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Identification of Maximum Road Friction Coefficient and Optimal Slip Ratio Based on Road Type Recognition

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Abstract: The identification of maximum road friction coefficient and optimal slip ratio is crucial to vehicle dynamics and control. However, it is always not easy to identify the maximum road friction coefficient with high robustness and good adaptability to various vehicle operating conditions. The existing investigations on robust identification of maximum road friction coefficient are unsatisfactory. In this paper, an identification approach based on road type recognition is proposed for the robust identification of maximum road friction coefficient and optimal slip ratio. The instantaneous road friction coefficient is estimated through the recursive least square with a forgetting factor method based on the single wheel model, and the estimated road friction coefficient and slip ratio are grouped in a set of samples in a small time interval before the current time, which are updated with time progressing. The current road type is recognized by comparing the samples of the estimated road friction coefficient with the standard road friction coefficient of each typical road, and the minimum statistical error is used as the recognition principle to improve identification robustness. Once the road type is recognized, the maximum road friction coefficient and optimal slip ratio are determined. The numerical simulation tests are conducted on two typical road friction conditions(single-friction and joint-friction) by using CarSim software. The test results show that there is little identification error between the identified maximum road friction coefficient and the pre-set value in CarSim. The proposed identification method has good robustness performance to external disturbances and good adaptability to various vehicle operating conditions and road variations, and the identification results can be used for the adjustment of vehicle active safety control strategies.

Keywords: maximum road friction coefficient, optimal slip ratio, road type recognition, recursive least square

1 Introduction

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Various vehicle active safety electronic control systems have gained their rapid development in the past few years, such as anti-lock braking system(ABS), traction control system(TCS) and electronic stability control(ESC), etc. Since the main purpose of such systems is to regulate the tire tangential force, which is restricted by the friction potential between tires and roads, the performance of vehicle active safety control strategies depends heavily on the use of road friction potential, in other words, the determination of the maximum road friction coefficient (μ_{max}) . However, μ_{max} and s_{op} (the slip ratio corresponding to μ_{max}) differ with the variation of road types and conditions [1].

The longitudinal road friction coefficient(μ_x) is defined as the ratio of the longitudinal force F_x and the normal force $F_z^{[2-3]}$. It is also named the normalized traction force in some papers^[4–5]. For simplicity, in this paper, μ is named the instantaneous longitudinal road friction coefficient, and

 μ_{max} represents the maximum value of μ , $\mu_{\text{max}} = \frac{max|F_x/F_z|}{max|F_x/F_z|}$.

Since many factors affect the road friction coefficient, such as road materials, road conditions, tire types and pressures, vehicle velocity, etc^[6], the accurate and robust identification of μ_{max} is always a difficult and challenging event in the automotive field.

Various approaches to identify μ_{max} have been developed^[7-27]. In HEDRICK K's philosophy, μ_{max} identification approaches are divided into two basic categories: the "cause-based" method and the "effect-based" method^[5]. The "cause-based" method mainly detects the physical parameters of road-tire contact patch which affect road friction conditions through vehicle equipped sensors(light, sound, microwave, etc) $[7-9]$. BREUER, et al^[7], presented a method to infer μ_{max} using optical sensors to measure road wetness by detecting the reflected light. TUONONEN[8] proposed a method to estimate μ_{max} using the tire carcass displacements measured by LED sensor units. Most of these methods could identify μ_{max} accurately, but they needed to add extra sensors, which prohibited its application for production vehicles because of the cost involved, thus many researchers mainly focused on the research on "effect-based" methods $[10-27]$. The "effect-based" method was to identify μ_{max} through estimating vehicle dynamics parameter response generated

