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Korteweg-de Vries-Burgers Equation in a Multi-Component Magnetized Plasma with Nuclei of Heavy Elements

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The nonlinear properties of ion-acoustic (IA) waves are investigated in a relativistically degenerate magnetized quantum plasma, whose constituents are non-degenerate inertial ions, degenerate electrons and immobile positively-charged heavy elements. For nonlinear studies, the well-known reductive perturbation technique is employed to derive the Korteweg-de Vries-Burger equation in the presence of relativistically degenerate electrons. Numerically, the amplitude, width, and phase speed are shown to be associated with the localized IA solitons, and shocks are shown to be significantly influenced by the various intrinsic parameters relevant to our model. The solitary and the shock wave properties have been to be influenced in the non-relativistic, as well as the ultra-relativistic, limits. The effects of the external magnetic field and the obliqueness are found to change the basic properties of IA waves significantly. The present analysis can be useful in understanding the collective process in dense astrophysical environments, like there of non-rotating white dwarfs, neutron stars, *etc.*

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I. INTRODUCTION

For several decades, intense theoretical and experimental research has been done on ion-acoustic waves, which is one of the basic wave processes in plasmas [1–16]. The ion acoustic (IA) soliton formation in an electronion (e-i) plasma has been extensively studied both theoretically and experimentally for a long time. The nonlinear propagation of IA waves has been investigated both in planar and non-planar geometries. IA waves were first predicted by Tonks and Langmuir by using a fluid analysis [17]. To study the characteristic of ion-acoustic solitons in an e-i plasma, Washimi and Taniuti developed a weakly nonlinear theory [1]. A fully nonlinear theory was presented by Sagdeev [18] to investigate the arbitrary amplitude ion-acoustic solitary waves in e-i plasmas. In the presence of some sort of dissipative mechanism, the balance between nonlinearity and dissipation is known to lead to the formation of shock structures. Thus, the search for the source of such a dissipation, which may responsible for the formation of shock structures is an important issue. Several different mechanisms, which may be responsible for the formation of the shock waves, have been studied by a number of authors who used different plasma models. Roy *et al.* [19] studied IA shock waves that formed due to effect of the ion kinematic viscosity in quantum pair plasmas. The propagation of IA shock waves in an unmagnetized collisionless plasma consisting of superthermal electrons has been studied by Sultana *et al.* [20]. They investigated the stability profile of the kink-shaped solutions of the Korteweg-de Vries (KdV)-Burger equation against external perturbations.

In a nonlinear dispersive medium, when nonlinear and dispersive effects have opposite signs, the wave equation gives rarefactive (dip-like) solitons. Many KdV equations, which have negative nonlinear effect and positive dispersive effect, result in rarefactive solitons. However, when both the nonlinear and the dispersive effects are positive, the KdV equation may result in a compressive (hump-like) soliton. A medium with dispersive and significant dissipative properties supports the existence of shock waves instead of solitons. In peculiar circumstances, the Burgers term in the KdV-Burger (KdVB) equation becomes negligibly small. The KdVB equation transforms to the KdV equation, which admits soliton solutions. This happens when the dissipation diminishes

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and the dispersion dominates. Nakamura *et al.* [21] observed solitary shock waves in unmagnetized dusty plasmas. They found that the development of a shock wave was due to the KdVB equation. The dust-ion-acoustic shock and solitary waves in dusty electronegative plasmas have been studied by Mamun and Tasnim [22]. They discussed the basic properties of the shock and solitary waves, that are associated with the presence of positive ion dynamics and dust charge fluctuations.

Recently, studies of relativistic degenerate plasmas have gained enormous attention due to their existence in interstellar compact objects such as white dwarfs and neutron stars and in intense laser plasma experiments. A white dwarf is a real example where degenerate electrons and heavy ions exist. The Pauli exclusion principle in quantum mechanics forbids electrons (and all fermions with half integer spin, including neutrons) occupying the same state. Basically, each electron must have a different energy when it is packed together with other electrons, as in a white dwarf. The number of available low-energy states is too small, and many electrons are forced into high-energy states. When this happens, the electrons are said to be degenerate. These high-energy electrons make a significant contribution to the pressure. Because this pressure arises from a quantum-mechanical effect, it is insensitive to temperature, *i.e.*, the pressure doesn't decrease as the star cools. This pressure is known as the electron degeneracy pressure, and it is the pressure that supports white dwarf stars against their own gravity [23–26]. Notably, the basic constituents of white dwarfs are mainly positively, and negatively-charged heavy elements like carbon, oxygen, and helium with an envelope of hydrogen gas. Heavy elements (positive and negative) are found to have been formed in a prestellar stage of the evolution of the universe when all matter was compressed to extremely high densities. The average number density of heavy particles is of the order of 10^{29} cm^{-3}, where distance between heavy particles is of the order of 10^{-10} cm (for white dwarfs) [27]. In white dwarfs, the degenerate number density of electrons can be of the order of 10^{30} cm^{-3} [28]. The equation of state for the degenerate electrons in such interstellar compact objects has been obtained by Chandrasekhar for two limits, namely, the nonrelativistic and the ultrarelativistic limits. Chandrasekhar found that the degenerate electron equation of state is given by ($P_e \propto {n_e}^{5/3}$) for nonrelativistic degenerate electrons and by ($P_e \propto {n_e}^{4/3}$) for ultrarelativistic degenerate electrons, where Pe is the degenerate electron pressure and n_e the electron number density [29–31]. Of note is that the degenerate electron pressure depends only on the electron number density.

