CHINESE JOURNAL OF MECHANICAL ENGINEERING
Vol. 26, No. 5, 2013

DOI: 10.3901/CJME.2013.05.900, available online at www.springerlink.com; www.cjmenet.com; www.cjmenet.com.cn

Optimal Load Balancing Leveling Method for Multi-leg Flexible Platforms

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Received May 17, 2012; revised May 9, 2013; accepted June 28, 2013

Abstract: The working platforms supported with multiple extensible legs must be leveled before they come into operation. Although the supporting stiffness and reliability of the platform are improved with the increasing number of the supporting legs, the increased overdetermination of the multi-leg platform systems leads to leveling coupling problem among legs and virtual leg problem in which some of the supporting legs bear zero or quasi zero loads. These problems make it quite complex and time consuming to level such a multi-leg platform. Based on rigid body kinematics, an approximate equation is formulated to rapidly calculate the leg extension for leveling a rigid platform, then a proportional speed control strategy is proposed to reduce the unexpected platform distortion and leveling coupling between supporting legs. Taking both the load coupling between supporting legs and the elastic flexibility of the working platform into consideration, an optimal balancing legs' loads(OBLL) model is firstly put forward to deal with the traditional virtual leg problem. By taking advantage of the concept of supporting stiffness matrix, a coupling extension method(CEM) is developed to solve this OBLL problem for multi-leg flexible platform. At the end, with the concept of supporting stiffness matrix and static transmissibility matrix, an optimal load balancing leveling method is proposed to achieve geometric leveling and legs' loads balancing simultaneously. Three numerical examples are given out to illustrate the performance of proposed methods. This paper proposes a method which can effectively quantify all of the legs' extension at the same time, achieve geometric leveling and legs' loads balancing simultaneously. By using the proposed methods, the stability, precision and efficiency of auto-leveling control process can be improved.

Key words: multi-leg platform, overdetermined problem, optimal balancing legs' loads, supporting stiffness matrix, static transmissibility matrix

1 Introduction

Many working platforms, such as truck-mounted radar system and truck–mounted missile launch system, must be leveled before they come into operation[1]. Their vertical supporting legs are extended to the ground, all the wheels are raised off the ground, then the working platform is adjusted to horizontal state by controlling the extension of each supporting leg. The working platform is required to keep this horizontal state with a very small inclination tolerance(usually much less than 0.1°) under each impossible operating mode. For some important civil equipments, e.g., wheeled cranes or heavy transportation platforms, the leveling process is also a key technical issue[2].

There are two kinds of leveling process: manual and automatic leveling[3–4]. The manual leveling process measures the inclination of working platform with quadrants or anti-air gun instruments while adjusting the extension of each supporting leg manually. The automatic leveling process is realized by the automatic control leveling hydraulic or electromechanical system, which usually takes only several minutes and can hold the state well in a long period.

KUROKAWA, et al[5], proposed a spring method to automatically level specimen surfaces for roughness measurement. The spring method used a part of tentative data points to calculate the surface inclination, so the surfaces were leveled with less distortion and large vertical measurement range can be achieved. LIU, et al[6], designed an automatic leveling and centering system of theodolite based on the algorithm of error correction and compensation. ZHANG, et al[7], put forward a circular auto-leveling strategy for a vehicle-borne platform with four legs. GAO, et al[8], proposed a "surface-adjust-surface" leveling technique, which used fuzzy decoupling algorithm to adjust the hydraulic legs. WU, et al[9], developed a novel intelligent leveling system of suspended access platform by adopting a same orientation pursuit strategy. FANG, et al[10], studied an automatic leveling method to calculate the regulating variables of each supporting point for the stage of a precision machining center.

Mounted on the top of an aircraft and utilizing a self-contained, coupled Inertial Navigation System-GPS, a

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This project is supported by Shandong Provincial Natural Science Foundation of China(Grant No. ZR2010EL003)

Stabilized Radiometer Platform(STRAP) was developed by BUCHOLTZ, et al[11], to deal with the offsets and fluctuation problem existing in solar and infrared irradiance measurements. The STRAP could actively keep a set of uplooking radiometers horizontally level within $\pm 0.02°$ for aircraft pitch and roll angles of up to approximately $\pm 10°$. By using an air-bearing rotary stage as the supporter of an electrolytic cell, HAN, et al[12], used the current feedback to characterize the levelness of the substrate of a scanning electrochemical microscopy. Tuning the adjusting screws of the tilt stage, substrate leveling can be completed in minutes by observing the decreased current amplitude. In order to control the deformation of the heavy-weight and long-span work positioner of automatic riveting system, ZHANG, et al[13], used the finite element method to analyze the deformations and the out normal vectors of the drilling and riveting points in multi-pose space, and developed an iterative compensating algorithm for the deformation errors.

Traditional three- or four-leg supporting platforms have been successfully used in many fields. Generally, the supporting legs are vertical to the platform at their fixed joint points. Since there are only three independent equations for a spatial parallel force system, the four-leg supporting platform is one time overdetermined. The conventional way to level a four-leg platform is to repeat trial and error with each individual leg one by one while measuring the inclination of the platform. It is time consuming and has poor precision[14].

