\frac{t+r}{a}\right)^6 e^{\frac{-1}{1+\left(\frac{t+r}{a}\right)^2}} \\
&- 192a^{18}a_0{}^2\left(\frac{t+r}{a}\right)^4 e^{\frac{-1}{1+\left(\frac{t+r}{a}\right)^2}} - 128a^{18}a_0{}^2\left(\frac{t+r}{a}\right)^2 e^{\frac{-1}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 66a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{12} e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} + 264a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{10} e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 400a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^8 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} + 64a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^8 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 276a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^6 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} + 256a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^6 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 81a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^4 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} + 390a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^4 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 12a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^2 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} + 264a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^2 e^{\frac{-2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 2a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{12} e^{\frac{4}{1+\left(\frac{t+r}{a}\right)^2}} + 2a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{16} e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&- 4a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^8 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} - 4a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{12} - 12a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{10} \\
&- 16a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^4 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} - 16a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^8 - 4a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^4 \\
&- 12a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^2 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} - 12a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^6 - 4a^{16}a_0{}^4 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 8a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{10} e^{\frac{4}{1+\left(\frac{t+r}{a}\right)^2}} + 8a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{14} e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 16a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^8 e^{\frac{4}{1+\left(\frac{t+r}{a}\right)^2}} + 16a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{12} e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 20a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^6 e^{\frac{4}{1+\left(\frac{t+r}{a}\right)^2}} + 20a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^{10} e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 16a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^4 e^{\frac{4}{1+\left(\frac{t+r}{a}\right)^2}} + 16a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^8 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 8a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^2 e^{\frac{4}{1+\left(\frac{t+r}{a}\right)^2}} + 8a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^6 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}} \\
&+ 2a^{16}a_0{}^4 e^{\frac{4}{1+\left(\frac{t+r}{a}\right)^2}} + 2a^{16}a_0{}^4\left(\frac{t+r}{a}\right)^4 e^{\frac{2}{1+\left(\frac{t+r}{a}\right)^2}}\Bigg]
\end{aligned}
\tag{A.2}
$$

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