Journal of the Korean Physical Society, Vol. 67, No. 3, August 2015, pp. 496∼501

Nonplanar Ion-acoustic Shock Waves in a Multi-ion Plasma with Nonextensive Electrons and Positrons

N. Jannat,* M. Ferdousi and A. A. Mamun
Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

(Received 12 February 2015, in final form 4 June 2015)

The basic features of ion-acoustic shock waves (IASHWs) in a multi-ion nonextensive plasma (containing positive light ions, negative heavy ions, as well as nonextensive electrons and positrons) have been rigorously investigated in a nonplanar geometry. The standard reductive perturbation method has been employed to derive the Modified Burgers (MB) equation. The combined effects of the electron and positron nonextensivity, and the ion kinematic viscosity significantly have been found to modify the basic properties of these electrostatic shock structures. The properties of the cylindrical and the spherical IASHWs are observed to differ significantly from those of one-dimensional planar waves. The findings obtained from this theoretical investigation may be useful in understanding the characteristics of IASHWs both in space and laboratory plasmas.

PACS numbers: 52.27.Ep, 52.35.Mw, 52.35.Tc
Keywords: Ion-acoustic shock waves, Nonextensive electrons and positrons, Multi-ion plasma, Modified Burgers equation.
DOI: 10.3938/jkps.67.496

I. INTRODUCTION

The so-called ion-acoustic (IA) waves are low-frequency longitudinal plasma density oscillations. In these oscillations, electrons and ions are propagating in the phase space [1,2]. The IA waves were predicted first by Tonks and Langumir based on fluid dynamics [3]. The first experimental observation of IA waves was reported in Ref. 4 [4]. Two models for the IA waves [5] are based on: the continuum models, in which the plasma is treated as a fluid and fluid dynamics is used for theoretical studies and models based on the kinetic equations in statistical theory, where the distribution functions are used to describe the properties of the IA waves.

Nowadays, research works on the nonlinear propagation of shock waves (SHWs) in electron-positron-ion (e-p-i) plasmas have received a considerable attention because of the importance of understanding the behavior of space plasmas viz. supernovas, pulsar environments, cluster explosions, and active galactic nuclei [6–8]. Some theoretical investigations [9–11] have been made on the nonlinear propagation of ion-acoustic shock waves (IASHWs) in e-p-i plasmas. Electron-positron (e-p) plasmas have been observed to behave differently as opposed to typical electron-ion (e-i) plasmas [12,13]. An interesting aspect of an e-p plasma in comparison to the usual e-i plasma is the fact that the components of an e-p plasma have the same mass and equal magnitude of the charge. Much research has been carried on e-p and e-p-i plasmas in the last few years [14–16]. For instance, Nejoh [14] investigated the effect of ion temperature on the large amplitude IA waves in e-p-i plasmas. Mushtaq and Shah [16] studied the effect of positron concentration on the nonlinear propagation of two-dimensional magnetosonic waves and found that the waves in an e-p-i plasma behaved quite differently from those in an ordinary e-i plasma. Recently, Ferdousi *et al.* [17, 18] studied the characteristics of planar and nonplanar IASHWs in an e-p-i plasma with nonextensive electrons and positrons.

Because in many cases, the wave structures observed in space or laboratory devices are certainly not infinite (unbounded) in one dimension [19], one should consider nonplanar geometries, specially cylindrical ($\nu = 1$) and spherical ($\nu = 2$). The nonplanar geometries of practical interest are capsule implosion (spherical geometry), shock tubes (cylindrical geometry), star formation, supernova explosions, *etc.* Some of the investigations on nonplanar IASHWs in e-p-i plasmas are reported here. Moslem *et al.* [20] used cylindrical geometry to study the propagation of nonlinear excitations in an e-p-i plasma in the inner region of the accretion disc. Masood *et al.* [21] studied the propagation of nonlinear IASHWs in planar and nonplanar geometries and found that the strength of IASHWs was maximum for spherical geometry, intermediate for cylindrical geometry, and minimum for planar geometry. These works [22–24] are concerned with nonplanar IASHWs in e-p-i plasmas.

At the present time, significant attention has been de-

*E-mail: nilimajannat74@gmail.com

voted to the study of wave propagation in multi-ion plasmas because of its (wave propagation) vital role in understanding different types of collective processes in space environments [25–27] as well as in laboratory devices [28–30]. In different situations like plasma processing reactors [31], neutral beam sources [32], low-temperature laboratory experiments [33], *etc.*, positive-negative ion plasmas have been found to exist. The presence of negative ions in Earth's ionosphere [34] and cometary comae [35] is well known. The importance of negative-ion plasmas to the field of plasma physics is growing because negative ions have been found to outperform positive ions in plasma etching.