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from different road friction conditions. In this area, researchers have pursued at least four types of μ_{max} identification approaches: acoustic methods, tire tread deformation methods, slip-based methods and model-based methods. EICHHORN, et al^[10], proposed the acoustic method which used optical and noise sensors to "listen" to the sound that tire made and inferred μ_{max} . In tire deformation methods, WANG, et al^[11], EDOARDO, et al^[12], GURKAN, et al^[13] and KANWAR, et al^[14], used a magnetic sensor embedded into the tire tread to measure the tire local strain as well as stress, because the detected signals from the tire inner surface reflected μ_{max} information. Slip-based methods were very popular in the μ_{max} identification. FREDRIK^[15] first proposed the μ_{max} estimation approach based on slip ratio, subsequently there were many methods emerging based on this theory $[16-19]$. In general, these methods utilized Kalman filter or recursive least square method to estimate the scope of road friction coefficient vs. slip ratio curve under a small slip ratio interval, then the maximum road friction coefficient was identified according to the slip scope value. RAJAMANI, et al^[19], gave the corresponding relationship between the maximum road friction coefficient and the slip scope. Refs. [20–22] described the model-based approaches. These approaches were based on the longitudinal road friction coefficient vs. slip ratio curves of typical roads, such as BURCKHARDT model, Magic Formula, and the maximum road friction coefficient was identified by estimating the fitting parameters of such models. LIU, et $al^{[20]}$, presented a recursive least square method to estimate the two parameters (p_1, p_2) which determined the distribution of μ_{max} and s_{op} in KIENCKE model. TANELLI, et al^[21], estimated the three parameters (c_1, c_2, c_3) in the BURCKHARDT model by the maximum likelihood method, thus μ_{max} was calculated from the estimated results. RICARDO, et $al^{[22]}$, used a linear parametrization approach based on a feedforward neural network to identify μ_{max} .

There were other approaches to identify μ_{max} except for four approaches introduced above. HEDRICK, et $al^{[23]}$, used a nonlinear equation to fit the test data of the road friction coefficient and took the peak value of the fitting curve as the estimated μ_{max} . ONE, et al^[24], presented a method using extended braking stiffness to identify μ_{max} . Ref. [25] used the rate of the road friction coefficient vs. slip ratio curve to extract the maximum value of μ when the rate is zero. In Ref. [26], a nonlinear observer based on unscented Kalman filter was proposed to estimate μ_{max} . HAHN, et al^[27], utilized a GPS system to estimate tire sideslip angles, and the lateral maximum road friction coefficient was identified based on vehicle lateral dynamics, which is effective during steering.

In general, many researchers have made great effort in the real-time and effective identification of μ_{max} , and great progress has been obtained from the review introduced above. But higher robustness and better adaptability to complicated vehicle operating conditions are always key research issues among current literatures and future work.

This paper tries to develop an effective μ_{max} and s_{op} identification approach based on road type recognition. The rest of the paper is organized as follows. Section 3 introduces the estimation method of the slip ratio and instantaneous road friction coefficient based on the single wheel model using the recursive least square with a forgetting factor method. In section 4, the proposed identification methodology of maximum road friction coefficient and optimal slip ratio is illustrated. In section 5, numerical simulation tests under different conditions are conducted on the basis of CarSim-Simulink program.

2 Estimation of Instantaneous Road Friction Coefficient and Slip Ratio

In the μ_{max} identification methodology of this paper, the instantaneous road friction coefficient and slip ratio are needed to estimate first. The tire slip ratio is estimated as $below^[2]$:

$$
s = \frac{\omega R - V_x}{\omega R}, \text{ vehicle is in traction}, \text{ (1)}
$$

$$
s = \frac{V_x - \omega R}{V_x}
$$
, vehicle is in braking, (2)

where V_x is the longitudinal vehicle velocity, which can be directly measured by sensors or estimated by Ref. [28]. *R* is the dynamic radius of tires, ω is the wheel angular speed which can be measured from the ABS wheel angular speed sensor.

The instantaneous road friction coefficient is estimated by the recursive least square with a forgetting factor method $[29]$ on basis of the single wheel rotational model shown in Fig. 1.

Fig. 1. Single wheel model

The wheel rotational dynamics equation is

$$
J_{\omega}\dot{\omega} = T - F_x R - T_f,\tag{3}
$$

where F_x —Tire longitudinal force,

 J_{ω} —Wheel moment of inertia,

- T_f —Rolling resistance torque,
- $\dot{\omega}$ —Wheel angular acceleration,

T —Tire traction torque of driving wheel(negative when braking).