For a large-scale platform which carries tens of or even hundreds of tons of equipments, it often spans over ten meters. If only three or four legs are used to support the working platform, its deformation deflection will exceed the allowable range. In order to improve its stiffness, six or more legs will be used. If all of the legs are vertical to the platform, the six-leg platform system is three time overdetermined. For multi-leg platform with more than six legs, it is overdetermined whatever the legs are or not vertical to the platform. The overdetermination will bring two problems:

(1) The coupling of rigid displacements and elastic deformations of the leg-platform system, and the elastic deformation varying with the legs' load distribution, which make the leveling process quite complex.

(2) When the platform is leveled in a horizontal state, the load of each leg is uncertain and variable, which very likely leads to virtual leg phenomenon and overload leg[1]. A virtual leg means that its load is zero or very small by contrast to that of other legs.

In present paper, an optimal load balancing leveling method for multi-leg flexible platforms is proposed, to achieve rapid leveling and load balancing of all the supporting legs. In section 2, the calculation of legs' extension to level a rigid platform is discussed. In section 3 an optimal balancing leg load (OBLL) model is proposed to calculate the ideal load of each leg. And then two kinds of calculation method of legs' extension to achieve OBLL are studied in section 4. Based on the above work, the optimal load balancing leveling method is developed in section 5. At the end, three numerical examples solved with the proposed methods are given out in section 6.

2 Leg Extension Calculation for Leveling a Rigid Platform

For the convenience of discussion, a global Cartesian coordinate system $Oxyz$ is placed at the center of mass of the platform in horizontal state, with the x-axis forward, the y-axis towards the left and the z-axis upward. Besides a body coordinate system $O'x'y'z'$ coincident with the global system in horizontal state is built to incline with the platform.

To simplify the problems, a rigid plane platform is firstly taken as an example for further study. The upper plot of Fig. 1 is the horizontal state of this rigid platform, and the lower plot is its two dimensional inclining state. Supposing the two dimensional inclination(TDI) is a result of first rotating the platform about x-axis with an angle α, then rotating the platform about y-axis with an angle β. α and β are following the right-hand rule, and their unit is radian.

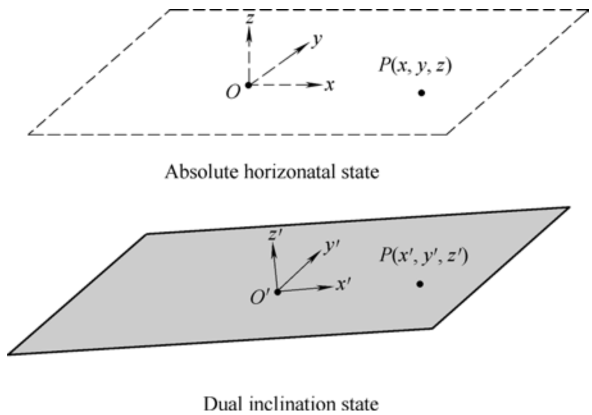

Fig. 1. Two states of a rigid platform

2.1 Relationship between the TDI and the plane normal vector

The base vector along the z-axis or z'-axis, i.e., the normal vector of the rigid plane platform, can be determined with three noncolinear points in the coordinate plane xOy or $x'O'y'$. Supposing that the unit normal vector of the absolute horizontal plane xOy is $\{0, 0, 1\}^{\mathrm{T}}$, and the unit normal vector of the plane $x'O'y'$ expressed in the global coordinate system $Oxyz$ is $\{a, b, c\}^{\mathrm{T}}$, according to rigid body kinematics, the relationship between the two normal vectors can be formulated as follows:

$$\begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{pmatrix} c_\beta & 0 & s_\beta \\ 0 & 1 & 0 \\ -s_\beta & 0 & c_\beta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{pmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}, \tag{1}$$

where $c_\beta = \cos\beta$, $s_\beta = \sin\beta$, $c_\alpha = \cos\alpha$, $s_\alpha = \sin\alpha$. Further rearranging Eq. (1) yields

$$\begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{pmatrix} c_\beta & s_\beta s_\alpha & s_\beta c_\alpha \\ 0 & c_\alpha & -s_\alpha \\ -s_\beta & c_\beta s_\alpha & c_\beta c_\alpha \end{pmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} s_\beta c_\alpha \\ -s_\alpha \\ c_\beta c_\alpha \end{Bmatrix}. \tag{2}$$

The TDI direction cosine matrix can be symbolized as

$$\boldsymbol{A} = \begin{pmatrix} c_\beta & s_\beta s_\alpha & s_\beta c_\alpha \\ 0 & c_\alpha & -s_\alpha \\ -s_\beta & c_\beta s_\alpha & c_\beta c_\alpha \end{pmatrix}. \tag{3}$$

$\boldsymbol{A}$ is an unit orthogonal matrix, i.e.,

$$\boldsymbol{A}^{-1} = \boldsymbol{A}^{\mathrm{T}}. \tag{4}$$

Solving Eq. (2) yields

$$\tan\beta = \frac{a}{c}, \quad \tan\alpha = -\frac{b}{c}\cos\beta. \tag{5}$$

Therefore,

$$\beta = \arctan\frac{a}{c}, \ \alpha = -\arctan\left(\frac{b}{c}\cos\beta\right). \tag{6}$$

If α and β are relatively small, Eq. (6) can be simplified as

$$\beta = \frac{a}{c}, \ \alpha = \frac{b}{c}. \tag{7}$$

If α and β are both less than 0.052 radian (3°), the relative error is no more than 0.1%.