The nonextensive distribution (q-distribution) [36] is the most generalized distribution for the study of the nonlinear properties of SHWs in different plasma systems. The study of nonextensive plasmas [36] has received a great deal of attention from plasma physics researchers due to their relevance to astrophysical and cosmological scenarios like protoneutron stars [37], stellar polytropes [38], hadronic matter and quark-gluon plasmas [39], dark-matter halos [40], *etc.* Different types of waves have been studied in nonextensive plasmas by many authors who considered one or two components to be nonextensive [17,41–44].

Therefore, in our present work, we have considered a four-component plasma system consisting of positive light ions, negative heavy ions, nonextensive electrons, and nonextensive positrons. The aim of this paper is to study the effects of a nonplanar geometry, the nonextensivity of electrons and positrons, and the kinematic viscosity of ions on the basic features (viz. polarity, amplitude, width, speed, *etc.*) of IASHWs in multi-ion nonextensive plasma systems. The manuscript is organized as follows: The governing equations are provided in Section II. The Modified Burgers (MB) equation is derived by using the reductive perturbation method in Section III. A brief discussion is given in Section IV.

II. GOVERNING EQUATIONS

We consider a nonplanar (cylindrical or spherical) geometry and nonlinear propagation of the IA waves in a four-component plasma system consisting of inertial positive light ions, negative heavy ions, noninertial nonextensive electrons, and nonextensive positrons. Thus, the equilibrium charge neutrality condition is $Z_i n_{i0} + n_{p0} = Z_h n_{h0} + n_{e0}$, where n_{s0} is the unperturbed number densities of the species s (here $s = i, h, e, p$ for positive light ions, negative heavy ions, electrons, and positrons, respectively) and Z_i (Z_h) is the number of light positive ions (heavy negative ions). The electrons and the positrons are assumed to obey nonextensive distributions on the IA wave's time scale, and their number densities are given by the following expressions, respectively:

$$n_e = n_{e0}[(1 + (q-1)\psi]^{\frac{1+q}{2(q-1)}},$$
$$n_p = n_{p0}[(1 - (q-1)\psi]^{\frac{1+q}{2(q-1)}},$$

where q is the nonextensive parameter describing the degree of nonextensivity, *i.e.*, $q = 1$ corresponds to a Maxwellian distribution and $q < (> 1)$ denotes the nonextensive distribution. The parameters n_e and n_p are the number densities of perturbed electrons and positrons. The normalized basic equations governing the dynamics of the IA waves in a nonplanar geometry, are given in dimensionless variables as follows:

$$\frac{\partial n_{i,h}}{\partial t} + \frac{1}{r^\nu}\frac{\partial}{\partial r}(r^\nu n_{i,h} u_{i,h}) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial r} = -\frac{\partial \psi}{\partial r} + \eta \frac{\partial^2 u_i}{\partial r^2}, \tag{2}$$

$$\frac{\partial u_h}{\partial t} + u_h \frac{\partial u_h}{\partial r} = \alpha \frac{\partial \psi}{\partial r} + \eta \frac{\partial^2 u_h}{\partial r^2}, \tag{3}$$

$$\frac{1}{r^\nu}\frac{\partial}{\partial r}\left(r^\nu \frac{\partial \psi}{\partial r}\right) = -n_i + \mu_e[1 + (q-1)\psi]^{\frac{(q+1)}{2(q-1)}} - \mu_p[1 - (q-1)\sigma\psi]^{\frac{(q+1)}{2(q-1)}} + (1 - \mu_e + \mu_p)n_h. \tag{4}$$

