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# Detailed Study of the TE Band Structure of Two Dimensional Metallic Photonic Crystals with Square Symmetry

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By virtue of the efficiency of the Dirichlet-to-Neumann map method, we have calculated, for H-polarization (TE mode), the band structure of 2D photonic crystals with a square lattice composed of metallic rods embedded in an air background. The rod in the unit cell is chosen to be circular in shape. Here, from a practical point of view, in order to obtain maximum band gaps, we have studied the band structure as a function of the size of the rods. We have also studied the flat bands appearing in the band structures and have shown that for frequencies around the surface plasmon frequency, the modes are highly localized at the interface between the metallic rods and the air background.

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## I. INTRODUCTION

In recent years, the study of periodic dielectric structures called photonic crystals (PCs) has received cosiderable interest because of their ability to prevent the propagation of electromagnetic (EM) waves in a certain frequency range known as the photonic band gap (PBG). PBG crystals enable the manipulation of EM waves in many elaborately designed ways [1–4]. Depending on the constituent materials, PCs can be categorized into two classes. One is dielectric photonic crystals (DPCs), and the second is dispersive photonic crystals such as metallic photonic crystals (MPCs). MPCs are interesting for different applications such as practical filters [5, 6], polarizers [7] or waveguides [8]. The band structure of photonic crystals has been extensively studied for many structures with different geometry of rods [9–12]. Several approaches such as the plane wave method (PWM) [13–15], the transfer matrix method (TMM) [16] and the finite difference time domain (FDTD) method [17–19], have been employed to calculate the photonic band structures for 2D and 3D DPCs. The PWM method is quite useful and is highly efficient for calculating the photonic band structure of PCs. In this method, the material is assumed to be independent of the frequency and an eigenvalue problem is formulated to solve for the eigenfrequencies of the given wave vector in the irreducible Brillouin zone. For dispersive materials, because of the frequency dependence of the dielectric constant, the characteristic matrix is frequency dependent, which consequently renders the conventional PWM difficult to handle. However, because of the convergency problem, calculating the H-polarization band structure of MPCs by using this method is impossible. Several theoretical methods, such as the revised plane wave method (RPWM) [20,21], the Korringa-Kohn-Rostoker (KKR) method [22,23], the multiple scattering method (MSM) [24,25] and the multiple multipole (MMP) method [26], have been employed to study the photonic band structures of dispersive PCs. More recently, the efficient Dirichlet-to-Neumann (DtN) map method, which is a powerful tool for calculating the band structure of photonic crystals composed of dispersive materials [27,28], especially for H-polarization, has been proposed. The DtN map is an operator that maps the wave field on the boundry of a unit cell to its normal derivative there. The main advantages of this method are as follows: (i) Eigenvalue problems contain relatively small matrices. (ii) Unlike other methods based on cylin-

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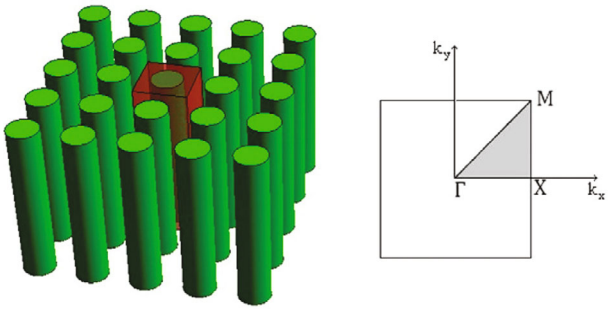


Fig. 1. (Color online) Two-dimensional photonic crystal with a square lattice of circular rods and the corresponding first Brillouin zone.

drical wave expansions, such as the KKR method, sophisticated lattice sum techniques are not needed.

In this paper, we consider 2D PCs with a square lattice composed of circular rods embedded in an air background. Here, the rod material is selected to be metallic so that it is dispersive and has a frequency-dependent dielectric constant. Because of the importance of the PBG in filtering and waveguide applications, we have studied how the size of the metallic material can affect the band gap properties for H-polarization. This work has already been done for E-polarization. Moreover, we discuss the properties of the flat bands appearing in the H-polarization band structures. The most significant feature of H-polarization is the existence of surface plasmon modes that are highly localized around the interface between the metal and the surrounding dielectric.

## II. MODEL AND METHOD OF CALCULATION

We consider a two-dimensional photonic crystal composed of circular metallic rods (parallel to the $z$-axis) with a dielectric constant $\varepsilon_a$ obeying the Drude model [29] in an air background, $\varepsilon_b = 1$. The geometry of the system is illustrated in Fig. 1. For in-plane propagation of an EM wave in 2D PCs, the H-polarization (the transverse electric (TE)) and the E-polarization (the transverse magnetic (TM)) modes are known to be independent eigenmodes that can be studied separately. Because the band structure of metallic photonic crystals for E-polarization has been extensively studied before, we here restrict ourselves to the H-polarization.

