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Optimal Design of Multiple Stresses Accelerated Life Test Plan Based on Transforming the Multiple Stresses to Single Stress

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Abstract: For planning optimum multiple stresses accelerated life test plans, a commonly followed guiding principle is that all parameters of the life-stress relationship should be estimated, and the number of the stress level combinations must be no less than the number of parameters of the life-stress relationship. However, the general objective of an accelerated life test(ALT) is to assess the *p*-th quantile of the product life distribution under normal stress. For this objective, estimating all model parameters is not necessary, and this will increase the cost of test. Based on the theoretical conclusion that the stress level combinations of the optimum multiple stresses ALT plan locate on a straight line through the origin of coordinate, it is proposed that a design idea of planning the optimum multiple stresses ALT plan through transforming the problem of designing an optimum multiple stresses ALT plan to designing an optimum single stress ALT plan. Moreover, a method of planning the optimum multiple stresses ALT plan which can avoid estimating all model parameters is established. An example shows that, the proposed plan which only has two stress level combinations could achieve an accuracy no less than the traditional plan, and save the test time and cost on one stress level combination at least; when the actual product life is less than the design value, even the deviation of the model initial parameters value is up to 20%, the variance of the estimation of the *p*-th quantile of the proposed plan is still smaller than the traditional plans approximately 25%. A design method is provided for planning the optimum multiple stresses ALT which uses the statistical optimum degenerate test plan as the optimum multiple stresses accelerated life test plan.

Keywords: multiple stresses, accelerated testing, optimum design, test planning

1 Introduction

Constant stress accelerated life test(CSALT) is the most commonly used method to assess the product life rapidly in engineering. In practice, most products work in a complex environment, and their failures are caused by the combined effects of several stresses, such as temperature, humidity and vibration. In order to make the test environment more consistent with the actual environment and to increase the credibility of life assessment, the multiple constant stresses accelerated life test(MCSALT), which uses more than one accelerating factor, is increasingly used widely in engineering with the development of the environment simulation technology.

Designing the optimum test plan is important to improve the testing efficiency, as it can improve the estimation accuracy, shorten the test time, and reduce the sample size.

On the design of single constant stress accelerated life test(SCSALT), CHERNOFF[1] made a widely discussion on the cases that the product life followed exponential distribution and the life-stress relationship followed linear function, quadratic function, and multiple linear function respectively. He provided a theoretical frame of the optimal design of ALT plan and pointed out that the optimum plan is depended on the initial estimation of the model parameters, which phenomenon is caused by the censored data, so that experimenters should use plans which are not sensitive to the deviation of the model parameters in practice. Subsequently, focusing on Weibull distribution, lognormal distribution and the linear life-stress relationship which were the most commonly used models in engineering, and taking minimizing the asymptotic variance of the maximum likelihood estimate(MLE) of *p*-th quantile of the product life distribution under normal stress as the optimization goal(called V-optimality), NELSON and MEEKER, et al[2–7], provided the laws that the stress levels, the sample location ratios and the asymptotic variance of the statistical optimum plan vary with the model parameters; and they proposed the method that

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introducing into the optimization model the constraints on the number of stress levels, the sample location ratios, the space of stress levels, and the expected number of units failing on some stress levels, for obtaining the compromise plans which were less sensitive to the deviation of the model parameters than the statistical optimum plan. Furthermore, they provided a method for a comprehensive comparison of various optimum plans on the bases of the estimation precise and the robustness through computer experiment. These studies have been considered as the "standard procedure" of the studies on optimal design of ALT plan. Thereafter, the studies mainly pointed to two directions.

(1) Extended the design idea proposed by NELSON and MEEKER to other models, such as the ALT under periodic inspection[8–11], Rayleigh distribution[10], Burry Type Ⅻ distribution[11], the Weibull distribution with nonconstant scale parameter[12], or to provide other compromise plans which maybe have better combination property[13–14]. In recent years, with the improvement of the relevant studies, the research focus gradually shifted to step stress ALT, progressive stress ALT and accelerated degradation test[15–16].

(2) Sought for methods different from NELSON and MEEKER's for designing test plans which had good estimation precise and robustness both, such as given the model parameters by interval estimation[17] or prior distribution[18], used the optimization objective function which included the effect of the misspecification of the distribution or the life-stress relationship[19–22], or developed the sequential ALT[23–24]. These methods can obtain better plans, but have not yet been widely used in engineering because of their complex theory.