2.2 Leveling displacements(leg extension) of reference point

In Fig. 1, $P(x, y, z)$ is an arbitrary point of the rigid platform in horizontal state, which moves to $P'(x', y', z')$ when the platform inclines. Here, the coordinates (x', y', z') is defined in global coordinate system. (Without otherwise specified, all the coordinates and displacements in this paper are defined in the global coordinate system.) If P is the fixed joint point of a supporting leg and the rigid platform, in the general situation there are $|x| \gg |z|$ and $|y| \gg |z|$. Based upon Eq. (3) and Eq. (4), the relationship between the coordinates of points P and P' can be outlined as

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{pmatrix} c_\beta & 0 & -s_\beta \\ s_\beta s_\alpha & c_\alpha & c_\beta s_\alpha \\ s_\beta c_\alpha & -s_\alpha & c_\beta c_\alpha \end{pmatrix} \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix}. \tag{8}$$

When the platform is leveled from the current inclining sate to the horizontal state by rotating it around a certain axis, the displacement of the point P' to point P can be obtained:

$$\boldsymbol{d} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} - \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{pmatrix} c_\beta - 1 & 0 & -s_\beta \\ s_\beta s_\alpha & c_\alpha - 1 & c_\beta s_\alpha \\ s_\beta c_\alpha & -s_\alpha & c_\beta c_\alpha - 1 \end{pmatrix} \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix}. \tag{9}$$

If α and β are relatively small, Eq. (9) can be simplified as

$$\boldsymbol{d} \approx -\begin{pmatrix} 0 & 0 & \beta \\ 0 & 0 & -\alpha \\ -\beta & \alpha & 0 \end{pmatrix} \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = -\begin{Bmatrix} \alpha \\ \beta \\ 0 \end{Bmatrix} \bullet \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{Bmatrix} -\beta z' \\ \alpha z' \\ \beta x' - \alpha y' \end{Bmatrix}. \tag{10}$$

Eq. (10) shows that the leveling process can be approximately achieved by one time Euler rotation around the axis $(\alpha, \beta, 0)^{\mathrm{T}}$ with an angle $\sqrt{\alpha^2 + \beta^2}$. Supposing that $|x'| \gg |z'|$ and $|y'| \gg |z'|$, and α and β are relatively small, from Eq. (10) we can outline

$$|d_x| \ll |d_z| \ll |x'|, \ |d_y| \ll |d_z| \ll |y'|. \tag{11}$$

Therefore, the displacements of P' can be proximately represented as

$$\boldsymbol{d} \approx \{0, 0, \beta x' - \alpha y'\}^{\mathrm{T}}. \tag{12}$$

If the absolute value of α or β is not very small, the horizontal displacements of point P' can not be ignored. For this case, we can divide α and β into several small angle steps, and repeat using Eq. (10) to calculate the displacement of the point P'.

For a real multi-leg supporting platform, the inclination angles α and β of current state are usually quite small. So only the vertical extensions of the supporting legs are taken into consideration, after ignoring its horizontal displacements and horizontal loads generated in the leveling process. If α and β are less than 0.052 radian (3°), the vertical extension error of Eq. (10) or Eq. (12) is no more than 0.1%, even if α and β are near to 0.175 radian (10°), the error is only about 1%.

If the ratio of extending speed to whole extending length of each leg is controlled synchronously to be same all the time, the fixed joint points of all the legs will maintain the initial geometric configurations during the whole leveling process, without leading to incompatible geometric constrains on the platform which often cause unexpected platform distortion. Another advantage of this proportional control strategy is that it can effectively reduce or even eliminate the leveling coupling between supporting legs.

3 Optimal Balancing Legs' Loads for a Flexible Platform

If there are more than three vertical legs to supporting a work platform, this multi-leg platform must be an overdetermined mechanical system, in that the load of each leg is not only influenced by the mass distribution of the system, but also related to the deformations of the platform and legs. So the geometric leveling of the platform is coupled with the load variation of supporting legs, which makes the leveling process quite complex. However, this problem is a double-edged sword for it provides a possibility of balancing the legs' load without significant changing the geometry state of the platform. It will improve the supporting stiffness and reliability, and reduce the practical requirements for structural strength of the working platform and supporting legs, and the operating pressure of the hydraulic system.