In order to describe the propagation of electromagnetic waves in a 2D PC, one can use the Helmholtz equation

$$\frac{\partial}{\partial x}\left(\frac{1}{\varepsilon}\frac{\partial \mathbf{H}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\varepsilon}\frac{\partial \mathbf{H}}{\partial y}\right)+k_0^2\mathbf{H}=0. \qquad (1)$$

Here, $k_0 = \omega/c$ is the vacuum wave number, $c$ is the speed of light in vacuum, and $\varepsilon$ is the dielectric constant, which is a periodic function with the periodicity of the lattice constant. For computing the band structure of our system, we use the recently-developed DtN map method, which is a highly efficient approach for calculating the band structure of dispersive photonic crystals [27]. In this method, one needs the matrix form of an operator that maps $\mathbf{H}$ on the boundary of the square unit cell to the normal derivative of $\mathbf{H}$ on the boundary. In our system, this matrix can be efficiently calculated by using the following cylindrical wave expansion:

$$\mathbf{H}(x,y)=\sum_{m=-\infty}^{+\infty} C_m\Phi_m(r,\theta) \quad , \quad \Phi_m(r,\theta)=\phi_m(r)e^{im\theta}, \qquad (2)$$

where $m$ is an integer, $r$ and $\theta$ are polar coordinates, and $\phi_m(r)$ is related to the Bessel functions $J_m$ and $Y_m$ as

$$\phi_m(r)=\begin{cases} A_mJ_m(k_0n_ar) & , \quad r<R, \\ B_mJ_m(k_0n_br)+Y_m(k_0n_br) & , \quad r>R. \end{cases} \qquad (3)$$

Here, $n_a = \sqrt{\varepsilon_a}$ , $n_b = 1$, and $R$ is the radius of the metallic rod. By using the Bloch theorem and applying the boundary conditions of the electromagnetic fields at the interface of the rods, we find that the method leads to the following generalized eigenvalue problem [27]:

$$\lambda\begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} V \\ U \end{bmatrix}+\begin{bmatrix} B & C \\ -I & 0 \end{bmatrix}\begin{bmatrix} V \\ U \end{bmatrix}=0. \qquad (4)$$

Here, the eigenvalue $\lambda$, which is related to the Bloch wave vector, and other quantities are introduced in Ref. 27. For a given frequency $\omega$, this generalized eigenvalue equation can be solved numerically to obtain the photonic band structure.

## III. RESULTS AND DISCUSSION

In this paper, the photonic band structures of 2D PCs for H-polarization are calculated using the Dirichlet-to Neumann map method. The considered photonic crystal with a square lattice is composed of metallic circular rods embedded in an air background. The metal is a dispersive material and has a frequency-dependent dielectric constant. The dielectric constant for a metal is given by the Drude model [29], and the plasma frequency is set to be $\omega_p a/2\pi c = 1$. In a practical point of view, metals with a high electron density and thus a high plasma frequency, such as silver (Ag), can be a proper candidate. The plasma frequency of Ag is $13.6 \times 10^{15} Hz$, and from the normalization relation, lattice constant of the metallic photonic crystal, $a$, is 0.138 $\mu$m. A total of 9 points on each edge of the unit cell were used in the calculations, which ensured sufficient convergence for the frequencies of interest. Our main goal here was to study the modification of the band gap spectrum when the size of the

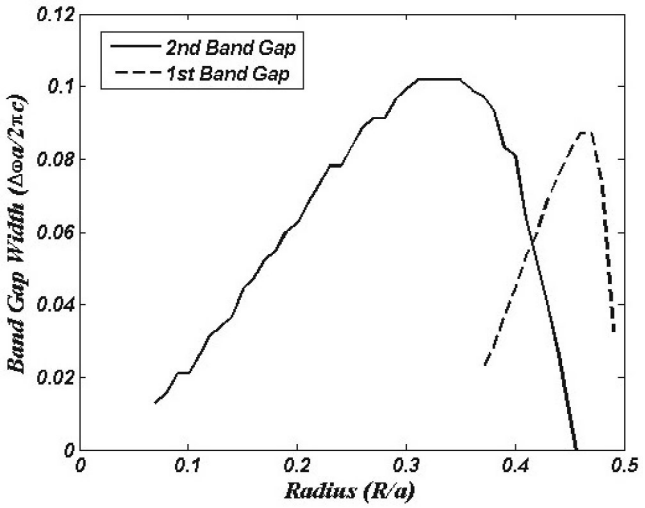


Fig. 2. Variation of the normalized value of band gap width with the radius of the rods, $R/a$.

scatterers was varied. Thus, the radius of the circular rods was treated as an adjustable parameter to obtain the maximum photonic band gap.