When a vehicle is in traction, *T* can be estimated by $T_i i_g i_0 \eta_g \eta_0 \eta_d$, where i_g is the gear ratio of transmission; i_0 is the ratio of final reduction; η_g is the efficiency of transmission; η_0 is the efficiency of final reduction; η_d is the efficiency of driveline.

For a vehicle equipped with MT(Manual Transmission), T_t is the engine torque which is obtained from engine map lookup table according to the throttle opening and the engine rotational speed. For a vehicle equipped with automatic transmission(AT), $T_i = \tau K_i \omega_e^2$, where ω_e is the engine speed; τ and K_t are the torque ratio and capacity factor of torque converters, which are the function of the speed ratio of turbines and pumps.

When a vehicle is in braking, *T* can be estimated by the product of the wheel cylinder pressure and braking efficacy factor, and the relationship between *T* and the wheel cylinder pressure is approximately linear. The wheel cylinder pressure can be estimated by the measured master cylinder pressure. The master cylinder pressure sensor is a standard sensor equipped in ESC.

To estimate the tire longitudinal force, Eq. (3) is written into the discrete form shown in Eq. (4):

$$
\frac{\omega(k+1) - \omega(k)}{\Delta t} - \frac{T(k)}{J_{\omega}} + \frac{T_f(k)}{J_{\omega}} = \frac{-F_x(k)R}{J_{\omega}},
$$
 (4)

where Δt is the sampling time and *k* is the time index. To use the least square method to estimate F_r , the output signal is given in Eq. (5):

$$
y(k) = \boldsymbol{\Phi}^{\mathrm{T}}(k)P(k),\tag{5}
$$

where

$$
y(k) = \frac{\omega(k+1) - \omega(k)}{\Delta t} - \frac{T(k)}{J_{\omega}} + \frac{T_f(k)}{J_{\omega}},
$$

$$
\Phi^{\mathrm{T}}(k) = -\frac{R}{J_{\omega}}, \hat{P}(k) = \hat{F}_x(k).
$$
(6)

The parameter vector in the recursive least square estimation is

$$
\hat{P}(k+1) = \hat{P}(k) + \Psi(k+1)\Gamma(k)\Phi(k+1) \times
$$

[y(k+1) - \Phi^T(k+1)\hat{P}(k)]. (7)

And the auxiliary matrices are updated according to Eqs. $(8)–(9)$:

$$
\Gamma(k+1) = \frac{1}{\lambda} [\Gamma(k) - \Psi(k+1)\Gamma(k) \times
$$

$$
\Phi(k+1)\Phi^{T}(k+1)\Gamma(k)],
$$
 (8)

$$
\Psi(k+1) = \frac{1}{1 + \Phi^{T}(k+1)\Gamma(k)\Phi(k+1)},
$$
\n(9)

where $0 < \lambda < 1$ is the forgetting factor.

Finally, the longitudinal force is estimated by Eqs. (7)–(9). The wheel normal force \hat{F}_z can be estimated by Eqs. $(10)–(11)$:

$$
\hat{F}_{zf} = \frac{Mgl_r}{2l} - \frac{Ma_x h}{2l},
$$
\n(10)

$$
\hat{F}_{zr} = \frac{Mgl_f}{2l} - \frac{Ma_x h}{2l},
$$
\n(11)

where *M* is the vehicle mass; *l* is the wheel base; *h* is the height of vehicle center of gravity(CG); a_x is the vehicle longitudinal acceleration, which can be estimated by vehicle velocity or measured by acceleration sensors; l_f and l_r are the distance from vehicle CG to the front axle and rear axle, respectively; \hat{F}_{zf} and \hat{F}_{zr} are the estimated normal force of front wheels and rear wheels, respectively.