On the design of optimum MCSALT plan, there are mainly two special problems caused by the increasing in the number of stress: 1) One must determine the position of the stress level combinations(called test points) in the test region; 2) Sometimes the stresses could not reach the highest stress levels simultaneously, and the test region becomes a non-rectangle. For the first problem, ESCOBAR, et al[25], demonstrated that for the linear-extreme value model and the type-I censored CSALT, the V-optimality test plans are not unique, if not restricting the position of the test points in the test region, all test points maybe locate on a straight line through the origin of coordinate(called the degenerate test plan, as shown in Fig. 1). One can not get all parameters of the life-stress relationship only depending on the life data obtained from a degenerate test plan. For obtaining all parameters, three types of methods for arranging the test points were proposed mainly: arranging the stress level combinations on the intersection points of the life-stress relationship contours and the test region boundary(called E-M method)[25], arranging the stress level combinations based on the orthogonal design[26–27], and arranging the stress level combinations based on the uniform design[28–29]. GAO, et al[30], made a comparison of these three methods on the bases of the estimation precision of the p-th quantile of life distribution under normal stress, the robustness of optimal test plans to misspecified model parameters and the estimation precise of model parameters through computer experiments, and pointed out that the test plans obtained from E-M method are the best. For the second problem, ESCOBAR, et al[25], presented an optimal design method for a special non-rectangle test region which is formed by the upper right corner of a rectangle test region being truncated by the life-stress relationship contour. And then, CHEN, et al[31], extended the E-M method to the design of the optimum MCSALT plan on the non-rectangle test region with arbitrary boundary.

The methods mentioned above follow a default guiding principle that one must assess all parameters of the life-stress relationship depending on the test data. As a result, the number of test points can not be less than three, and the test points can not be collinear. However, if the objective of an ALT is to assess the p-th quantile of the product life distribution under normal stress, to obtain all model parameters is not necessary, and will increase the cost of test. From this point of view, based on the properties demonstrated in Refs. [25, 31], this paper proposes a new method for designing the optimum MCSALT plan on the test region with arbitrary boundaries by transforming the problem of designing the optimum MCSALT plan to designing the optimum SCSALT plan.

2 Model Assumptions and Standardization

2.1 Model assumptions

(1) For each stress level combination in the test region Ω, the natural logarithm of product life θ follows extreme value distribution(the product life follows Weibull distribution), and the probability function is

$$F(\theta) = 1 - \exp\{-\exp[(\theta - \mu) / \sigma]\}, \tag{1}$$

where μ is the location parameter, σ is the scale parameter.

(2) On the test region, the relationship of the location parameter μ and the stresses(or the transformed stresses) x_1, x_2 satisfies

$$\mu(x_1, x_2) = a + bx_1 + cx_2,\ (x_1, x_2) \in \Omega,\ b, c < 0. \tag{2}$$

(3) The scale parameter σ is a constant, which is independent with x_1 and x_2.

(4) The lifetimes of the test units are s-independent.

(5) The type-I censored MCSALT is considered here, and the censoring time of each stress level combination is τ.

The statistical model of most electromechanical products can be transformed to the above linear-extreme value model, and the examples are shown in Refs. [28–31].

2.2 Standardization

In order to simplify the problem and make the result more general, x_1 and x_2 should be transformed to the standardized stresses ξ_1 and ξ_2 as follows[25]:

$$\xi_1=\frac{x_1-x_{1,0}}{x_{1,\mathrm{H}}-x_{1,0}};\ \ \xi_2=\frac{x_2-x_{2,0}}{x_{2,\mathrm{H}}-x_{2,0}}, \tag{3}$$

where $x_{1,0}$ and $x_{2,0}$ are the normal levels of the stresses x_1 and x_2 respectively; $x_{1,\mathrm{H}}$ and $x_{2,\mathrm{H}}$ are the highest levels of x_1 and x_2 respectively in the test. Substitute Eq. (3) into Eq. (2), the standardized life-stress relationship is obtained:

$$\mu(\xi_1,\xi_2)=\gamma_0+\gamma_1\xi_1+\gamma_2\xi_2\ ,\ \xi_1,\xi_2\in[0,\ 1], \tag{4}$$

where

$$\gamma_0=a+bx_{1,\,0}+cx_{2,\,0};\gamma_1=b(x_{1,\,\mathrm{H}}-x_{1,\,0});$$
$$\gamma_2=c(x_{2,\,\mathrm{H}}-x_{2,\,0}). \tag{5}$$

Discussions below are based on the standardized model.

3 Method for Planning Optimum MCSALT Plan

3.1 Theoretical basis of the method

As shown in Fig. 1(a), in the Cartesian coordinates system $\xi_1O\xi_2$ with the standardized stresses ξ_1 and ξ_2 as the axes, the coordinates of points A and B are (1, 0) and (0, 1) respectively, the boundary $\partial\Omega$ of the non-rectangular test region Ω contain the segments OA, OB and an arbitrary curve S_{AB}[31]. The design method proposed in this paper is based on following property:

Property: For the V-optimality, the optimum test plans are not unique[25]. One of the optimum test plans is the optimum degenerate test plan[25]. In the optimum degenerate test plan, all the test points locate on a straight line l^* through the origin of the coordinate system $\xi_1O\xi_2$[25], another point on the line l^* is the point M which is on the test region boundary $\partial\Omega$ and has the maximum failure probability until censored time τ[31].