We note that $\nu = 0$ for a 1 dimensional ($1D$) planar geometry, and $\nu = 1(2)$ for a nonplanar cylindrical (spherical) geometry. Here, $n_{i,h}$ are the number densities of light positive ions and heavy negative ions normalized by their equilibrium values $n_{i0,h0}$, $u_i(u_h)$ is the positive (negative) ion fluid speed normalized by $C_i = (k_B T_e/m_i)^{1/2}$, ψ is the electrostatic wave potential normalized by $k_B T_e/e$, and η is the viscosity coefficient normalized by $m_i n_{io} \omega_{pi} \lambda_D^2$. The time variable t and the space variable r are normalized by $\omega_{pi}^{-1} = (m_i/4\pi n_{io} e^2)^{1/2}$ and $\lambda_D = (k_B T_e/4\pi e^2 n_{io})^{1/2}$, respectively, where k_B is the Boltzmann constant, T_e is the electron temperature, and e is the magnitude of the electric charge. We have defined the following parameters: $\mu_e = n_{eo}/n_{io}$ (electron number density to ion number density), $\mu_p = n_{po}/n_{io}$ (positron number density to ion number density), $\sigma = T_e/T_p$ (electron temperature to positron temperature), and $\alpha = Z_h m_i/Z_i m_h$, where m_i (m_h) is the mass of light positive ions (heavy negative ions).

III. FORMATION OF SHOCK WAVES

To study the finite amplitude electrostatic IASHWs by analyzing the ingoing SHWs of Eqs. (1)−(4), we employ the reductive perturbation method (RPM) [45]. The RPM is mostly applied to small amplitude nonlinear waves [46]. This method rescales both space and time in the governing equations of the system in order to introduce space and time variables, which are appropriate

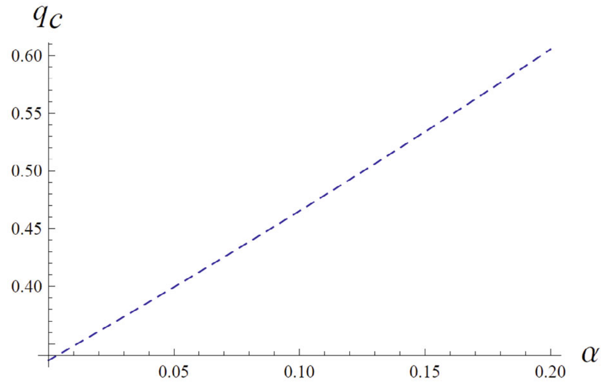

Fig. 1. (Color online) The $A = 0$ graph which represents the variation of q_c with α, where q_c is the critical value of q above or below which positive or negative shock structures are formed.

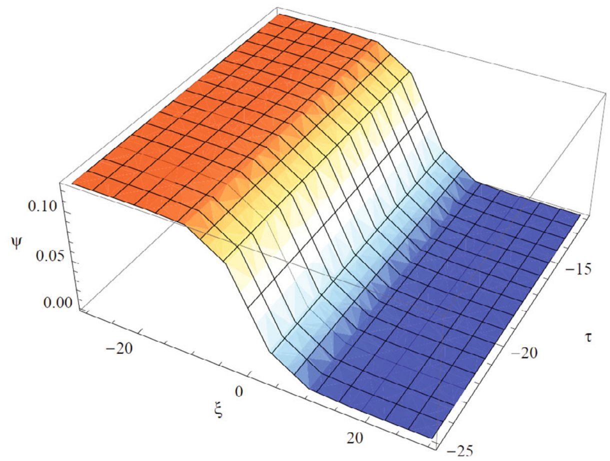

Fig. 2. (Color online) Numerical solution of Eq. (13) in a planar geometry ($\nu = 0$) for a positive potential shock profile with $q = 0.9$, $\mu_p = 0.3$, $\mu_e = 0.6$, $\alpha = 0.3$, $\sigma = 0.1$, $\eta = 0.1$, and $U_0 = 0.01$.

for the description of long-wavelength phenomena. According to this method, the independent variables are stretched as

$$\xi = \epsilon(r - V_p t), \quad \tau = \epsilon^2 t, \tag{5}$$

where ϵ is a smallness parameter measuring the weakness of the dispersion ($0 < \epsilon < 1$) and V_p is the phase speed of the IA waves. We can expand the perturbed quantities $n_{i,h}$, u_i, u_h, and ψ asymtotically about the equilibrium values in power series of ϵ as

$$n_{i,h} = 1 + \epsilon n_{i,h}^{(1)} + \epsilon^2 n_{i,h}^{(2)} + \cdots, \tag{6}$$

$$u_i = 0 + \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots, \tag{7}$$

$$u_h = 0 + \epsilon u_h^{(1)} + \epsilon^2 u_h^{(2)} + \cdots, \tag{8}$$

$$\psi = 0 + \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \cdots, \tag{9}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, Eqs. (1)-(4) give

$$u_i^{(1)} = \frac{\psi^{(1)}}{V_p}, \;\; u_h^{(1)} = -\alpha \frac{\psi^{(1)}}{V_p}, \tag{10}$$

$$n_i^{(1)} = \frac{\psi^{(1)}}{V_p^2}, \;\; n_h^{(1)} = -\alpha \frac{\psi^{(1)}}{V_p^2}, \tag{11}$$

$$V_p = \sqrt{\frac{2(1 + \alpha - \alpha\mu_e + \alpha\mu_p)}{(q+1)(\mu_e + \mu_p\sigma)}}. \tag{12}$$