Supposing a working platform in horizontal state is supported with n vertical hydraulic legs, as shown in Fig. 2. The weight of the platform is G and the weight of all the legs is ignored. The horizontal coordinates of the leg i is (x_i, y_i). As mentioned above, the origin of the global coordinate system is located at the center of mass of the platform.

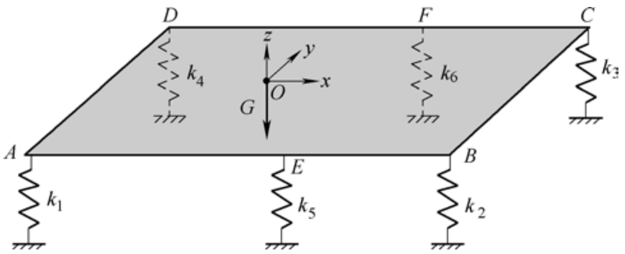

Fig. 2. Multi-leg platform

The ideal legs' loads distribution is all the legs' loads equal to the mean load $\overline{R}=G/n$. In fact this distribution is almost impossible to be achieved in a real system. But we can take this distribution as an ideal goal to build an optimization model as follows:

$$\begin{cases} \min & \dfrac{1}{2}\displaystyle\sum_{i=1}^{n}(R_i-\overline{R})^2, \\ \text{s.t.} & \displaystyle\sum_{i=1}^{n}R_i-G=0, \\ & \displaystyle\sum_{i=1}^{n}R_i x_i=0, \\ & \displaystyle\sum_{i=1}^{n}R_i y_i=0, \end{cases} \tag{13}$$

where the three constrain equations are the static equilibrium equations of the spatial parallel force system, and R_i is the vertical load of the leg i. This is an optimization problem with equality constraint, and can be solved with the Lagrange multiplier method. A Lagrange function can be outlined as

$$L=\frac{1}{2}\sum_{i=1}^{n}(R_i-\overline{R})^2+\lambda_1\left(\sum_{i=1}^{n}R_i-G\right)+\lambda_2\sum_{i=1}^{n}R_i x_i+\lambda_3\sum_{i=1}^{n}R_i y_i. \tag{14}$$

The optimal solution must be the roots of the following equations:

$$\begin{cases} \dfrac{\partial L}{\partial R_i}=0, \ \ i=1,2,\cdots,n, \\ \dfrac{\partial L}{\partial \lambda_j}=0, \ \ j=1,2,3. \end{cases} \tag{15}$$

Substitute Eq. (14) into Eq. (15), we can obtain

$$\begin{cases} R_i^*+\lambda_1+\lambda_2 x_i+\lambda_3 y_i=\overline{R}, \ \ i=1,2,\cdots,n, \\ \displaystyle\sum_{i=1}^{n}R_i^*=G, \\ \displaystyle\sum_{i=1}^{n}R_i^* x_i=0, \\ \displaystyle\sum_{i=1}^{n}R_i^* y_i=0, \end{cases} \tag{16}$$

where R_i^*, the balancing load of leg i, is the optimal solution of optimization problem (13). Eq. (16) can be rewritten in matrix form:

$$\begin{pmatrix} 1 & 0 & \cdots & 0 & 1 & x_1 & y_1 \\ 0 & 1 & \cdots & 0 & 1 & x_2 & y_2 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 1 & x_n & y_n \\ 1 & 1 & \cdots & 1 & 0 & 0 & 0 \\ x_1 & x_2 & \cdots & x_n & 0 & 0 & 0 \\ y_1 & y_2 & \cdots & y_n & 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} R_1^* \\ R_2^* \\ \vdots \\ R_n^* \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{Bmatrix} = \begin{Bmatrix} \overline{R} \\ \overline{R} \\ \vdots \\ \overline{R} \\ G \\ 0 \\ 0 \end{Bmatrix}. \tag{17}$$

Eq. (17) is a definite linear system of equations, and its solution $\boldsymbol{R}^*=\left\{R_1^*, R_2^*, \cdots, R_n^*\right\}^{\mathrm{T}}$ is just the optimal balancing legs' loads (OBLL). Next, we will study the calculation procedure for the extension of each supporting leg to achieve OBLL.

4 Leg Extension Calculation for OBLL

It is assumed that the multi-leg platform system stands on the ground with all the supporting legs keeping in touch with the ground, which means the load of each leg is greater than or equal to zero, and no tension force exists in any leg. The current load R_i^{C} of leg i ($i=1,2,\cdots,n$) can be measured with a force sensor or oil pressure gauge, and the difference between this current load R_i^{C} and the corresponding OBLL R_i^* is

$$\Delta R_i = R_i^* - R_i^{\mathrm{C}}. \tag{18}$$

There are two ways to calculate the extension of each leg: the first only uses each individual leg's stiffness, referred to as individual extension method (IEM), and the second takes the coupling stiffness between legs into consideration, referred to as coupling extension method (CEM).