Recently, a number of authors have theoretically investigated the nonlinear propagation of electrostatic waves in degenerate quantum plasmas. Those investigations were mainly based on the electron equation of state, which is only valid in the nonrelativistic limit. Some investigations have addressed the nonlinear propagation in a degenerate dense of electrostatic waves plasma which are mainly based on the degenerate electron equation of state valid for ultrarelativistic limit [32–36]. Shah *et al.* [37–40] studied an unmagnetized degenerate quantum plasma and investigated the effects of relativistic degenerate electrons and positrons and plasma particle number densities on the propagation of positron-acoustic solitary or shock waves. Sabary *et al.* [41] studied the ion-acoustic waves (IAWs) in a plasma with two distinct ion species. Gill *et al.* [42] discussed the properties of IAWs in a plasma consisting of warm positive and negative ions with differences concentrations, charged states, and nonthermal electrons. Shukla *et al.* [43] theoretically investigated the nonlinear propagation of electrostatic waves in degenerate quantum plasmas. They considered strongly coupled nondegenerate ions and degenerate electron fluids in an unmagnetized dense plasma and studied the basic properties of solitary and shock structures. Sultana *et al.* [44] investigated obliquely propagating IA of arbitrary amplitude solitons in a magnetized e-i plasma with superthermal electron. Finite amplitude IA waves, ion cyclotron waves, and IA solitons have also been studied in a warm-ion magnetized plasma by Yashvir *et al.* [45]. Hossen *et al.* [46–49] investigated the formation and propagation of shocks and solitons in an unmagnetized, ultradense plasma by considering different models. They found that the relativistic factors and degeneracy are significantly affected by these nonlinear structures. Masood *et al.* [50] investigated ion-acoustic shock waves in electron-positron-ion plasmas. Pakzad [51] derived the KdVB equation, and analyzed the properties of IA shock waves in a plasma model. Shah and Saeed [52] derived the KdVB equation for IA shock waves in a weakly-relativistic electron-positron-ion plasma. They found that the amplitude and the steepness of the shock wave decreased with increasing relativistic streaming factor and positron concentration. They showed that increasing the coefficient of kinematic viscosity increased the amplitude and the steepness of shock structure.

In this paper, we investigate ion-acoustic shock waves in a relativistically degenerate magnetized quantum plasma containing both degenerate electron and ion fluids and positively-charged static heavy elements. An analytical solution of the KdVB equation is studied as a function of plasma parameters such as the obliqueness (via δ), the heavy-ion to ion number density ratio (via μ), the heavy-ion charge state (via Z_h), the external magnetic field (via B_0) and the relativistic factor (via γ).

II. THEORETICAL MODEL AND BASIC EQUATIONS

We consider a three-component magnetized quantum plasma system consisting of non-degenerate inertial ions, both non relativistic and ultra relativistic degenerate electrons and positively-charged immobile heavy elements. Thus, the equilibrium condition reads $n_{i0} - n_{e0} + Z_h n_{h0} = 0$, where n_{s0} is the unperturbed number densitY of the species s (here s=i, e, h for inertial positive ion, degenerate electron, and immobile heavy element, respectively) and Z_h is the number of positive ions residing on the heavy elements surface. The positively-charged static heavy elements participate only in maintaining the quasi-neutrality condition at equilibrium. The electron's inertia can, in fact, be neglected if we assume that the IA electrostatic waves move at a phase velocity V_p that is much higher than the ion thermal speed, but much lower than the electron thermal speed: $V_{th,i} << V_p << V_{th,e}$. The dynamics of nonlinear IA waves in the presence of an external magnetic field $\mathbf{B} = \hat{z}B_0$ is governed by the momentum equation

$$\nabla\phi - \frac{K}{n_e}\nabla {n_e}^{\gamma} = 0, \tag{1}$$

and the non-degenerate inertial ion equations composed of the ion continuity and ion momentum equations are given by

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i \mathbf{u}_i) = 0, \tag{2}$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + (\mathbf{u}_i.\nabla)\mathbf{u}_i = -\nabla\phi + \eta\nabla^2.\mathbf{u}_i + \omega_{ci}(\mathbf{u}_i \times \hat{z}), \tag{3}$$

Poissons equation is

$$\nabla^2\phi = (1 + Z_h\mu)n_e - n_i - Z_h\mu, \tag{4}$$

where n_i (n_e) is the ion (electron) number density normalized by its equilibrium value n_{i0} (n_{e0}), $\mathbf{u}_i$ is the ion fluid speed normalized by $C_i = (m_e c^2/m_i)^{1/2}$, with m_e (m_i) being the electron (ion) rest mass, c is the speed of light in vacuum, and ϕ is the electrostatic wave potential normalized by $m_e c^2/e$. Here, μ ($= n_{h0}/n_{i0}$) is the heavy element to ion number density ratio. The nonlinear propagation of usual IA waves in an electron-ion (EI) plasma can be recovered by setting $\mu = 0$. The time variable (t) is normalized by $\omega_{pi} = (4\pi n_{i0}e^2/m_i)^{1/2}$, and the space variable (x) is normalized by $\lambda_s = (m_e c^2/4\pi n_{i0}e^2)^{1/2}$. The coefficient of viscosity η is a normalized quantity given by $\omega_{pi}\lambda_s^2 m_s n_{s0}$. We have defined the parameter K in Eq. (1) as $K = n_{e0}^{\gamma-1}K_e/m_e c^2$.