So far, the instantaneous road friction coefficient can be estimated according to the definition.

$$
\mu = \frac{\hat{F}_x}{\hat{F}_z}.\tag{12}
$$

3 Maximum Road Friction Coefficient Identification Approach

In this paper, the maximum road friction coefficient is identified based on the real-time recognition of the current road type on which a vehicle is traveling. The road type is recognized by comparing the estimated road friction coefficient with the standard road friction coefficient of each typical road. The minimum statistical error is used as the recognition principle to improve identification robustness. It is assumed that roads are classified into several typical types, and each typical road corresponds to a specified maximum road friction coefficient and an optimal slip ratio. Once the current road type is recognized, the maximum road friction coefficient and optimal slip ratio of the current road are identified simultaneously.

As introduced above, typical road friction coefficient databases are needed. They can be derived from some high precision and widely used μ -s model. BURCKHARDT model, which is proposed by BURCKHARD $T^{[30]}$ and developed on the basis of experimental data fitting on some typical roads, is widely used in vehicle dynamics studies. In this paper, with the application of BURCKHARDT model, roads are classified into six typical types, and the μ -s curves of typical roads in the BURCKHARDT model are composed of standard road friction coefficient databases for the foundation of the identification algorithm.

The BURCKHARDT model is expressed in Eq. (13):

$$
\mu = c_1 (1 - \exp(-c_2 s)) - c_3 s,\tag{13}
$$

where c_1 , c_2 and c_3 are parameters in the model. Table 1 shows the values of them on six typical roads. In Fig. 2, the shapes of road friction coefficient vs. slip ratio $(\mu - s)$ curves of six different road types are depicted according to the values shown in Table 1.

Table 1. Parameters of typical roads in BURCKHARDT model

Road type	Model	Model	Model
	parameter c_1	parameter c_2	parameter c_3
Ice	0.05	306.39	$_{0}$
Snow	0.195	94.13	0.0646
Wet cobblestone	0.400	33.71	0.12
Wet asphalt	0.857	33.82	0.35
Dry cement	1.197	25.17	0.54
Dry asphalt	1.280	23.99	0.52

The maximum road friction coefficient and optimal slip ratio of various typical roads are obtained by calculating the extreme value of the road friction coefficient in μ –*s* curves, as shown in Table 2.

Table 2. Typical road characteristics

Road type	Road number	Maximum friction coefficient μ_{max}	Optimal slip ratio s_{op}
Ice		0.05	0.031
Snow	2	0.19	0.060
Wet cobblestone	3	0.38	0.140
Wet asphalt	4	0.8	0.131
Dry cement	5	1.08	0.16
Dry asphalt	6	117	0.17

There are three steps in the proposed μ_{max} identification approach, as shown in Fig. 3. Firstly we need to estimate and collect the samples of the estimated road friction coefficient and tire slip ratio in a small time interval $[t-T, t]$, where *t* is the current time, *T* is a constant small time interval. Then the current road type is recognized by comparing the estimated μ with the standard μ of each typical road using the principle of minimum statistical error, finally μ_{max} and s_{op} are determined according to the recognized road type.

(1) Collect samples. When a vehicle is in traction or braking, the instantaneous road friction coefficient and slip ratio are estimated by the method introduced in section 2. The estimated μ and *s* can be grouped in a set of *N* samples (s_i, μ_i) in a small time interval $[t-T, t]$, $i = 1, 2, \dots, N$. The sample data are updated with the time progressing.

Fig. 3. Diagram of μ_{max} and s_{on} identification

(2) Road type recognition. After collecting *N* samples of μ_i and s_i at the current time, the total statistical error of the instantaneous road friction coefficient deviating from standard μ_b under each typical road is calculated simultaneously, and the recognized road type is the one corresponding to the minimum error.

The total statistical error $(\sigma^2)^j$ is

$$
(\sigma^2)^j = \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_b^j(s_i))^2, \qquad (14)
$$

where *N* is the sample number in $[t-T, t]$, *j* is the typical road type index, μ_i and s_i are the estimated road friction coefficient and slip ratio at sampling points, respectively. $\mu_b^{\ j}(s_i)$ is the standard road friction coefficient corresponding to the sampling slip ratio of the *j*th typical road, and $\mu_b^j(s_i)$ is calculated from the BURCKHARDT model according to Eq. (13). *Q* denotes the identified road type number. Obviously, *Q* is the road type corresponding to the minimum value of $(\sigma^2)^j$:

$$
Q = \text{Min}((\sigma^2)^j), j = 1, 2, \cdots, 6. \tag{15}
$$

(3) Identification of the maximum road friction coefficient and optimal slip ratio

After recognizing the current road type by step (2), μ_{max}

and *sop* are identified by lookup table in Table 2 according to the recognized road type number.