Equation (12) represents the linear dispersion relation for the IA waves significantly modified by the electron and the positron nonextensivity. To the next higher order of ϵ, *i.e.*, taking the coefficients of ϵ^3 from both sides of Eqs. (1)−(3) and ϵ^2 from both sides of Eq. (4), one may obtain another set of simultaneous equations for $\psi^{(1)} = \psi$, $\psi^{(2)}$, $n_i,_h^{(2)}$, $u_i^{(2)}$, and $u_h^{(2)}$. Invoking this different set of equations which contain coupled sets of parameters viz. as the first-order and second-order perturbed quantities of the ion density, as well as the ion fluid speed, and characterizing wave potential, we finally deduce the cylindrical and the spherical MB equation for the propagation of IA waves in the considered plasma system as

$$\frac{\partial \psi}{\partial \tau} + \frac{\nu}{2\tau}\psi + A\psi\frac{\partial \psi}{\partial \xi} = B\frac{\partial^2 \psi}{\partial \xi^2}, \tag{13}$$

where the nonlinear coefficient A and the dissipative coefficient B are given by

$$A = \frac{V_p^3}{2(\alpha - \alpha\mu_e + \alpha\mu_p + 1)}\left[\frac{3}{V_p^4} - \frac{3\alpha^2}{V_p^4}(1 - \mu_e + \mu_p) \right.$$
$$\left. \times \frac{1}{4}(q+1)(q-3)(\mu_e - \mu_p\sigma^2)\right], \tag{14}$$

$$B = \frac{\eta}{2}. \tag{15}$$

Equation (13) is the MB equation modified by an extra term (viz. $\frac{\nu}{2\tau}\psi$) arising due to the effect of the nonplanar cylindrical ($\nu = 1$) or spherical ($\nu = 2$) geometry.

An exact analytic solution of Eq. (13) is not possible. Therefore, we have numerically solved Eq. (13) and have studied the effects of cylindrical ($\nu = 1$) and spherical ($\nu = 2$) geometries on the time-dependent IASHWs in the presence of nonextensive electrons and nonextensive positrons. We have already mentioned that $\nu = 0$ corresponds to a $1D$ planar geometry which reduces Eq. (13) to a standard Burgers equation. Obviously from Eq. (13), the nonplanar geometrical effect is important when $\tau \to 0$ and weaker for larger values of $\mid \tau \mid$. At first, we consider a $1D$ planar geometry ($\nu = 0$) and examine the basic features of the shock wave solution of the MB equation. The stationary shock wave solution of Eq. (13) in a planar geometry ($\nu = 0$) without the term ($\frac{\nu}{2\tau}\psi$) is

$$\psi(\nu = 0) = \psi_m[1 - \tanh(\xi/\Delta)], \tag{16}$$

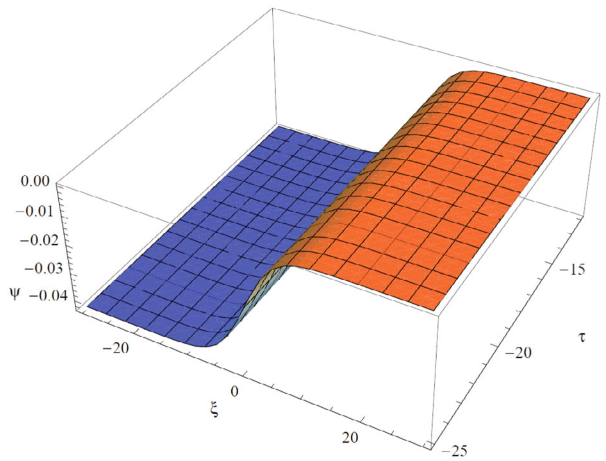

Fig. 3. (Color online) Numerical solution of Eq. (13) in a planar geometry ($\nu = 0$) for a negative potential shock profile with $q = 0.4$, $\mu_p = 0.3$, $\mu_e = 0.6$, $\alpha = 0.3$, $\sigma = 0.1$, $\eta = 0.1$, and $U_0 = 0.01$.