III. DERIVATION OF MAGNETIZED KDVB EQUATION

To observe the dynamics of small-amplitude stationary IA waves, we shall adopt stretched coordinates:

$$\xi = \epsilon^{1/2}(l_x\hat{x} + l_y\hat{y} + l_z\hat{z} - V_p t), \tag{5}$$

$$T = \epsilon^{3/2}t, \tag{6}$$

where ϵ is a smallness parameter ($0 < \epsilon < 1$) measuring the amplitude of the perturbation, V_p is the wave's phase velocity normalized by the IA speed (C_i), and l_x, l_y, and l_z are the directional cosines of the wave vector k along the x, y, and z axes, respectively, so that $l_x^2 + l_y^2 + l_z^2 = 1$. We note here that x, y, and z are all normalized by the Debye length λ_D and that T is normalized by the inverse of the ion plasma frequency (ω_{pi}^{-1}). We may expand n_s, u_s, and ϕ in power series of ϵ as

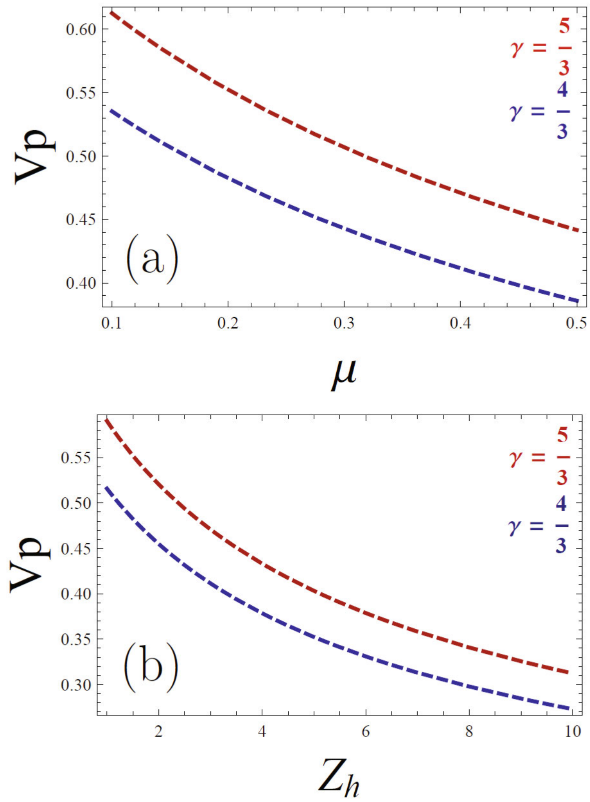

Fig. 1. (Color online) Variation of the phase speed V_p with (a) the heavy ion-to-ion number density ratio μ and (b) the number of positive ions residing on the heavy ion's surface Z_h, for fix values $U_0 = 0.1$, $\omega_{ci} = 0.5$, and $\delta = 10^0$. The red dashed line represents the non relativistic case, and the blue dashed line represents the ultra relativistic case.

$$n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \cdots, \tag{7}$$

$$u_{ix,y} = 0 + \epsilon^{3/2}u_{ix,y}^{(1)} + \epsilon^2 u_{ix,y}^{(2)} + \cdots, \tag{8}$$

$$u_{iz} = 0 + \epsilon u_{iz}^{(1)} + \epsilon^2 u_{iz}^{(2)} + \cdots, \tag{9}$$

$$\phi = 0 + \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \cdots. \tag{10}$$

A weak damping situation has been considered in terms of a small ion kinematic viscosity. This leads one to

$$\eta \approx \epsilon^{1/2}\eta_0, \tag{11}$$

where η_0 is a finite parameter of the order of unity. Now substituting Eqs. (5) - (11) into Eqs. (1) - (4)

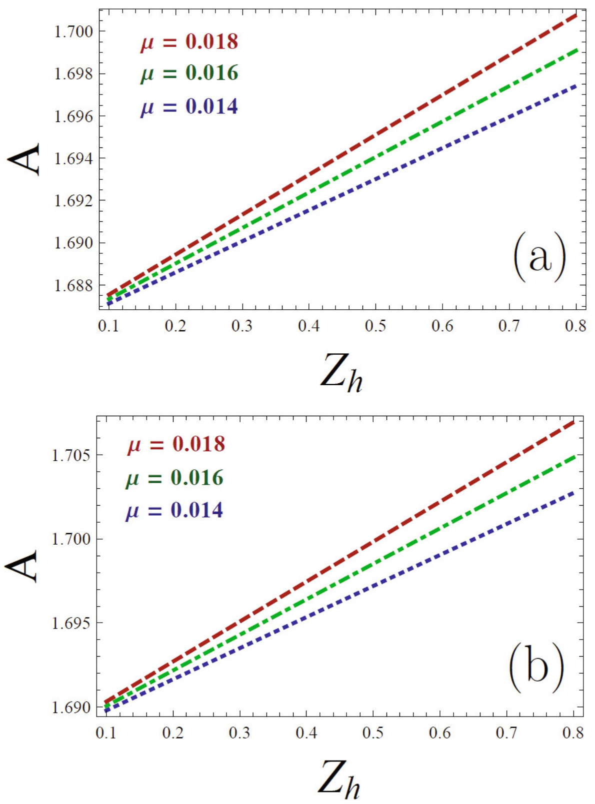

Fig. 2. (Color online) Variation of nonlinearity coefficient A with Z_h and μ (a) for the non relativistic case and (b) for the ultra relativistic case. The other plasma parameters are fixed at $U_0 = 0.1$, $\omega_{ci} = 0.5$, and $\delta = 10^0$.