The identification approach is adaptive to varying road conditions. If the current road varies in the next time point $t(k+1)$, the distribution of μ_i and s_i will change accordingly. For example, when road varies from dry cement road to wet asphalt road shown in Fig. 4, μ_i declines and s_i increases simultaneously. Steps (2) and (3) will quickly track the change of them, and the μ_{max} after the road variation will be identified soon.

Fig. 4. Identification of varying roads

4 Simulation and Experiment Results

To evaluate the effectiveness and performance of the proposed maximum road friction coefficient identification approach, the simulation experiment is necessary before actual testing on vehicles. The effectiveness is examined by comparing the identified results with the outputs in CarSim under two typical road friction conditions(single-friction and joint friction). CarSim is a commercial software with high precision and is widely used in vehicle virtual simulations. The vehicle model is provided by CarSim, and the μ_{max} and s_{op} identification algorithm is achieved in MATLAB/Simulink. Both of them consist of the complete simulation model. During actual vehicle traveling, there are noise and disturbances in the measurement of vehicle signals. To simulate these disturbances, the white noise is added to the signals of wheel velocity and traction torque, et al in the simulation model.

4.1 Simulation results and analysis on a single friction road

In this virtual experiment, the effectiveness of the proposed μ_{max} identification approach is validated by both a braking test and a driving test, respectively. The virtual vehicle for experiment in CarSim is a B-Class hatchback equipped with ABS and AT with four gears.

Figs. 5–10 are the simulation results of μ_{max} identification and vehicle dynamic parameters during an emergency braking after 0.2 s on a high friction road. The internal tire model in CarSim is chosen and used in the virtual vehicle, and any road friction coefficient can be pre-set in the model conveniently. The maximum road friction coefficient of the test road surface in CarSim is set to 1.074 and the initial braking speed is 120 km/h. Because four wheels brake on the same friction road, the results of four wheels are similar and the left front wheel is used as an example to analyze. A simple slip ratio threshold ABS control strategy is used in the CarSim program.

Fig. 6. Estimated instantaneous road friction coefficient

Fig. 7. Road friction coefficient-slip ratio curve

Fig. 8. The recognized road type

Fig. 10. Longitudinal velocity of vehicle

Figs. 5–6 are results of the estimated slip ratio, the road friction coefficient, respectively. Fig. 7 presents the relationship between the estimated road friction coefficient and slip ratio. It can be seen from Fig. 7 that the μ -s curve from test is close to the μ -s curve of dry cement road. Fig. 8 shows that the recognized road type is dry cement whose μ_{max} is 1.08. Although both μ_i and s_i are oscillating during the whole braking, the identified μ_{max} through the proposed identification approach has little error with the real value in CarSim, as shown in Fig. 9. It indicates that the identification approach has adaptive resistance to the oscillations of μ_i and s_i and has good robustness to external disturbances. However, μ_{max} cannot be identified correctly in the initial period of braking. The reason is that there is little difference among μ_i of each typical road when the slip ratio is extremely low, which can be seen from Fig. 2. The total error $(\sigma^2)^j$ is similar with each other. However, with the growth of braking strength, μ_i and s_i increase quickly. (σ^2) ^{*j*} of each road becomes obvious and the identified μ_{max} is precise and stable later. The identified μ_{max} is 1.08, and the identification error is 0.005 6. The identification approach has good performance of accuracy and robustness.