3.2 Design idea of the method

The objective of an ALT is to assess the p-th quantile y_p of the product life distribution under normal stress, and the relationship of the MLE of y_p and the model parameters is

$$\hat{y}_p=\hat{\gamma}_0+z_p\hat{\sigma}, \tag{6}$$

where z_p is the p-th quantile of the standard extreme value distribution, $\hat{y}_p$, $\hat{\gamma}_0$ and $\hat{\sigma}$ are the MLE of y_p, γ_0 and σ respectively.

According to Eq. (6), if $\hat{\gamma}_0$ and $\hat{\sigma}$ are estimated, then $\hat{y}_p$ can be obtained easily. According to the "property", based on the geometric characteristics of the optimum degenerate test plan, one can get $\hat{\gamma}_0$ without estimating all parameters of the life-stress relationship. The design idea is as follows.

As shown in Fig. 1(b), in the Cartesian coordinates system $O\xi_1\xi_2\mu$ with the location parameter μ and the standardized stresses ξ_1, ξ_2 as the axes, the equation of the plane Π is the life-stress relationship shown in Eq. (4). Given the initial assessed values of γ_0, γ_1, γ_2 and the boundary of test region Ω, according to the "property", the line l^* which the stress level combinations of the optimum degenerate test plan local on can be determined. According to Eq. (4), when test point (ξ_1, ξ_2) moves along the line l^*, the locus of point (ξ_1, ξ_2, $\mu(\xi_1,\ \xi_2)$) is the line L^* which belongs to the plane Π. Obviously, the point (0, 0, μ(0, 0)) belongs to the line L^* and μ(0, 0) equals to γ_0. Therefore, one only needs to proceed the test according to the degenerate test plan and assess the parameters of line L^*, then $\hat{\gamma}_0$ could be get, and y_p could be assessed by Eq. (6). The asymptotic variance of $\hat{y}_p$ can be minimized by adopting the optimum degenerate test plan. As described above, the problem of designing the optimum MCSALT plan on the test region Ω is transformed into the problem of designing the optimum SCSALT plan on the line l^*.

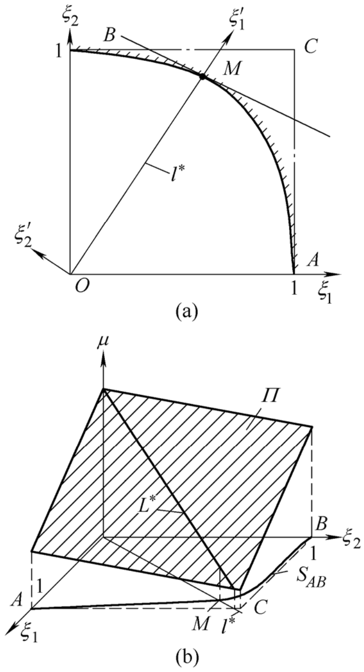

Fig. 1. Schematic diagram of the design method

3.3 Specific steps of the method

Firstly, determine the line l^* which the stress level combinations of the optimum degenerate test plan local on. The key point is to determine the point M which locates on the test region boundary $\partial\Omega$ and has the maximum failure probability until censored time τ. According to the shape of the curve S_{AB}, this problem can be divided into four cases, and Ref. [31] proposes the specific method for solving it.

Secondly, transform the problem of designing the optimum MCSALT plan on the test region Ω into designing the optimum SCSALT plan on the line l^*.

As shown in Fig. 1, define the coordinate of the point M as (ξ_{1M}, ξ_{2M}) and the slope of the line l^* as s_M, rotate the coordinate system $O\xi_1\xi_2\mu$ around the axis $O\mu$ with the angle $\theta = \arctan s_M$. That make $O\xi_1$ coincide with l^* and form a new coordinate system $O\xi_1'\xi_2'\mu$.

The equation of the coordinate transformation is

$$\begin{cases}\xi_1 = \xi_1'\cos\theta - \xi_2'\sin\theta,\\ \xi_2 = \xi_1'\sin\theta + \xi_2'\cos\theta,\end{cases}$$

in $O\xi_1'\xi_2'\mu$, the equation of the plane Π is

$$\mu(\xi_1', \xi_2') = \gamma_0 + \gamma_1'\xi_1' + \gamma_2'\xi_2',$$

where $\gamma_1' = \gamma_1\cos\theta + \gamma_2\sin\theta$, $\gamma_2' = -\gamma_1\sin\theta + \gamma_2\cos\theta$.