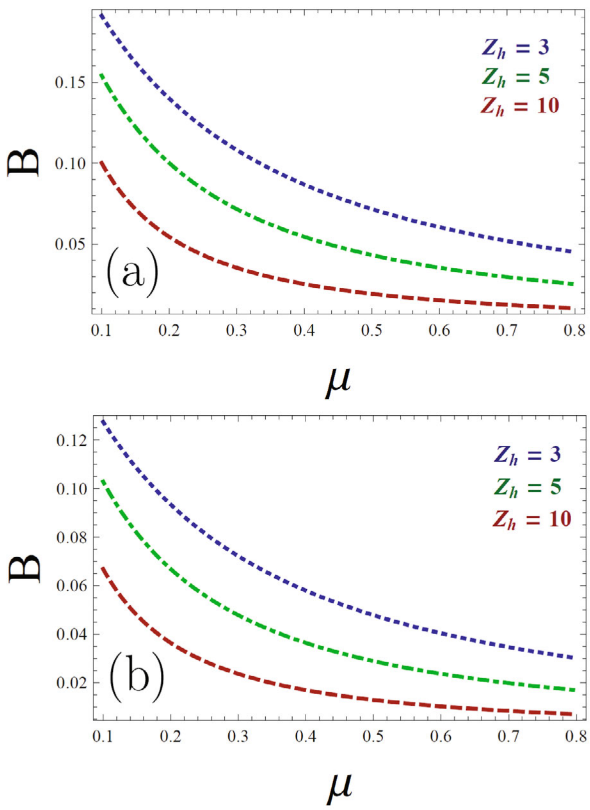

Fig. 3. (Color online) Variation of dispersive coefficient B with μ and Z_h (a) for the non relativistic case and (b) for the ultra relativistic case. The other plasma parameters are fixed at $U_0 = 0.1$, $\omega_{ci} = 0.5$, and $\delta = 10^0$.

and taking the lowest order coefficient of ϵ, we obtain, $u_{iz}^{(1)} = l_z\phi^{(1)}/V_p$, $n_i^{(1)} = l_z^2\phi^{(1)}/V_p^2$, and $n_e^{(1)} = \phi^{(1)}/K_1$. The linear dispersion relation can be obtained from these equations as

$$V_p = l_z\sqrt{\left(\frac{K_1}{1+Z_h\mu}\right)}. \tag{12}$$

To the lowest order of x- and y-component of the momentum equation (3) we get,

$$u_{iy}^{(1)} = \frac{l_x}{\omega_{ci}}\frac{\partial\phi^{(1)}}{\partial\xi}, \tag{13}$$

$$u_{ix}^{(1)} = -\frac{l_y}{\omega_{ci}}\frac{\partial\phi^{(1)}}{\partial\xi}. \tag{14}$$

Now substituting Eqs. (5) - (14) into Eq. (3), we obtain the following from the higher-order series in ϵ of the momentum and Poisson's equations:

$$u_{iy}^{(2)} = \frac{l_y V_P}{\omega_{ci}^2}\frac{\partial^2\phi^{(1)}}{\partial\xi^2}, \tag{15}$$

$$u_{ix}^{(2)} = \frac{l_x V_P}{\omega_{ci}^2}\frac{\partial^2\phi^{(1)}}{\partial\xi^2}, \tag{16}$$

$$\frac{\partial^2\phi^{(1)}}{\partial\xi^2} = (1+Z_h\mu)n_e^{(2)} - n_i^{(2)}. \tag{17}$$

Using the same process, we get the next-higher-order continuity equation, as well as the z-component of the momentum equation. Now, combining these higher-order equations together with Eqs. (13) - (17) and considering $\phi^{(1)} = \psi$, we obtain

$$\frac{\partial\psi}{\partial T} + A\psi\frac{\partial\psi}{\partial\xi} + B\frac{\partial^3\psi}{\partial\xi^3} = C\frac{\partial^2\psi}{\partial\xi^2}, \tag{18}$$

where A, B, and C represent the coefficients of nonlinearity, dispersion, and dissipation, respectively. These