When the vehicle is in traction on the same road, the performance of the μ_{max} identification is examined in the following experiment. Figs. 11, 12 show the throttle opening input of the driver and the gear status during driving. The simulation results are presented in Figs. 13–16. Fig. 13 and Fig. 14 show the estimated slip ratio and road friction coefficient, respectively. In the simulation, there are some oscillations of them due to the white noise added to the signal of wheel velocity and traction torque. Fig. 15

presents the result of the recognized road type. Fig. 16 shows the identification result of μ_{max} and the real value pre-set in CarSim. It can be seen from Figs. 11–12 that there is a common period when the throttle opening declines to a low value $(15%)$ and the transmission gear is in high status, the μ_{max} identification is not effective, as shown in Fig. 16. This is caused by the extremely low slip ratio and used road friction coefficient in this period, as shown in Figs. 13, and 14. However, μ_{max} identification is accurate under most periods and conditions, and the identification result is also 1.08, which is identical with the result in the previous braking test.

Fig. 16. μ_{max} identification result

4.2 Simulation results and analysis on a joint friction road

The simulation experiments in this section mainly focus on the validation and application of the proposed identification approach on varying friction roads.

The results of a braking simulation experiment on a joint friction road are presented in Figs. 17–21. The initial braking velocity is 140 km/h. Each wheel slip ratio is controlled by using PID algorithm to achieve optimal slip ratio control based on the real-time optimal slip identified through the proposed identification algorithm. The pre-set road condition in CarSim is the dashed line shown in Fig. 18. Fig. 17 and Fig. 18 present the results of the recognized road type and the identified μ_{max} , respectively. Both of them indicate that when road varies, it takes little time to track and identify the varied road type and the maximum road friction coefficient. The identified μ_{max} has little error compared with the real value. In Fig. 19, benefited from the accurate identification of road characteristics, the actual slip ratio are well controlled around the optimal slip ratio of the identified road, and the instantaneous road friction coefficient shown in Fig. 20 approximates to the maximum road friction coefficient, indicating that the maximum friction potential of different roads has been fully used.

The results in the simulation experiment show that ABS based on slip ratio control can benefit a lot from the identified μ_{max} and s_{op} . When road varies, the control target (optimal slip ratio) can be modified according to the identified optimal slip ratio of the varied road, which helps ABS or TCS make full use of the maximum road friction potential of different roads and enhances the braking and driving performance on variable roads.

It can be concluded from the simulation results and analysis above that no matter the vehicle drives or brakes on a single friction road or a variable friction road, the proposed approach can effectively identify the maximum friction coefficient and optimal slip ratio accurately and quickly, which can be used for the adjustment of vehicle active safety control strategies. But when the slip ratio is extremely $low(s<0.01)$, the difference of road friction coefficient on different roads is tiny, which causes the typical roads cannot be differentiated. This drawback can be solved by pre-setting a minimum slip ratio threshold value used for the judge of μ_{max} identification. When slip ratio doesn't reach the threshold at the beginning of driving or braking, the slip ratio control target of the controller adopts the optimal slip ratio of dry asphalt (0.17) . Since the target slip ratio is high, the actual slip ratio will soon cross the threshold and the identification will quickly effect. When the actual slip ratio declines and is less than the threshold value, the identified maximum road friction coefficient keeps the value of last circle. The value of minimum slip ratio threshold is related to the noise level of signals required. In this paper, the minimum slip ratio threshold is 0.01.

5 Conclusions

(1) The proposed identification approach could effectively identify and predict μ_{max} as well as s_{op} of current road at any conditions through the samples of the estimated road friction coefficient and slip ratio, and the samples are in a small time interval before the current time, updating with time progressing.

(2) Due to measurement noise and disturbances of required signals, both instantaneous road friction coefficient and slip ratio oscillate largely. However, the proposed identification approach can overcome the influences of them and doesn't need signal filters. The identification approach has good robustness performance to external disturbances.

(3) The identification has good adaptability to vehicle operating conditions and road variations. No matter vehicles drive or brake on a single friction road or a variable friction road, the reliable identification results can be conveniently used for the adjustment of vehicle active control strategies.

(4) Accurate and detailed road characteristic databases are the foundation of effective identification. The μ -s model considering the factors of different tire types will be more accurate. Building more typical road databases of standard μ -s curves with consideration of different tire types will be the emphasis of future researches.

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