4.1 Individual extension method (IEM)

IEM calculates the leg extension by dividing the load difference ΔR_i, which is obtained from Eq. (18), with the corresponding equivalent leg stiffness. The own axial stiffness k_i of each leg can be taken as equivalent stiffness, then the extension of the leg i can be calculated as

$$e_i = \frac{\Delta R_i}{k_i}. \tag{19}$$

This method is hereinafter referred to as IEM1.

We can also take the flexibility of the working platform at the fixed joint points into consideration, and quantitatively define the equivalent leg stiffness k_i^{E} as follows.

Definition 1: Ignoring any external loads (including weight), constraining the bottom points of all the supporting legs but leg *i*, a certain vertical force is applied at the bottom point of leg *i* to result in a unit vertical displacement at the bottom of leg *i*, the applied force is just the equivalent leg stiffness k_i^{E}.

According to the IEM, the extension of leg i can be calculated as

$$e_i = \frac{\Delta R_i}{k_i^{\mathrm{E}}}. \tag{20}$$

This method is referred to as IEM2.

IEM1 and IEM2 use the same assumption that the extension of a certain leg only leads to the load variation of its own, and has no influence on all of the other legs' loads. It is not entirely true in real platform system, so we can not get the exact solution to achieve OBLL with only one time calculation of IEM. But we can improve the solution by iteration with IEM. Because IEM2 take the elastic deformation of working platform, it seems to be more precise than IEM1, however, it tends to diverge in iterative computation. An example is illustrated in section 6.2.

4.2 Coupling extension method(CEM)

The main idea of CEM to calculate the extension of each leg is: not only the legs' own stiffness but also the elastic flexibility of the working platform being taken into consideration, and not only the influence on its own load but also the coupling influence on all the other legs' load caused by the extension of each leg being taken into consideration.

Definition 2: Ignoring any external loads (including weight), a set of vertical forces are applied at the bottom points of all the supporting legs but leg *j* to constraint these bottom points fixed in *z*-direction, a certain vertical force is applied at the bottom point of leg *j* to result in a unit vertical displacement at the bottom of leg *j*, the force applied at the bottom of leg *i* is the stiffness coefficient k_{ij}. According to the Maxwell reciprocity, $k_{ij} = k_{ji}$. All stiffness coefficients constitute the supporting stiffness matrix(SSM) of a multi-leg platform system:

$$\boldsymbol{K} = \begin{pmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{pmatrix}. \tag{21}$$

Noting that the summation of each row or column of the supporting matrix $\boldsymbol{K}$ is equal to zero, this matrix is singular. Further analysis shows that if all of the legs are vertical to the working platform, and only the vertical linear displacements are taken into consideration, the whole leg-platform system will have three rigid DOF: translation along *z*-axis, rotation about *x*-axis and rotation about *y*-axis. For this case, the rank of the supporting stiffness matrix $\boldsymbol{K}$ is $n-3$.

The procedure of CEM is as follows: Firstly taking the platform weight into consideration, the vertical reaction force column vector $\boldsymbol{R}^{\mathrm{C}}$ of all legs in current state is calculated or measured; Then the extension of each leg to achieve OBLL is solved form Eq. (22):

$$\boldsymbol{K}\boldsymbol{e} = (\boldsymbol{R}^* - \boldsymbol{R}^{\mathrm{C}}), \tag{22}$$

where $\boldsymbol{e} = \{e_1, e_2, \cdots, e_n\}$ is the extension column vector of each leg.

Because the supporting stiffness matrix $\boldsymbol{K}$ is singular with a rank $n-3$, its singularity can be eliminated by assuming any three extension elements of the vector $\boldsymbol{e}$ equal to zero. For example, assuming the last three extension elements equal to zero, so the Eq. (22) can be rewritten as

$$\boldsymbol{K}_{1:n,1:n-3}\boldsymbol{e}_{1:n-3,1} = (\boldsymbol{R}^* - \boldsymbol{R}^{\mathrm{C}})_{1:n,1}. \tag{23}$$

This is an quasi-overdetermined linear system of equations, and the least square solutions can be obtained with the generalized matrix inverse method[15]. Another way to let Eq. (23) become a determined problem is just to remove any three equations from it. For example, removing the last three equations, Eq. (23) can be rewritten as

$$\boldsymbol{K}_{1:n-3,1:n-3}\boldsymbol{e}_{1:n-3,1} = (\boldsymbol{R}^* - \boldsymbol{R}^{\mathrm{C}})_{1:n-3,1}. \tag{24}$$

Generally, the solutions of Eq. (23) have less error for the whole multi-leg platform system than that of Eq. (24).