In $O\xi_1'\xi_2'\mu$, line L^* is the intersecting line of Π and the coordinate plane $\xi_1'O\mu$. The equation of L^* is

$$\mu(\xi_1') = \gamma_0 + \frac{1}{\sqrt{1+s_M^2}}(\gamma_1 + s_M\gamma_2)\xi_1' . \quad (7)$$

Eq. (7) is the life-stress relationship of single stress optimum test plan in the coordinate plane $\xi_1'O\mu$, and the highest stress level is $\xi_m = \| OM \| = \xi_{1M}(1+s_M^2)^{1/2}$. Set $\bar{\gamma}_1 = \xi_{1M}(\gamma_1 + s_M\gamma_2)$, $\bar{\xi} = \xi_1'/\xi_m$, Eq. (7) is transformed to the standard form of the single stress life-stress relationship[7]:

$$\mu(\bar{\xi}) = \gamma_0 + \bar{\gamma}_1\bar{\xi} . \quad (8)$$

Then, the problem of finding out the optimum degenerate test plan on the line l^* is transformed into finding out the optimum single stress test plan in $\xi'_1O\mu$: Specified the censored time τ, the total sample size N and the number of stress levels K, find out the stress levels $\bar{\xi}_i$ and the sample location ratios $p_i (i = 1, 2, \cdots, K)$ which minimize the asymptotic variance of the MLE of the p-th quantile of life distribution under normal stress. The optimization model can be described as follows[7]:

$$\begin{aligned}\text{Q1:}\ \min\ &\mathrm{Var}[\hat{y}_p] = (1,0,z_p)\boldsymbol{F}_d^{-1}(1,0,z_p)^{\mathrm{T}} = \sigma^2 V_K / N,\\ \text{s.t.}\ &0 \leqslant \bar{\xi}_1 < \cdots < \bar{\xi}_{i-1} < \bar{\xi}_i < \cdots < \bar{\xi}_K = 1;\\ &p_i \geqslant 0,\ \sum_{i=1}^{K} p_i = 1,\ i = 1, 2, \cdots, K,\end{aligned}$$

where V_K is the variance factor, $\boldsymbol{F}_d$ is the Fisher information matrix which is[7]

$$\boldsymbol{F}_d = \frac{N}{\sigma^2}\sum_{i=1}^{K} p_i \begin{pmatrix} A_d(\zeta_i) & \bar{\xi}_i A_d(\zeta_i) & B_d(\zeta_i)\\ \bar{\xi}_i A_d(\zeta_i) & \bar{\xi}_i^2 A_d(\zeta_i) & \bar{\xi}_i B_d(\zeta_i)\\ B_d(\zeta_i) & \bar{\xi}_i B_d(\zeta_i) & C_d(\zeta_i)\end{pmatrix}, \quad (9)$$

where

$$\zeta_i = [\ln\tau - (\gamma_0 + \bar{\gamma}_1\bar{\xi}_i)]/\sigma;\ A_d(\zeta_i) = 1 - \exp(-\exp\zeta_i);$$

$$B_d(\zeta_i) = \int_0^{\exp\zeta_i} u\ln u\exp(-u)\mathrm{d}u + \zeta_i\exp(\zeta_i - \exp\zeta_i);$$

$$C_d(\zeta_i) = \int_0^{\exp\zeta_i} u\ln^2 u\exp(-u)\mathrm{d}u + \zeta_i^2\exp(\zeta_i - \exp\zeta_i) + A_d(\zeta_i).$$

The problem Q1 has been solved by NELSON and MEEKER[2–7]: If the censored time is not too long, then in the optimum plan(called statistical optimum plan), the number of stress levels K^* is 2, the highest stress level $\bar{\xi}_2^*$ is 1, and the lowest stress level $\bar{\xi}^*_1$ and the best sample location ratios p_1^* on $\bar{\xi}_1^*$ relate to the initial assessed values of the model parameters γ_0, γ_1, γ_2.

Finally, transform the solution of the problem Q1 to the practical test plan. In the test region Ω, the coordinate of the i-th$(i = 1, 2, \cdots, K^*)$ stress level combination $(x_{1,i}^*, x_{2,i}^*)$ is $x_{1,i}^* = x_{1,0} + \bar{\xi}_i^*\xi_{1M}(x_{1,\mathrm{H}} - x_{1,0})$ and $x_{2,i}^* = x_{2,0} + \bar{\xi}_i^*\xi_{2M}(x_{2,\mathrm{H}} - x_{2,0})$,, and the sample location ratio on $(x_{1,i}^*, x_{2,i}^*)$ is p_1^*.

The above method is also suitable for the MCSALT with more than two factors. The general form is described in appendix A.1, and the corresponding data analysis method is described in appendix A.2. Methods for determining the initial assessed values of the model parameters are provided in appendix A.3.

3.4 Some discussions about the method

(1) For the objective of estimating y_p, one only needs to obtain the life-stress relationship as shown in Eq. (8). On one hand, finding out all parameters of Eq. (4) seems to obtain more information of the model, but it needs to carry out the test on three test points at least. That consumes more test time and occupies more resources. For the engineering, consuming resource or time for obtaining the unnecessary information is equivalent to the loss of test cost. According to the Ockham's razor[32], "Entities should not be multiplied unnecessarily", using the statistical optimum degenerate test plan is suggested. On the other hand, if the objective of an ALT is to obtain Eq. (4), using V-optimality as the objective function is not appropriate, and other objective functions such as D-optimality[33] are more appropriate.