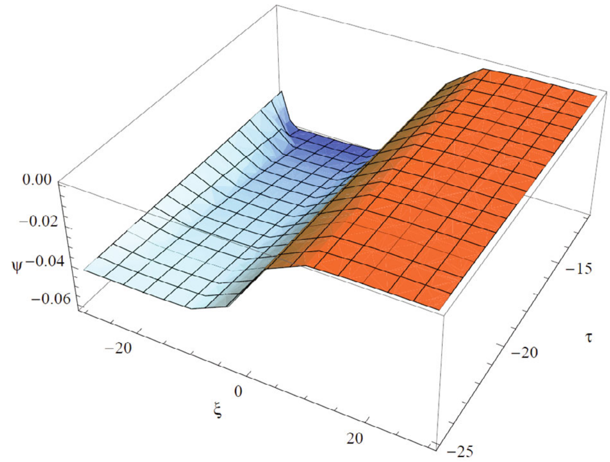

Fig. 5. (Color online) Numerical solution of Eq. (13) in a cylindrical geometry ($\nu = 1$) for a negative potential shock profile with $q = 0.4$, $\mu_p = 0.3$, $\mu_e = 0.6$, $\alpha = 0.3$, $\sigma = 0.1$, $\eta = 0.1$, and $U_0 = 0.01$.

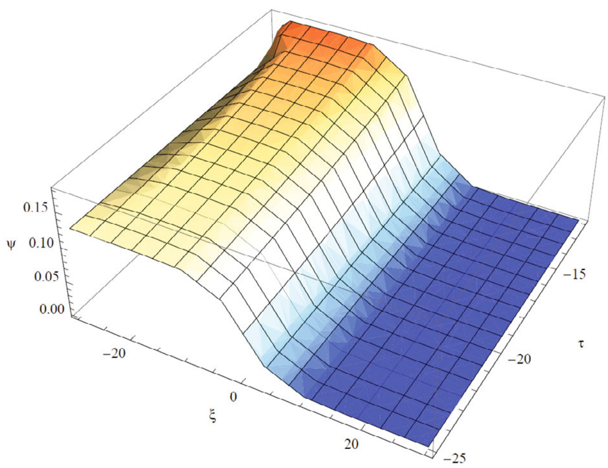

Fig. 4. (Color online) Numerical solution of Eq. (13) in a cylindrical geometry ($\nu = 1$) for a positive potential shock profile with $q = 0.9$, $\mu_p = 0.3$, $\mu_e = 0.6$, $\alpha = 0.3$, $\sigma = 0.1$, $\eta = 0.1$, and $U_0 = 0.01$.

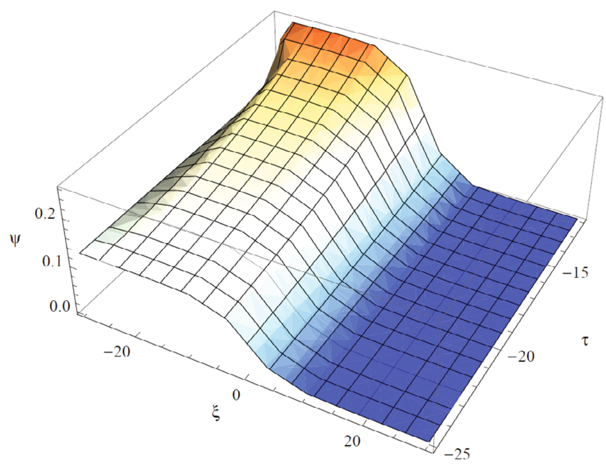

Fig. 6. (Color online) Numerical solution of Eq. (13) in a spherical geometry ($\nu = 2$) for a positive potential shock profile with $q = 0.9$, $\mu_p = 0.3$, $\mu_e = 0.6$, $\alpha = 0.3$, $\sigma = 0.1$, $\eta = 0.1$, and $U_0 = 0.01$.