5 OBLL Leveling Method for a Flexible Platform

It is assumed that there are n vertical legs supporting a flexible working platform, and m reference points on the platform surface are selected to evaluate the levelness of the platform, generally $m \geqslant 3$. Under the small inclination assumption, only the z-axis translation displacements are considered, and the theoretical displacement Δz_i of each reference points required to level the platform can be calculated with Eq. (9) or Eq. (10). All of the Δz_i constitute the displacements vector of reference points:

$$\Delta z = \left\{\Delta z_1, \Delta z_2, \cdots, \Delta z_m\right\}^{\mathrm{T}}. \tag{25}$$

Definition 3: Ignoring any external loads (including weight), constraining the bottom points of all the supporting legs but leg j, a certain vertical force is applied at the bottom point of leg j to result in a unit vertical displacement at the bottom of leg j, in that way the vertical displacements of reference point i is the static transmissibility coefficient t_{ij} from point j to point i. All the static transmissibility coefficients constitute the static transmissibility matrix(STM) of a multi-leg platform system:

$$\boldsymbol{T} = \begin{pmatrix} t_{11} & t_{12} & \cdots & t_{1n} \\ t_{21} & t_{22} & \cdots & t_{2n} \\ \vdots & \vdots & & \vdots \\ t_{m1} & t_{m2} & \cdots & t_{mn} \end{pmatrix}. \tag{26}$$

The static transmissibility coefficient does not satisfy the Maxwell reciprocity, i.e., $t_{ij} \neq t_{ji}$.

The required extension of each leg, to level the platform which is evaluated from the displacements vector of reference points in Eq. (25), and to achieve the OBLL solved from Eq. (17), is the solution $\boldsymbol{e}$ of the following linear system of equations

$$\begin{pmatrix} \boldsymbol{K} \\ \boldsymbol{T} \end{pmatrix}_{(m+n)\times n} \{\boldsymbol{e}\}_{n\times 1} = \begin{Bmatrix} \boldsymbol{R}^{*} - \boldsymbol{R}^{\mathrm{C}} \\ \Delta z \end{Bmatrix}_{(m+n)\times 1}. \tag{27}$$

The rank of the supporting stiffness matrix $\boldsymbol{K}$ is $n-3$, which means that there must be three rows of the matrix are linear dependent to other rows. On the other hand three reference points can exactly define a plane to evaluate the levelness of the platform, which is used to measure the levelness of platform. If $m=3$, Eq. (27) is a determined problem and its solution is just satisfied the geometric leveling requirements and achieves the OBLL at the same time. In practical application, more than three reference points may be used to comprehensively and objectively evaluate the levelness of the platform. In this case, Eq. (27) will be overdetermined and only the least square solution or the weighted least square solution can be obtained.

In military or civil engineering, most of the multi-leg working platforms adopt hydro-cylinder legs. It is more precise and reliable to control a hydro-cylinder leg to stretch out than to draw it back. For this case, we can increase all the elements of the solution vector $\boldsymbol{e}$ resulted from Eq. (27) with a same positive value Δe to make the extension of each leg nonnegative, which only vertically translates the whole platform without changing its levelness and the load of each leg. Usually, the solution of Eq. (27) can be modified with the following equation:

$$\boldsymbol{e}^{*} = \boldsymbol{e} - \min(\boldsymbol{e}), \tag{28}$$

where $\boldsymbol{e}^{*}$ is no longer the solution of Eq. (27).

6 Numerical Examples

6.1 OBLL calculation example

As demonstrated in Fig. 2, a steel plane platform with size 2 000 mm×600 mm×5 mm is supported with six legs, which are vertically fixed with the platform at point A through F respectively. With the coordinate system defined in section 2, the horizontal coordinates and stiffness of each leg are listed in Table 1. Only the gravity of the platform is taken into consideration. The gravity acceleration is $g = 9.8\ \mathrm{m/s^2}$. This example is just to calculate the OBLL.

Table 1. XY coordinates and stiffness of each supporting leg

Leg	Coordinate x/mm	Coordinate y/mm	Stiffness k/(N • mm^{-1})
1	−1 000	−300	10
2	1 000	−300	15
3	1 000	300	12
4	−1 000	300	20
5	200	−300	8
6	200	300	14

In ANSYS platform, using the element type SHELL63 to mesh the plane platform and COMBIN14 to simulate the six legs, the current load of each leg without any extension can be calculated using FEA. And the OBLL of this multi-leg platform system is calculated by solving Eq. (17). All theses results are listed in Table 2. The OBLL is more balancing than the current load distribution.

Table 2. Current load and OBLL of each leg

Leg	Current load R_i^{C} / N	OBLL R_i^{*} / N
1	68.45	84.49
2	59.10	69.40
3	30.49	69.40
4	67.72	84.49
5	101.77	75.43
6	131.11	75.43

6.2 Leg extension calculation examples for OBLL

Taking the OBLL result listed in Table 2 as the objective, the leg extensions are calculated with IEM1, IEM2 and

CEM respectively. The leg extension results are taken as placement boundary conditions applied at the bottom node of each COMBIN14 element of the finite element model shown in Fig. 3. Then the leg loads are verified by FEA.