(2) There are two doubts about the statistical optimum test plan usually: 1) The statistical optimum test plan only has two stress levels and can't test the correctness of the life-stress relationship; 2) The robustness of the statistical optimum plan is inferior to the test plans with more than two stress levels[6–7]. For the first doubt, in our view, the premise of applying the statistical optimum plan is that the form of the life-stress relationship and the failure life distribution are determined, and the correctness testing of

the statistical model is not necessary. If the correctness of the statistical model is doubted, using non-parametric or semi-parametric models or the optimal objective functions and restrictions which include the misspecification of the model may be more appropriate. For the second doubt, in our view, although the robustness of the statistical optimum plan is less than the test plans with 3 or 4 stress levels, it still has higher estimate accuracy for y_p in certain area of the change range of the model parameters. In such area, the statistical optimum plan is still the best plan for the objective of assessing y_p. This characteristic will be shown in the following example.

4 Example

Under the combined effect of temperature and vibration, the electrical contact life of a certain type of electrical connectors follows two parameter Weibull distribution, and the life-stress relationship follows the generalized Eyring model[34]:

$$\eta(T,S) = A \bullet S^{-\alpha} \exp[\Delta E/k(T+273.15)], \tag{8}$$

where $\eta(T, S)$ is the scale parameter of Weibull distribution, T /℃ is environment temperature, $S/(\mathrm{g}^2 \bullet \mathrm{Hz}^{-1})$ is random vibration stress, $k = 0.867\ 1\times10^{-4}\ \mathrm{eV/K}$ is the Boltzmann constant, A and α are unknown parameters.

Let the life be logarithmic, the Weibull-generalized Eyring model is transformed into the linear-extreme value model[34], and Eq. (10) is transformed into Eq. (2), where, $a = \ln A$, $b = -\Delta E/k$, $c = -\alpha$, $x_1 = -1/(T + 273.15)$, $x_2 = \ln S$. According to the engineering practice, the normal stress levels of electrical connectors are $T_0 = 45$℃, $S_0 = 0.04\mathrm{g}^2/\mathrm{Hz}$[34], the highest stress levels are $T_\mathrm{H} = 150$℃, $S_\mathrm{H} = 1.0\mathrm{g}^2/\mathrm{Hz}$, and the transformed stresses are $x_{1,0} = -3.14\times10^{-3}$, $x_{2,0} = -3.22$, $x_{1,\mathrm{H}} = -2.36\times10^{-3}$, $x_{2,\mathrm{H}} = 0$. According to the preliminary test data, obtain the initial estimation of the model parameters[34] $\tilde{a} = -3.922\ 9$, $\tilde{b} = -1.393\ 1\times10^3$, $\tilde{c} = -1.795\ 1$ and $\tilde{\sigma} = 0.353\ 9$. Set the test censored time $\tau = 100$h. Transform Eq. (2) into Eq. (4) according to Eq. (3). According to Eq. (5), the initial estimation of the parameters in Eq. (4) are $\tilde{\gamma}_0 = 6.231\ 7$, $\tilde{\gamma}_1 = -1.086\ 6$ and $\tilde{\gamma}_2 = -5.780\ 2$. As both of temperature and vibration stresses can reach the highest level simultaneously, the test region is a rectangle as shown in Fig. 1(a), the boundary curve S_{AB} is the polygonal line ACB.

4.1 Optimum test plan design

On the polygonal line ACB, the point with the maximum failure probability is $C(1,1)$[25, 31], the line l^* which the stress level combinations of the optimum degenerate test plan local on is the diagonal of the test region OC. Rotate the coordinate system $O\xi_1\xi_2\mu$ around the axis $O\mu$ with the angle $\theta = 45°$, make $O\xi_1$ coincide with l^* and form a new coordinate system $O\xi_1'\xi_2'\mu$. In Eq. (8), the initial assessed value of $\bar{\gamma}_1$ is $\tilde{\bar{\gamma}}_1 = \tilde{\gamma}_1 + \tilde{\gamma}_2 = -6.866\ 8$. Set the value of p is 0.5(medium life), substitute τ, $\tilde{\gamma}_0$, $\tilde{\bar{\gamma}}_1$ and $\tilde{\sigma}$ into problem Q1, then $\bar{\xi}_1^* = 0.263\ 3$, $p_1^* = 0.807\ 4$ can be obtained, and the value of corresponding variance factor $V_{K,1}^*$ is 3.5572. The stress level combinations corresponding to $\bar{\xi}_1^*$ and $\bar{\xi}_2^*$ are $(T_\mathrm{L}, S_\mathrm{L}) = (67.6$℃, $0.09\mathrm{g}^2/\mathrm{Hz})$ and $(T_\mathrm{H}, S_\mathrm{H}) = (150$℃, $1.0\mathrm{g}^2/\mathrm{Hz})$ respectively.