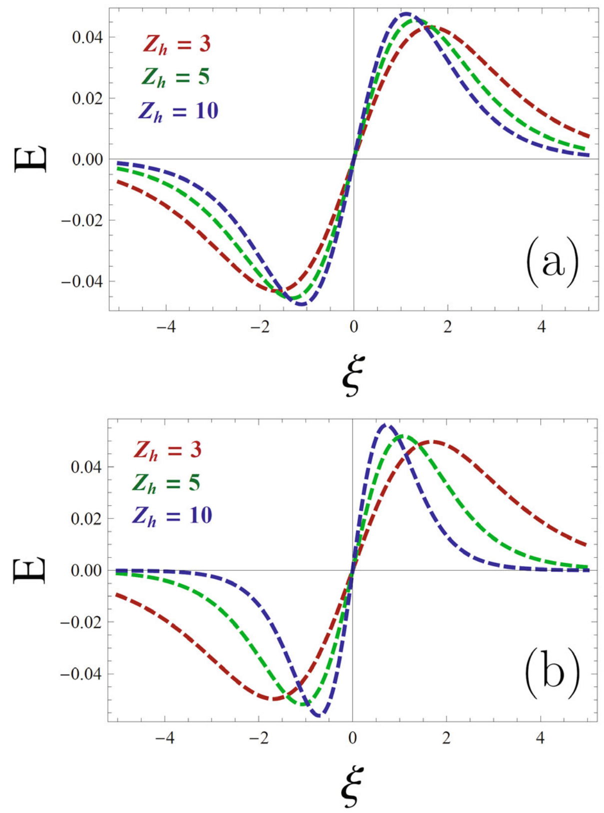

Fig. 4. (Color online) Variation of corresponding electric field E with Z_h (a) for the non relativistic case and (b) for the ultra relativistic case. The other plasma parameters are fixed at $\mu = 0.1$, $U_0 = 0.1$, $\omega_{ci} = 0.5$, and $\delta = 10^0$.

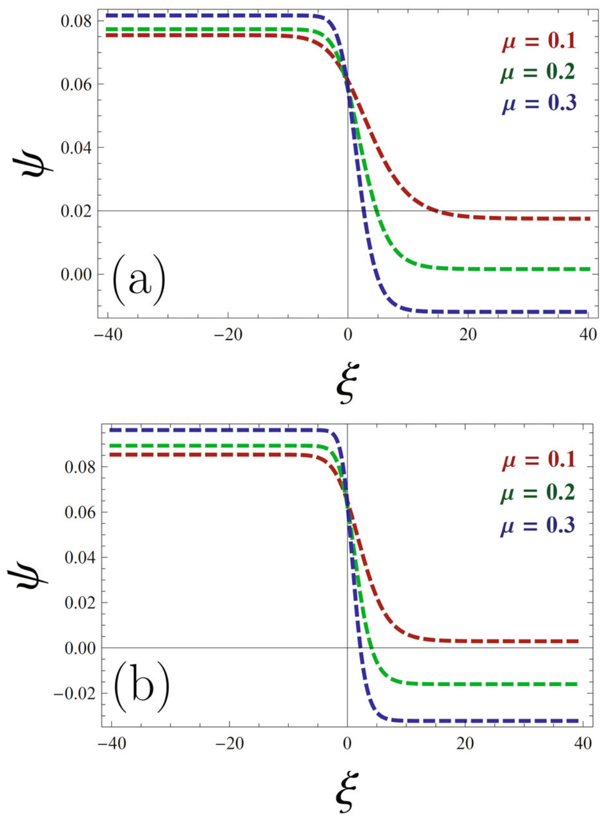

Fig. 5. (Color online) Variation of the shock profile with μ (a) for the non relativistic and (b) for the ultra relativistic case. The other plasma parameters are fixed at $Z_h = 5$, $U_0 = 0.1$, $\omega_{ci} = 0.5$, $\eta_0 = 0.4$, and $\delta = 20^0$.

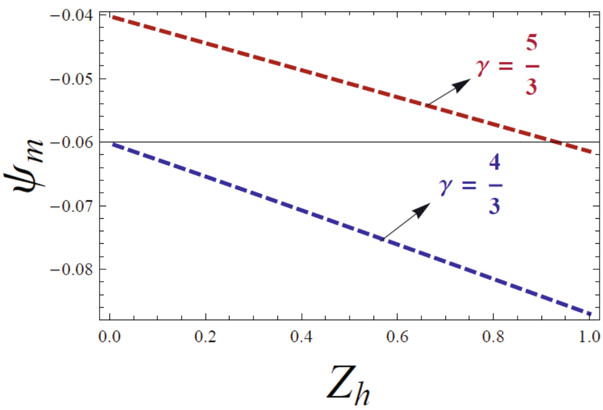

Fig. 6. (Color online) Variation of the shock wave amplitude with Z_h for the fixed plasma parameters $\mu = 0.1$, $U_0 = 0.1$, $\omega_{ci} = 0.5$, $\eta_0 = 0.4$, and $\delta = 20^0$. The red dashed line represents the non-relativistic case, and the blue dashed line represents the ultra-relativistic case.

coefficients are given by the relations

$$A = \frac{V_p^3}{2l_z^2}\left[\frac{l_z^2(\gamma-2)}{V_p^2 K_{11}} + \frac{3l_z^4}{V_p^4}\right], \tag{19}$$

$$B = \frac{V_p^3}{2l_z^2}\left[1 + \frac{(1-l_z^2)}{\omega_{ci}^2}\right], \tag{20}$$

$$C = \frac{\eta_0}{2}. \tag{21}$$

Equation (18) represents the KdV-Burgers equation, which describes the nonlinear evolution of obliquely propagating IA shock waves in a magnetized plasma. The nonlinear coefficient A and the dispersion coefficient B are seen to be influenced by the obliqueness. We note that in the absence of the ion kinematic viscosity, the dissipation coefficient vanishes, and Eq. (18) reduces to the well-known KdV equation.