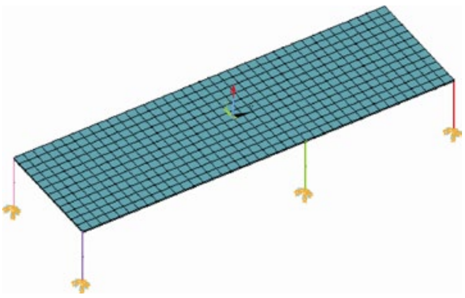

Fig. 3.　FEA model of a multi-leg platform

Although the results obtained from IEM1 do not converge to the exact solutions with only one time computation, by taking the boundary conditions applied with calculated extension displacement as a new current state, the results can be gradually improved by carrying on several iterations. Further study shows that IEM1 is linearly convergent. In Fig. 4, the first results are the current legs' load without any extension, the others are the results of each iterations. The results of the last three iterations are very close to the OBLL. The iteration results of IEM2 are shown in Fig. 5. Obviously they are divergent.

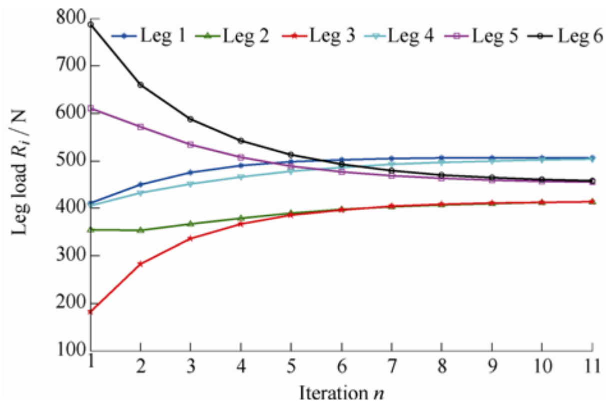

Fig. 4.　Iteration of IEM1

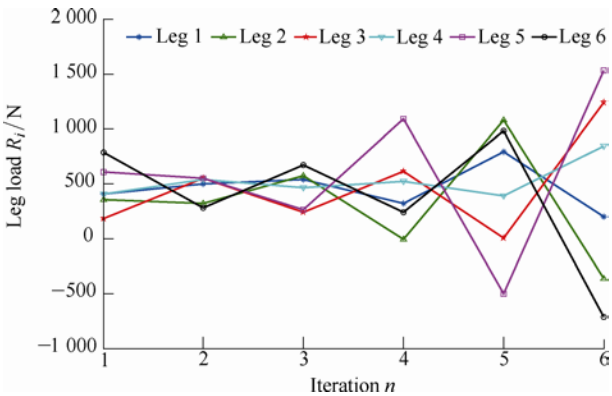

Fig. 5.　Iteration of IEM2

To use CEM to calculate the leg extensions, the supporting stiffness matrix (SSM) listed in Table 3 is calculated firstly in ANSYS according to the Definition 1.

Table 3.　SSM of a six-leg platform　N/mm

Leg	1	2	3	4	5	6
1	2.149 8	−0.576 4	1.013 0	−1.858 7	−1.573 4	0.845 7
2	−0.576 4	2.758 1	−1.903 9	1.145 8	−2.181 7	0.758 1
3	1.013 0	−1.903 9	2.899 1	−0.349 6	0.890 9	−2.549 5
4	−1.858 7	1.145 8	−0.349 6	2.389 5	0.712 8	−2.039 9
5	−1.573 4	−2.181 7	0.890 9	0.712 8	3.755 1	−1.603 7
6	0.845 7	0.758 1	−2.549 5	−2.039 9	−1.603 7	4.589 5

Supposing the last three legs' extension are zero, the one time computation results with CEM are compared with the last iterations results of IEM1 in Table 4.

Table 4.　Results of IEM1 and CEM

Leg	OBLL	IEM1		CEM	
	R_i^* / N	Extension e_i / mm	Load R_i / N	Extension e_i / mm	Load R_i / N
1	84.49	22.09	84.46	0.18	84.49
2	69.40	19.37	68.90	23.77	69.40
3	69.40	47.99	69.03	28.97	69.40
4	84.49	17.83	83.94	0.00	84.49
5	75.43	−63.95	75.97	0.00	75.44
6	75.43	−66.62	76.35	0.00	75.44

6.3　Optimal load balancing leveling example

Taking the point (−100, −50, 0) as rotation center, firstly rotating the platform demonstrated in Fig. 2 about an axis parallel to the global x-axis positive direction with an angle 1°, and then rotating it about an axis parallel to the global y-axis positive direction with an angle 2°, then the key points of A through F are moved to A' through F', and the key point initially coinciding with the global origin O is moved to O'. Still taking the global coordinate system as reference, and supposing all the legs remain vertical state (along with the global z-axis direction), i.e., the whole structure obeys the small displacement hypothesis, the coordinates of each key points after inclination are listed in Table 5. We now take the key points E', F' and O' as references to level the platform, in other words, the three key points should have the same z coordinate in the end.