4.2 Robustness of the optimum test plans to misspecified model parameters

For comparison purposes, find out the best compromise test plan firstly. The best compromise test plan has three stress levels and equally spaced test stresses, and the sample location ratio on the highest, middle and lowest test stress be 4: 2: 1[6]. Based on the same τ and the same initial estimation of the parameters $\tilde{\gamma}_0$, $\tilde{\bar{\gamma}}_1$, $\tilde{\sigma}$, it can be solved that $p^*_1 = 0.571\ 4$, $\bar{\xi}_1^* = 0.266\ 6$, $\bar{\xi}_3^* = 1$, and the variance factor is $V_{K,2}^* = 4.931\ 6$. The stress level combinations corresponding to $\bar{\xi}_1^*, \bar{\xi}_2^*, \bar{\xi}_3^*$ are $(T_\mathrm{L}, S_\mathrm{L}) = (67.9$℃, $0.09\mathrm{g}^2/\mathrm{Hz})$, $(T_\mathrm{M}, S_\mathrm{M}) = (104.8$℃, $0.31\mathrm{g}^2/\mathrm{Hz})$ and $(T_\mathrm{H}, S_\mathrm{H}) = (150$℃, $1.0\mathrm{g}^2/\mathrm{Hz})$ respectively.

Let $\Delta_1 = |\tilde{\gamma}_0 - \gamma_0|/\gamma_0$, $\Delta_2 = |\tilde{\bar{\gamma}}_1 - \bar{\gamma}_1|/\bar{\gamma}_1$ and $\Delta_3 = |\tilde{\sigma}_1 - \sigma_1|/\sigma_1$ be the relative deviation of the initial estimations $\tilde{\gamma}_0$, $\tilde{\bar{\gamma}}_1$, $\tilde{\sigma}$ to their truth values $\gamma_0, \bar{\gamma}_1$, σ respectively. The comparison of the robustness of the optimal test plans to misspecified model parameters between the statistical optimum plan and the best compromise test plan is shown in Table 1. As the estimation precise of γ_0 and σ are usually lower than $\bar{\gamma}_1$, let $\Delta = \Delta_1 = \Delta_2 = \Delta_3 + 5\%$, where, $\Delta = 10\%, 15\%, 20\%$. In Table 1, "the actual effect of the nominal optimum plan" is the variance factor of actualizing the test according to the optimum plans corresponding to the parameters $\tilde{\gamma}_0$, $\tilde{\bar{\gamma}}_1$, $\tilde{\sigma}$. "The effect of the actual optimum plan" is the variance factor of the optimum plan corresponding to the truth value of the parameters γ_0, $\bar{\gamma}_1$, σ. In the square brackets below the numbers in the first column, the three bit of the encode correspond to the sign in front of Δ_1, Δ_2 and Δ_3, where "1" represent positive, and "0" represent negative.

Table 1 shows the following result.

(1) When $\gamma_0 < \tilde{\gamma}_0$ (see the rows which the serial number of the 1st column is from 1 to 4), $V_{K,1}$ is less than $V_{K,2}$, and that means, even consider the misspecification of the initial estimation of the model parameters, the statistical optimum plan is still better than the best compromise test plan. From the engineering point of view, γ_0 is the location parameter of the life distribution on the normal stress level, and it can be assessed according to the design life or reliability of product. For example, let $\tilde{\gamma}_0 = [y_{0.5}] - \tilde{\sigma} \bullet z_{0.5}$, where $[y_{0.5}]$ is the design median life of product. If $\gamma_0 < \tilde{\gamma}_0$, that means the actual life of the product on the normal stress level is less than the assessed value, and the product life do not meet the requirements of design. In that case, as it is important to assess the value of $y_{0.5}$ more precise for product improvement, using the statistical optimum plan is more appropriate than the best compromise test plan.

(2) When $\gamma_0 > \tilde{\gamma}_0$ (see the rows which the serial number of the 1st column is from 5 to 8), $V_{K,1}$ is greater than $V_{K,2}$. The reason is that, as the actual product life on the normal stress level is longer than the assessed value, actualizing the test according to the statistical optimum plan causes a lot of censored sample and amount of information loss. Contrarily, if actualizing the test according to the best compromise test plan, the failure sample on the middle stress level can provide remedies and decrease the information loss. To solve this problem, propose design the optimum plan with larger $\tilde{\gamma}_0$. For example, let $\tilde{\gamma}_0$ be 10%–20% larger than $[y_{0.5}] - \tilde{\sigma} \cdot z_{0.5}$ to ensure $\gamma_0 < \tilde{\gamma}_0$ and the statistical optimum plan is more precise. If the sample censor is still serious, there is a reason to believe that the actual product life is too long, and it could be considered to shorten the life appropriate for reducing product cost.