Our calculations show that some PBGs exist in the band structures. We have chosen the most important PBGs, which have the largest widths. Figure 2 presents the normalized width of these PBGs as a function of the size of the rods. This figure shows that the maximum width of the first PBG has a magnitude of $\triangle\omega = 0.0872(2\pi c/a)$ at $R = 0.46a$ and the second PBG has a magnitude of $\triangle\omega = 0.1016(2\pi c/a)$ at $R = 0.35a$. The photonic band structures of these optimum PBGs are displayed in Fig. 3. We consider Fig. 3(b) to be the photonic band structure with the largest PBG. This frequency spectrum shows that the PBG lies between frequencies 0.8663 and 0.9653 in units of $(2\pi c/a)$.

As this figure shows, in addition to the PBG, there is a flat band region. These flat bands, which disappear in the case of the E-polarization mode, are located in the vicinity of $\omega a/2\pi c = 0.7071$, known as the surface plasmon frequency $\omega_{sp}$. In this frequency range, the dielectric constant of the rods approaches to minus one ($\varepsilon_a = -1$). When the surface plasmon frequency $\omega_{sp}$ is approached, the number of flat bands is increased, and a dense region is formed. To provide more insight, we illustrate the field distributions of the two surface plasmon modes in Fig. 4. Clearly, the **H** field distributions are highly localized around the interface between the metallic rods and the air background, and when the frequencies of the modes approach the surface plasmon frequency, the localization length (decay length along the normal to the interface) of these modes decreases and the number of field nodes increases.

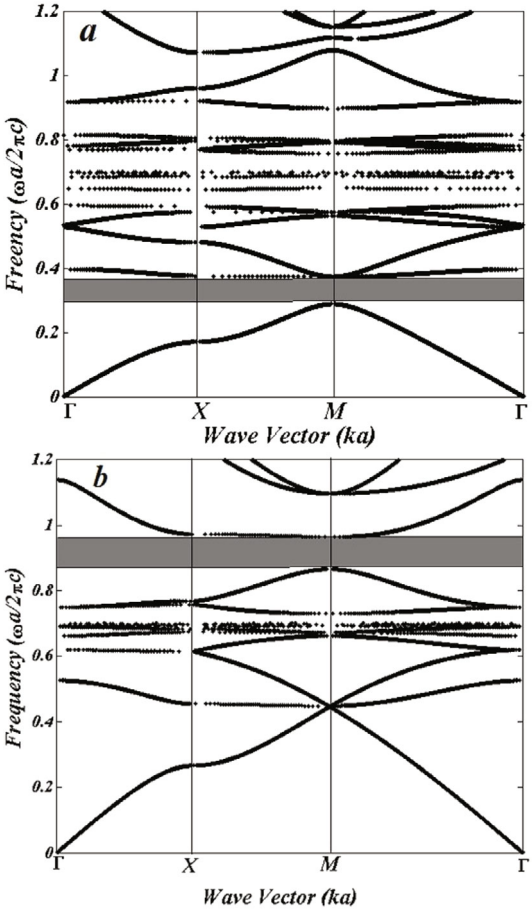


Fig. 3. Photonic band structures for H polarization at optimum values of (a) $R = 0.46a$ and (b) $R = 0.35a$.

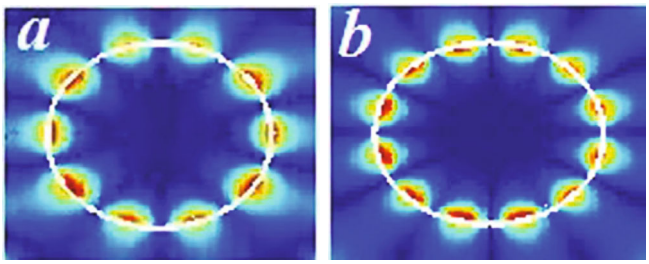


Fig. 4. (Color online) **H** field distribution of surface plasmon modes with frequencies (a) 0.6835 and (b) 0.6931 in units of $2\pi c/a$ at the $X$ and the $M$ points, respectively.

## IV. CONCLUSIONS

In conclusion, we have used the Dirichlet-to-Neumann method to perform a detailed numerical analysis of the photonic band structures of 2D MPCs with a square lattice composed of metallic rods embedded in an air background. The rods have circular cross sections. In our calculations, H-polarization is considered. We have investigated how the size of scatterers can affect the photonic band structures. We have also demonstrated the

existence of two major PBGs. Among these PBGs, the largest PBG has a magnitude of $\triangle\omega = 0.1016(2\pi c/a)$ at $R = 0.35a$. This result could be useful in designing 2D photonic crystals with large PBGs in frequency ranges of interest. We have also studied the flat band region appearing in the band structure corresponding to the maximum PBG. We have shown that the field distributions are highly localized around the interface between the metallic rods and the air background, and when the frequency of the modes approaches the surface plasmon frequency, the localization length (decay length along the normal to the interface) of these modes decreases and the number of field nodes increases.

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