IV. SOLITARY WAVE SOLUTION

In order to study the IA solitary solution of the KdV-Burgers equation, we first consider the conservative case (*i.e.*, $\eta = 0$). In the absence of dissipation, Eq. (18) reduces to the KdV equation. Therefore, the solitary wave solution is given by

$$\psi = \psi_m sech^2\left[\frac{\xi - U_0 T}{L}\right], \tag{22}$$

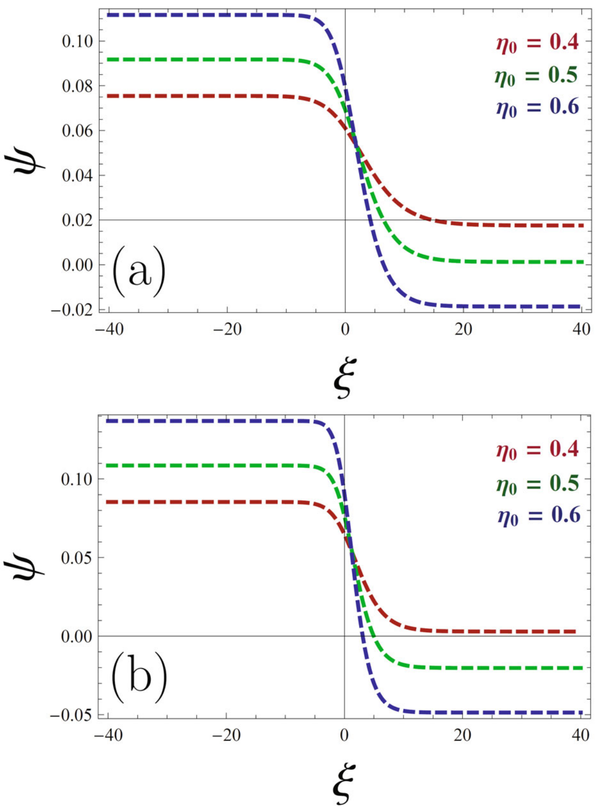

Fig. 7. (Color online) Variation of the shock profile with the viscosity coefficient η_0 (a) for the non relativistic and (b) for the ultra relativistic case. The other plasma parameters are fixed at $\mu = 0.1$, $Z_h = 5$, $\omega_{ci} = 0.5$, $U_0 = 0.1$, and $\delta = 20^0$.

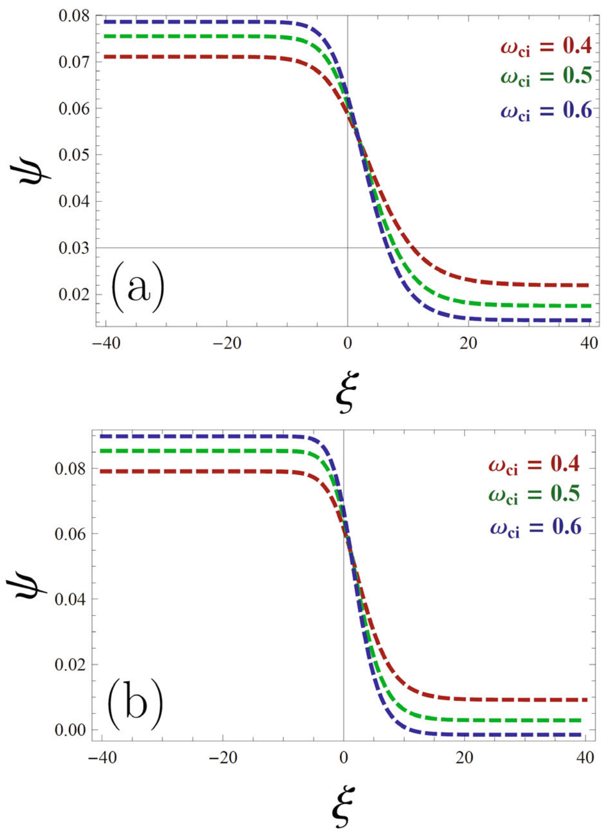

Fig. 8. (Color online) Variation of the shock profile with ion cyclotron frequency ω_{ci} (a) for the non-relativistic and (b) for the ultra-relativistic case. The other plasma parameters are fixed at $\mu = 0.1$, $Z_h = 5$, $\eta_0 = 0.4$, $U_0 = 0.1$, and $\delta = 20^0$.

where the maximum pulse amplitude and width of the soliton solution in the absence of the Burger's term are given by $\psi_m = 3U_0/A$ and $L = (4B/U_0)^{1/2}$, respectively. The properties of the nonlinear coefficient A and dispersive coefficient B represented by Eq. (19) and Eq. (21) are shown in Figs. 2 and 3, respectively. Here, U_0 is the solitary wave speed at equilibrium. The associated electric field is obtained from the relation

$$\mathbf{E} = -\nabla\psi; \tag{23}$$

then, the electric field is given by

$$E = \frac{6U_0}{AL} sech^2(\frac{\xi - U_0T}{L}) \tanh(\frac{\xi - U_0T}{L}). \tag{24}$$

here, E represents a bipolar electric field excitation which are illustrated in Fig. 4.