Table 1. Comparison of the robustness between the statistical optimum plan and the best compromise test plan

Type	Deviation of model initial parameters value Δ	Truth value of the parameters: Intercept of life-stress relationship γ_0	Truth value of the parameters: Slope of life-stress relationship $\bar{\gamma}_1$	Truth value of the parameters: Scale parameter σ	Actual effect of the nominal optimum plan: Variance factor of the statistical optimum plan $V_{K,1}$	Actual effect of the nominal optimum plan: Variance factor of the best compromise plan $V_{K,2}$	Effect of the actual optimum plan: Variance factor of the statistical optimum plan $V^*_{K,1}$	Effect of the actual optimum plan: Variance factor of the best compromise plan $V^*_{K,2}$
1 [111]	10%	5.665 2	−6.539 8	0.321 7	3.324 1	4.556 3	2.657 8	3.683 3
	15%	5.418 9	−6.242 5	0.307 7	3.324 0	4.556 2	2.367 3	3.297 1
	20%	5.193 1	−5.971 1	0.294 9	3.324 0	4.556 2	2.114 1	2.969 2
2 [110]	10%	5.665 2	−6.539 8	0.393 2	3.325 1	4.559 1	2.708 6	3.769 5
	15%	5.418 9	−6.242 5	0.416 4	3.324 1	4.556 5	2.432 2	3.411 2
	20%	5.193 1	−5.971 1	0.442 4	3.324 0	4.556 2	2.187 1	3.103 6
3 [101]	10%	5.665 2	−7.228 2	0.321 7	3.324 0	4.556 2	2.488 8	3.449 1
	15%	5.418 9	−7.629 8	0.307 7	3.324 0	4.556 2	2.140 5	2.986 5
	20%	5.193 1	−8.078 6	0.294 9	3.324 0	4.556 2	1.891 7	2.665 5
4 [100]	10%	5.665 2	−7.228 2	0.393 2	3.324 1	4.556 2	2.534 2	3.526 6
	15%	5.418 9	−7.629 8	0.416 4	3.324 0	4.556 2	2.194 2	3.081 1
	20%	5.193 1	−8.078 6	0.442 4	3.324 0	4.556 2	1.949 7	2.770 8
5 [011]	10%	6.924 1	−6.539 8	0.321 7	29.106 3	18.873 9	5.810 5	8.235 9
	15%	7.331 4	−6.242 5	0.307 7	140.058 3	34.955 7	8.552 4	12.439 1
	20%	7.789 6	−5.971 1	0.294 9	704.176 6	44.692 0	14.282 4	21.485 1
6 [010]	10%	6.924 1	−6.539 8	0.393 2	19.879 1	15.598 9	5.951 4	8.439 7
	15%	7.331 4	−6.242 5	0.416 4	62.230 9	26.793 0	8.877 0	12.858 2
	20%	7.789 6	−5.971 1	0.442 4	155.666 9	36.591 8	14.957 7	22.121 8
7 [001]	10%	6.924 1	−7.228 2	0.321 7	15.257 5	13.308 8	4.952 3	6.951 7
	15%	7.331 4	−7.629 8	0.307 7	47.898 6	23.692 4	5.747 0	8.141 3
	20%	7.789 6	−8.078 6	0.294 9	148.178 6	35.297 9	6.722 2	9.630 5
8 [000]	10%	6.924 1	−7.228 2	0.393 2	11.566 6	11.434 2	5.067 7	7.126 2
	15%	7.331 4	−7.629 8	0.416 4	25.113 2	17.579 7	5.955 5	8.450 9
	20%	7.789 6	−8.078 6	0.442 4	50.800 1	24.533 2	7.058 7	10.122 5

5 Conclusions

(1) When the objective of the MCSALT is to assess the p-th quantile of the product life distribution under normal stress, the statistical optimum degenerate test plan which only have two stress level combinations could be used. The proposed plan could achieve estimation accuracy no less than the traditional plan, but saves the test time and cost on one stress level combination at least.

(2) The example shows, when the actual product life is less than the design value, even the deviation of the model initial parameters value being up to 20%, the variance of the estimation of the p-th quantile is still smaller than the traditional plans approximately 25%. Here, as the product life doesn't meet the design requirements, it is very important to assess the product life more precise for product improving, and the proposed plan is more conducive to guide the reliability growth of the product.

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Appendix

A.1 Case of n-factor linear life-stress relationship

Assume that, on the test region Ω, the relationship of the location parameter μ and the standardized stresses ξ_i ($i = 1, 2, \cdots, n$) satisfies

$$\mu(\xi_1,\xi_2,\cdots,\xi_n)=\gamma_0+\sum_{i=1}^{n}\gamma_i\xi_i\ ,\ (\xi_1,\xi_2,\cdots,\ \xi_n)\in\Omega,\ \gamma_i<0. \tag{A-1}$$