Table 5.　Coordinates of key points after inclination

Point	Coordinate x/mm	Coordinate y/mm	Coordinate z/mm
A'	−997.78	−300.57	61.901
B'	997.35	−299.35	−77.602
C'	997.72	300.56	−67.136
D'	−997.41	299.34	72.366
E'	199.30	−299.84	−21.801
F'	199.67	300.07	−11.336
O'	−0.030 5	−0.007 6	−2.618

According to Definitions 1 and 2, the supporting stiffness matrix (SSM) and static transmissibility matrix (STM) after inclination are listed in Tables 6 and 7 respectively. All the coefficients are obtained with FEA.

Table 6. SSM of the six-leg platform N/mm

Leg	1	2	3	4	5	6
1	2.155 2	−0.579 3	1.017 0	−1.863 3	−1.575 9	0.846 3
2	−0.579 3	2.764 4	−1.907 2	1.150 7	−2.185 1	0.756 5
3	1.017 0	−1.907 2	2.905 8	−0.351 3	0.890 2	−2.554 4
4	−1.863 3	1.150 7	−0.351 3	2.396 3	0.712 6	−2.044 9
5	−1.575 9	−2.185 1	0.890 2	0.712 6	3.761 1	−1.602 8
6	0.846 3	0.756 5	−2.554 4	−2.044 9	−1.602 8	4.599 3

Table 7. STM of the six-leg platform mm/mm

Point	1	2	3	4	5	6
E'	0.197 0	0.273 1	−0.111 3	−0.089 1	0.529 9	0.200 3
F'	−0.060 5	−0.054 0	0.182 5	0.146 1	0.114 5	0.671 5
O'	0.116 4	0.064 7	−0.008 4	0.087 8	0.318 8	0.420 6

The current loads and OBLL after inclination are recalculated, the leg extensions are calculated with Eq. (27) and Eq. (28), and the leg loads and the z coordinate position of the reference points are verified with FEA in ANSYS.

The leg loads before and after leveling are listed in Table 8, and the z coordinate position of the reference points before and after leveling are listed in Table 9, where uz is the displacement of reference points before leveling caused by elastic deformation, $z+uz$ is the vertical position of reference points before leveling caused by rigid rotation and elastic deformation, uz' is the displacement of reference points after leveling resulted from leg extensions and elastic deformation, and $z+uz'$ is the vertical position of reference points after leveling resulted from rigid rotation, leg extensions and elastic deformation. The z-axis deformation contour and y-direction view of the multi-leg platform ofter leveling are shown in Fig. 6. From y-direction view, the plane, defined by the reference point E', F' and O', is placed in the horizontal state.

Table 8. Extension and load of each leg

Leg	Current load R_i^{C} /N	OBLL R_i^* /N	Extension e_i /mm	Final load R_i /N
1	69.97	84.49	14.69	84.49
2	59.18	69.40	138.60	69.40
3	30.53	69.40	129.29	69.40
4	67.78	84.49	0.00	84.49
5	101.68	75.43	74.79	75.44
6	131.02	75.43	60.28	75.43

Table 9. Location of reference points mm

Point	Displacement before leveling uz	Location before leveling $z+uz$	Displacement after leveling uz'	Location after leveling $z+uz'$
E'	−12.71	−34.51	65.36	43.56
F'	−9.36	−20.69	54.90	43.56
O'	−12.62	−15.24	46.18	43.56

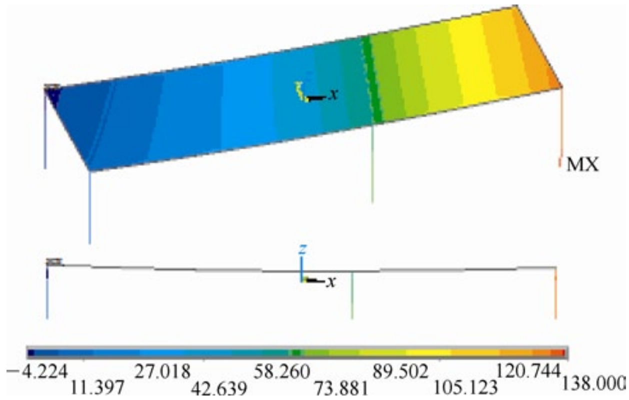

Fig. 6. z-axis deformation of the six-leg platform (mm)

7 Conclusions

(1) With the small rotation angle synthesis, the displacement of reference or leg extension can be approximately calculated from Eq. (10) or Eq. (12). For large rotation angle case, the rotation angle can be divided into several small angle increments and calculated iteratively. Controlling the extending speed of each leg proportion to its corresponding needed whole extension synchronously will reduce or even eliminate the unexpected platform distortion and leveling coupling between supporting legs.

(2) For overdetermined structures, the reaction force distribution can be optimized by slightly regulating the initial displacement of the boundary DOFs, in which the key problem is to determine the supporting stiffness matrix of the boundary DOFs.

(3) Combining the supporting stiffness matrix and static transmissibility matrix, the geometry leveling and OBLL problem can be solved simultaneously with Eq. (27). And this method can be further applied to general overdetermined structures to achieve controlling the displacements of reference DOFs and optimizing the reaction forces distribution.

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