V. SHOCK WAVE SOLUTION

In the presence of dissipation, *i.e.*, when $\eta \neq 0$, the total energy of the system is not conservative. To study the stationary solution of the KdV-Burgers equation, Eq. (18). in terms of shock excitations, we first consider the solution in the form

$$\psi = \sum_{n=0}^{N} a_n \tanh^n(\xi - U_0T), \tag{25}$$

where the coefficients a_n and N have to be determined. Now, we find the solution (omitting necessary calculations) given by

$$\begin{aligned} \psi(\xi, T) &= \frac{U_0}{A} + \frac{3C^2}{25AB} sech^2(\frac{\xi - U_0T}{\Delta}) \\ &\quad -\frac{6C^2}{25AB} \tanh^2(\frac{\xi - U_0T}{\Delta}), \end{aligned} \tag{26}$$

where the shock wave's amplitude is given by

$$\psi_m = \frac{12C^2}{25AB}. \tag{27}$$

The amplitude variation of the shock profiles are shown in Fig. 6 and Fig. 7. The width of the shock structure can be represented as

$$\Delta = \frac{10B}{C}, \tag{28}$$

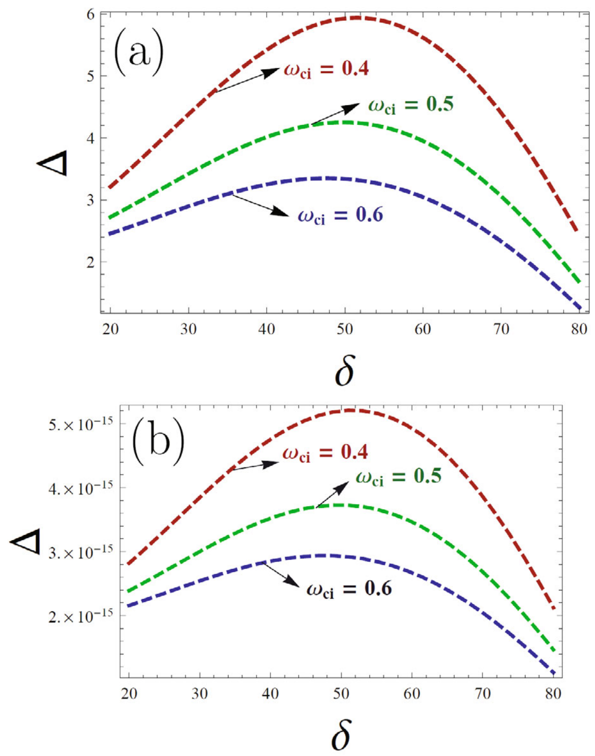

Fig. 9. (Color online) Variation of the shock wave width Δ with δ and ω_{ci} (a) for the non relativistic and (b) for the ultra relativistic case. The other plasma parameters are fixed at $\mu = 0.1$, $Z_h = 5$, $U_0 = 0.1$, $\eta_0 = 0.4$.

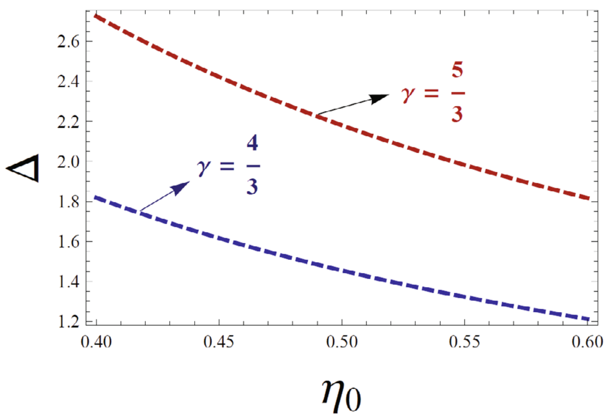

Fig. 10. (Color online) Variation of the shock wave width Δ with η_0 for the fixed plasma parameters $\mu = 0.1$, $Z_h = 5$, $U_0 = 0.1$, $\omega_{ci} = 0.5$, $\eta_0 = 0.4$, and $\delta = 20^0$. The red dashed line represents the non relativistic case, and the blue dashed line represents the ultra relativistic case.

which introduces the argument of the hyperbolic function. The variations in the width Δ are depicted in Fig. 9 and Fig. 10. The shock wave speed is given by

$$U_0 = \frac{6C^2}{25B}. \tag{29}$$

Here, $U_0 > 0$, so the solution corresponds to a shock structure traveling in the $+\xi$ direction. The amplitude, width, and speed of the shock waves are also seen to be the functions of ω_{ci}, δ, η_0, Z_h, and μ.

Noteworthy as that, because the electric potential is associated with an electric field $\mathbf{E} = -\nabla\psi$, the constant term of Eq. (26) can be omitted without the loss of physical meaning. However, for the sake of mathematical generality, we have chosen to retain the general expression above in Eq. (26). We can now simply find the expression for electric field from Eq. (26) as

$$E = \frac{6C^2}{25AB\Delta} sech^2(\frac{\xi - U_0T}{\Delta})[1 + \tanh(\frac{\xi - U_0T}{\Delta})], \tag{30}$$

which represents an inverse-ball-shaped monopolar localized excitation for the electric field [20].