Assume the coordinate of point M which locates on the test region boundary $\partial\Omega$ and has the maximum failure probability is $(\xi_{1,M}, \xi_{2,M}, \cdots, \xi_{n,M})$. Let point O be the origin of the coordinates and OM be the positive direction, establish the coordinate axis $O\xi$. When a stress level combination $(\xi_1, \xi_2, \cdots, \xi_n)$ locates on the segment OM, ξ_i could be expressed as

$$\xi_i=\left(\xi_{i,M}\Big/\sqrt{\sum_{i=1}^{n}\xi_{i,M}^2}\right)\xi, \tag{A-2}$$

where ξ is the distance from $(\xi_1, \xi_2, \cdots, \xi_n)$ to point O. Substitute Eq. (A-2) into Eq. (A-1), the following equation can be derived:

$$\mu(\xi)=\gamma_0+\left(\sum_{i=1}^{n}\gamma_i\xi_{i,M}\right)\left(\xi\Big/\sqrt{\sum_{i=1}^{n}\xi_{i,M}^2}\right),\ \xi\in\left[0,\sqrt{\sum_{i=1}^{n}\xi_{i,M}^2}\right]. \tag{A-3}$$

Let

$$\overline{\gamma}_1=\sum_{i=1}^{n}\gamma_i\xi_{i,M}\ ;\ \overline{\xi}=\xi\Big/\sqrt{\sum_{i=1}^{n}\xi_{i,M}^2}\ ,$$

then Eq. (A-3) transforms to Eq. (8). It indicates that the proposed method is not affected by the dimension of the life-stress relationship.

A.2 Corresponding test data analysis method

In the problem Q1, assume the optimum number of test stress levels be K^*, the optimum test stress levels be $\overline{\xi}_i^*$ ($i = 1, 2, \cdots, K^*$), the sample location ratios be p^*_i, the stress level combination corresponding to $\overline{\xi}_i^*$ be $(x_{1,i}^*, x_{2,i}^*)$, and the total sample size be N. Let the sample size on the i-th($i = 1, 2, \cdots, K^*$) stress level combination $(x_{1,i}^*, x_{2,i}^*)$ be $N_i = [Np^*_i]$, write the life data of the j-th sample as (y_{ij}, δ_{ij})(when the sample is censored, $\delta_{ij} = 0$, $y_{ij} = \ln\tau$; when the sample is failure, $\delta_{ij} = 1$, $y_{ij} = \ln t_{ij}$), and the log-likelihood function of the test could be written as

$$\ln L=\sum_{i=1}^{K}\sum_{j_i=1}^{N_i}\{\delta_{ij_i}\ln f_i(y_{ij_i})+(1-\delta_{ij_i})\ln[1-F_i(\tau)]\}, \tag{A-4}$$

where

$$f_i(y_{ij})=\frac{1}{\sigma}\exp\left[\frac{y_{ij}-(\gamma_0+\overline{\gamma}_1\overline{\xi}_i^*)}{\sigma}\right]\exp\left[-\exp\left(\frac{y_{ij}-(\gamma_0+\overline{\gamma}_1\overline{\xi}_i^*)}{\sigma}\right)\right],$$

$$F_i(\tau)=1-\exp\left\{-\exp\left[\frac{\ln\tau-(\gamma_0+\overline{\gamma}_1\overline{\xi}_i^*)}{\sigma}\right]\right\}.$$

Let Eq. (A-4) reach its maximum, $\hat{\gamma}_0$ and $\hat{\sigma}$ can be found out, and $\hat{y}_p$ can be obtained by substituting $\hat{\gamma}_0$ and $\hat{\sigma}$ into Eq. (6).

A.3 Methods for determining the initial estimation of model parameters

Method 1: Determine the initial estimation depending on the test data obtained from the test for testing the correctness of the life-stress relationship and the life distribution. A test for testing the correctness of the statistical model(called model-test experiment) always needs more test points and sample size than a test for assessing the product life[34]. As a result, relatively accurate initial estimation can be obtained according to the test data obtained from the model-test experiment. From this point of view, the provided method of planning the optimum MCSALT plan is more suitable for the routine reliability assessment and test for mass production products.

Method 2: Determine the initial estimation depending on the design life of product and the test data which are obtained from the diagnostic test[34]. A diagnostic test is usually used to assess the lower bound of the test time and to examine whether the failure mechanism of product at the highest stress level remains the same as the normal stress level before an ALT. Let $\tilde{\mu}_{\rm H}$ be the value of the location parameter of product life distribution on the highest stress level, assess $\tilde{\sigma}$ and $\tilde{\mu}_{\rm H}$ depending on the test data of the diagnostic test, and determine the design p-precentile life of product $[y_p]$, then $\tilde{\gamma}_0$ can be obtained by $\tilde{\gamma}_0=[y_p](1.1-1.3)-\tilde{\sigma}\bullet z_p$ (where, the coefficient 1.1–1.3 describes the margin of the p-precentile life) and $\tilde{\overline{\gamma}}_1$ can be estimated by $\tilde{\overline{\gamma}}_1=\tilde{\mu}_{\rm H}-\tilde{\gamma}_0$.