where the shock wave's amplitude $\psi_m = U_0/A$, and its width $\Delta = 2B/U_0$. Obvious from $\psi_m = U_0/A$, $\psi_m \to \infty$ as $A \to 0$. This means that our theory is not valid when $A \sim 0$, which makes the amplitude extremely large and breaks down the validity of the reductive perturbation method. We note here that the nonlinearity coefficient A is a function of μ_e, μ_p, σ, α, and q. There, the parametric regions corresponding to $A = 0$, we have to express one (viz. q_c) of these parameters in terms of the other (viz. μ_e, μ_p, α, and σ). Therefore, A leads to the critical value of q (long expression $\to$ omitted here), where q_c is the critical value of q above (below) which the SHWs with a positive (negative) potential exists. We have found numerically the critical value of q ($q = q_c = 0.6$) for $\mu_p = 0.3$, $\mu_e = 0.6$, $\sigma = 0.1$, and $\alpha = 0.2$. The parametric region for this set of values is shown in Fig. 1. We choose our initial pulse at $\tau = -25$ and have observed that for a large value of time, $\tau = -12$, the cylindrical and the spherical SHWs are similar to $1D$ SHWs.

The shock structures are depicted in Figs. (2)–(7), which show how the effects of planar ($\nu = 0$), cylindrical ($\nu = 1$), and spherical ($\nu = 2$) geometries modify the time-dependent IA shock structures. Figures 2, 4, and 6 show the positive potential shock profiles with τ and ξ, respectively, for planar ($\nu = 0$), cylindrical ($\nu = 1$), and spherical ($\nu = 2$) geometries with $q = 0.9$, $\mu_p = 0.3$, $\mu_e = 0.6$, $\alpha = 0.3$, $\sigma = 0.1$, $\eta = 0.1$, and $U_0 = 0.01$. Figures 3, 5, and 7 show the negative potential shock profile with τ and ξ, respectively, for planar ($\nu = 0$), cylindrical ($\nu = 1$), and spherical ($\nu = 2$) geometries with $q = 0.4$, $\mu_p = 0.3$, $\mu_e = 0.6$, $\alpha = 0.3$, $\sigma = 0.1$, $\eta = 0.1$, and $U_0 = 0.01$. Figure 8 shows the variation of

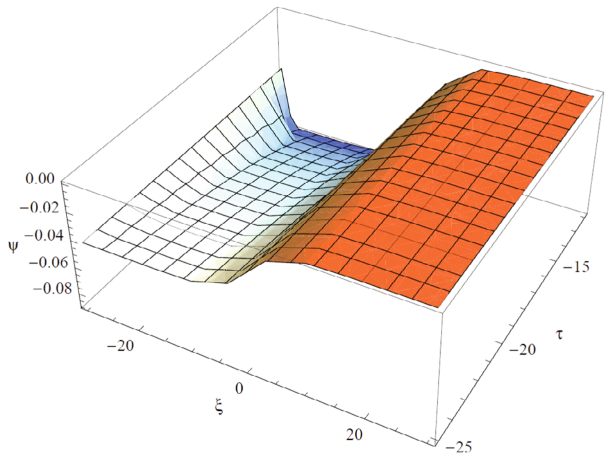

Fig. 7. (Color online) Numerical solution of Eq. (13) in a spherical geometry ($\nu = 2$) for a negative potential shock profile with $q = 0.4$, $\mu_p = 0.3$, $\mu_e = 0.6$, $\alpha = 0.3$, $\sigma = 0.1$, $\eta = 0.1$, and $U_0 = 0.01$.

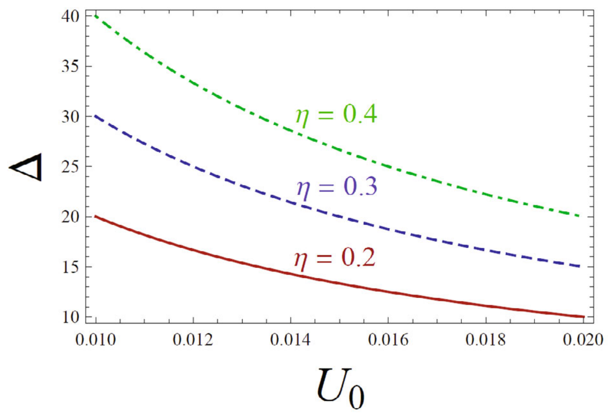

Fig. 8. (Color online) Variation of the shock's width (△) with η and U_0.

the width (△) of the SHWs with the kinematic viscosity (η).