V. NUMERICAL OBSERVATION AND RESULTS

In order to study a new physical approach, we have considered a magnetized degenerate plasma system (containing non-degenerate inertial ions, both non relativistic and ultra relativistic degenerate electrons, and positively-charged immobile heavy elements) and have studied IA waves by deriving the KdVB equation. We have used the well-known reductive perturbation method to derive the partial differential equation and found two types of solutions, viz., solitary and shock wave solutions. We observed that the relativistic effect and the degenerate pressure may be a great contributions to the amplitude, phase velocity, and width illustrated from the non relativistic ($P_e \propto {n_e}^{5/3}$) to the ultra relativistic ($P_e \propto {n_e}^{4/3}$) region. The parametric values considered here are associated with relativistic dense plasmas, especially dense astrophysical objects such as white dwarfs, neutron stars, *etc.* [53]. Important to mention is that, we find the solitary and shock profiles traveling in the $+\xi$ direction because the nonlinear wave velocities are greater than zero. The results of our theoretical exploration are illuminated in Figs. 1 - 10. The heavy element to ion number density ratio (via μ), the number of positive ions residing on the heavy element's surface (via Z_h) and the electron degeneracy have important effects on the phase speed (V_p) of IAWs. The variations in the phase speed V_p with μ and Z_h are depicted in Figs. 1(a) and 1(b), respectively for both the non relativistic and ultra relativistic cases. The phase speed V_p is seen to decrease with increasing of μ and Z_h. Also, the phase speed V_p is observed to be always greater for non relativistic case than ultra relativistic case. The (weak) kinematic viscosity η_0 has no effect on the dynamics at the linear level. Figure 2 shows the variation of nonlinearity coefficient A with Z_h for different values of μ for (a) the non relativistic and (b) the ultra relativistic case. The nonlinearity of the plasma system is observed to increase with increasing Z_h and to decrease with decreasing of μ for both the non relativistic and the ultra relativistic cases. Also, the nonlinearity coefficient A is observed to be larger for the ultra relativistic case than for the non relativistic case. The effects of μ and Z_h on dispersive coefficient B are illustrated in Fig. 3 for (a) the non relativistic and (b) the ultra relativistic case. The dispersive coefficient B is seen to decrease with the increasing μ and Z_h. For the non relativistic case the dispersive coefficient B is larger than this for the ultra relativistic case. Figure 4 presents the evolution of the associated electric field E with ξ for different values of Z_h for (a) the non relativistic and (b) the ultra relativistic cases. The electric field E is observed to increase sharply with increasing Z_h and the value is slightly larger for non-relativistic case. The Shock profiles of the IAWs are affected significantly by μ are shown in Fig. 5 for (a) the non relativistic and (b) the ultra relativistic cases. An increasing value of μ causes the shock profile to be taller and sharper for both the non relativistic and ultra relativistic case. Figure 6 presents the variation of the maximum amplitude of the shock wave, ψ_m, with Z_h for both the non relativistic and the ultra relativistic case. The amplitude is noticed to decrease with the increasing of Z_h and the amplitude is smaller for the ultra relativistic case in comparison to non-relativistic case. Figure 7 illustrates the effect of the kinematics viscosity on the shock profile of the IAWs for (a) the non relativistic and (b) the ultra relativistic cases. An important note is that, as a weak damping situation has been considered, we can stretch the viscosity coefficient η without any loss of physical meaning. A strong (larger amplitude and sharper) shock profile is found for a higher value of viscosity coefficient η_0. The external magnetic field B_0 has an important effect on the profile of the shock wave. Figure 8 illuminates the effect of the external magnetic field B_0 for (a) the non relativistic and (b) the ultra relativistic cases. With the increasing ion cyclotron frequency, the shock profile's amplitude is observed to increase for both the non relativistic and the ultra relativistic case. The effects of the obliqueness δ and the external magnetic field B_0 (viz., ω_{ci}) on the width of the shock excitations Δ for (a) the non relativistic and (b) the ultra relativistic cases are depicted in Fig. 9. The shock wave width Δ is seen to increase almost linearly for the lower range, of δ (from 0^0 to about 55^0), but above this range Δ decreases with increasing δ. The width goes to zero as $\delta \longrightarrow 90^0$; thus, the amplitude goes to ∞. Again, the applied magnetic field B_0 has a great influence on the width of the shock profile. Increasing the value of ω_{ci} decreases the width of the shock profile. Thus, the external magnetic field causes the shock profile to become more spiky. From Fig. 9 the width of the shock profile for the ultra relativistic case is observed to be very much smaller than that for the non relativistic case. Figure 10 presents the variation of the width Δ with the viscosity coefficient η_0 for both the non relativistic and the ultra relativistic cases. The width Δ decreases with increasing of viscosity coefficient η_0.

VI. DISCUSSION

A theoretical investigation has been carried out to study the nonlinear wave propagation in a magnetized, collisionless dense plasma containing non degenerate inertial ions, both non relativistic and ultra-relativistic degenerate electrons, and positively-charged immobile heavy elements by deriving the Korteweg-de Vries-Burgers (KdVB) equation. We obtained the Korteweg-de Vries (KdV) equation by considering the conservative ($C = 0$) case. In the presence of the Burgers term (dissipation) gives an exact solution via the tanh approach. The solution shown a monotonic kink-shaped structure, which is unstable with respect to an external perturbation. In our numerical analysis, we have shown the influences of the obliqueness, the external magnetic field, the kinematic viscosity and other plasma parameters on the basic features (phase speed, amplitude, polarity, width, *etc.*) of the IAWs, which makes our present work significant for understanding the localized electrostatic disturbances in many space and astrophysical plasma environments (viz. white dwarfs, neutron stars, compact planets like massive Jupiter, other exotic dense stars, and black holes). We have observed that, the nonlinear wave properties for the non relativistic case are extremely different from those for the ultra relativistic case. In conclusion, our present investigation could be rigorously important for global nonlinear models of astrophysical compact objects where the effects of dispersion, dissipation, degenerate pressure and positively-charged heavy elements play a crucial role.

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