IV. DISCUSSION

We have considered a four-component plasma system (consisting of light positive ions, heavy negative ions, nonextensive electrons, and nonextensive positrons) and studied the effects of the nonextensivity of electrons and positrons on the IASHWS in nonplanar geometries. We have derived the modified Burgers equation by using the reductive perturbation method and have numerically analyzed that equation. The results that have been found from our investigation can be summarized as follows:

1. The nonextensive plasmas under consideration support finite-amplitude shock structures, whose basic features (viz. polarity, amplitude, width, *etc.*) strongly depend on different plasma parameters, particularly, the electron-number-density to ion-number-density ratio (via μ_e), the positron-number-density to ion-number-density ratio (via μ_p), the electron-temperature to positron-temperature ratio (via σ), the ion's kinematic viscosity η, and the nonextensive index q.
2. The critical value of q, *i.e.*, q_c, is found to be 0.6 for $\mu_p = 0.3$, $\mu_e = 0.6$, $\sigma = 0.1$, and $\alpha = 0.2$. The variation of q_c with α is shown in Fig. 1.
3. IASHWs with a positive potential exist for $q > q_c$ while those with a negative potential exist for $q < q_c$. These are obvious from Figs. 2−7.
4. The time evolution of the nonplanar IASHWs is observed to differ from that of the $1D$ planar IASHWs. The characteristics of the SHWs are also found to be influenced by time for both cylindrical and spherical cases.
5. Equation (13) shows that $\frac{\nu}{2\tau}\psi$ goes to infinity when $\tau \to 0$. Therefore, this term is singular at $\tau = 0$. For large values of τ, this term vanishes, and we have the usual Burgers equation. In the direction of time, we can start from a sufficient large τ (like $\tau = -25$) where the term $\frac{\nu}{2\tau}\psi$ is negligible. Obviously from Eq. (13), the nonplanar geometrical effect is important when $\tau \to 0$ and weaker for larger value of τ.
6. The numerical solutions of Eq. (13) reveal that for a large value of τ (*e.g.*, $\tau = 15$), the planar and the nonplanar IASHWs are identical, but the amplitudes of both cylindrical and spherical IASHWs increase with decreasing of the value of τ. However, as τ decreases, the term $\frac{\nu}{2\tau}\psi$ becomes dominant, and cylindrical and spherical SHWs differ from $1D$ planar ones. The amplitudes of cylindrical IASHWs are found to be larger than those of $1D$ planar ones, but smaller than those of spherical ones. The amplitudes of both cylindrical and spherical IASHWs increase with decreasing τ (displayed in Figs. 4-7).
7. The height and the steepness of cylindrical shock structures are larger than those of $1D$ shock structures, but smaller than those of spherical shock structures. (depicted in Figs. 2−7).
8. Figure 8 shows the variation of the width (△) with U_0 for different values of η, where △ increases with the increasing η and decreases with increasing of U_0.

The important findings of our results are applicable in various astrophysical objects like quasars, pulsars, and active galactic nuclei [7], which contain e-p-i plasmas (for example, in the form of jets) in their vicinities and may lead to the form stable shock structures

[47]. We finally emphasize that the results of our investigation should be useful in understanding the nonlinear features of localized electrostatic disturbances in laboratory and space plasmas, in which positively-charged and negatively-charged ions, as well as nonextensive electrons and nonextensive positrons are the major plasma species.

ACKNOWLEDGMENTS

N. Jannat and M. Ferdousi are grateful to the Bangladesh Minstry of Science and Technology for awarding the National Science and Technology (NST) Fellowship.

REFERENCES

[1] L. Ding and C. Yinhua, *Plasma physics* (Higher Education Press, Beijing, 2006).
[2] X. Jialuan and J. Shangxian, *Plasma physics* (Nuclear Energy Press, Beijing, 1981).
[3] L. Tonks and I. Langmuir, Phys. Rev. **33**, 195 (1929).
[4] R. W. Revans, Phys. Rev. **44**, 798 (1933).
[5] B. D. Fried and R. W. Gould, Phys. Fluids **4**, 139 (1961).
[6] M. C. Begelman, R. D. Blanford and M. J. Rees, Rev. Mod. Phys. **56**, 255 (1984).
[7] H. R. Miller and P. J. Witta, *Active Galactic Nuclei* (Springer, Berlin, 1987).
[8] M. Tribeche, K. Aoutou, S. Younsi and R. Amour, Phys. Plasmas **16**, 072103 (2009).
[9] S. I. Popel, S. V. Vladimirov and P. K. Shukla, Phys. Plasmas **2**, 716 (1995).
[10] P. K. Shukla, J. T. Mendonca and R. Bingham, Phys. Scr. **113**, 133 (2004).
[11] S. Mahmood and H. Ur-Rehman, Phys. Lett. A **373**, 2255 (2009).
[12] F. B. Rizatto, J. Plasma Phys. **40**, 288 (1988).
[13] M. Y. Yu, Astrophys. Space Sci. **177**, 203 (1985).
[14] Y. N. Nejoh, Phys. Plasmas **3**, 1447 (1996).
[15] F. Verheest, M. A. Hellberg, G. J. Gary and R. L. Mace, Astrophys. Space Sci. **239**, 125 (1995).
[16] A. Mushtaq and H. A. Shah, Phys. Plasmas **12**, 072306 (2005).
[17] M. Ferdousi, S. Yasmin, S. Ashraf and A. A. Mamun, Chin. Phys. Lett. **32**, 015201 (2015).
[18] M. Ferdousi, S. Yasmin, S. Ashraf and A. A. Mamun, IEEE Trans. Plasma Sci. **43**, 643 (2015).
[19] P. K. Shukla and M. Rosenberg, Phys. Plasmas **6**, 1038 (1999).
[20] W. M. Moslem, I. Kourakis, P. K. Shukla and R. Schlickeiser, Phys. Plasmas **14**, 102901 (2007).
[21] W. Masood, N. Jehan, M. A. Mirza and P. H. Sakanaka, Phys. Lett. A **372**, 4279 (2008).
[22] B. Sahu and R. Roychoudhury, Phys. Plasmas **12**, 052106 (2005).
[23] N. Jehan, S. Mahmood and A. M. Mirza, Phys. Scr. **76**, 661 (2007).
[24] R. Sabry, W. M. Moslem and P. K. Shukla, Eur. Phys. J. D **51**, 233 (2009).
[25] D. A. Mendis and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32**, 419 (1994).
[26] P. K. Shukla and A. A. Mamun, *Introduction to dusty plasma physics* (IOP Publishing Ltd, Bristol, 2002).
[27] V. N. Tsytovich, G. E. Morfill and H. Thomas, Plasma Phys. Rep. **28**, 623 (2002).
[28] A. Barkan, R. L. Merlino and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[29] A. Homann, A. Melzer, S. Peters and A. Piel, Phys. Rev. E **56**, 7138 (1997).
[30] V. E. Fortov, A. V. Ivlev, S. A. Khrapak, A. G. Khrapak and G. E. Morfill, Phys. Rep. **421**, 1 (2005).
[31] R. A. Gottscho and C. E. Gaebe, IEEE Trans. Plasma Sci. **14**, 92 (1986).
[32] M. Bacaland and G. W. Hamilton, Phys. Rev. Lett. **14**, 1538 (1979).
[33] J. Jacquinot, B. D. Mcvey and J. E. Scharer, Phys. Rev. Lett. **39**, 88 (1977).
[34] H. Massey, *Negative Ions* (Cambridge University Press, Cambridge, 1976).
[35] B. A. Millar, J. Blevitt, L. Gerace, K. Sadhu, C. Featherstone and P. Russell, Natl. Acad. Sci. **88**, 349 (1991).
[36] C. Tsallis, J. Stat. Phys. **52**, 479 (1988).
[37] A. Lavagno and D. Pigato, Eur. Phys. J. A **47**, 52 (2011).
[38] A. R. Plastino and A. Plastino, Phys. Lett. A **174**, 384 (1993).
[39] G. Gervino, A. Lavagno and D. Pigato, Central Eur. J. Phys. **10**, 594 (2012).
[40] C. Feron and J. Hjorth, Phys. Rev. E **77**, 022106 (2008).
[41] M. Ferdousi, S. Yasmin, S. Ashraf and A. A. Mamun, Astrophys. Space Sci. **352**, 579 (2014).
[42] M. Ferdousi and A. A. Mamun, Braz. J. Phys. **45**, 89 (2015).
[43] M. Ferdousi, S. Sultana and A. A. Mamun, Phys. Plasmas **22**, 032117 (2015).
[44] M. Ferdousi, *Ion-acoustic Waves in Nonextensive Electron-positron-ion Plasmas* (LAP LAMBERT Academic Publishing, Germany, 2015), ISBN: 978-3-659-67001-5.
[45] H. Washimi and T. Taniuti, Phys. Rev. Lett. **17**, 996 (1966).
[46] T. Taniuti and C. C. Wei, J. Phys. Soc. **24**, 941 (1968).
[47] S. Hussain, H. Ur-Rehman and S. Mahmood, Astrophys. Space Sci. **351**, 573 (2014).