ACOUSTICS AND HYDROACOUSTICS

Dust Acoustic Solitary Waves with Vortex-Like Ion Distribution in Two-Dimensional Spherical Geometry[1]

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Abstract—Nonplanar two-dimensional (2D) spherical dust acoustic solitary waves (DASWs) in unmagnetized, collisionless, Boltzmann distributed electrons, negatively charged dust fluid and trapped ions following vortex-like ion distribution, in a dusty plasma were investigated theoretically. Using standard reductive perturbation technique, which is valid for a small but finite amplitude limit condition, nonlinear spherical modified Korteweg−de Vries (K-dV) equation was achieved. Two motions are observed in the radial and angular directions, with transverse perturbations in the angular direction. It is found that the properties of the DASWs in a 2D spherical geometry differ from 1D spherical geometry where transverse perturbations and unidirectional waves are observed for 2D spherical geometry. The effects of dusty plasma parameters and vortex-like ion distribution on the properties (such as amplitude and width) of spherical DASWs were theoretically investigated. These numerical investigations show that under such specific conditions, only compressive DASWs can exist.

1. INTRODUCTION

Dusty plasmas can be found in astrophysics, space and laboratory plasmas [1−3] in the form of mixtures of ordinary plasma particles and dust charge grains. The dust particles are massive and heavier than ion. These dust grains not only change the characteristics of plasma, but also present new types of waves from which dust acoustic wave (DAW) is an important one. Theoretically Rao et al. [4] predicted an extremely low frequency (typically a few Hz) DAW. Barkan et al. [5] have observed DAWs in laboratory experiments. The first observation of waves in Saturn's rings was theoretically modeled by Bilokh and Yarashenko [6]. In the DAWs, the inertia comes from the dust particle mass and the restoring force is provided by the pressure of the electrons and ions. The DASWs have been investigated using various models. Most of these models are limited to planar geometry and one-dimensional (1D) geometry. However, considering 1D geometry in laboratory devices and in space may not be a realistic. Franz et al. [7] have shown that 1D geometry may not be a correct model in space, especially at higher polar altitudes. The differences between the characteristic

properties of DASWs in bounded nonplanar geometry and those in unbounded planar geometry were also reported in [8−10].

However, such reports cover only some limited conditions, for example, their studies are limited to the radial symmetry case while unidirectional waves and transverse perturbation, which expected to occur in the higher dimensional system, are ignored. In fact, the transverse perturbations occur in the higher dimensional system. Therefore, further investigations are required to understand DASWs in higher dimensional nonplanar geometry. Sahu et al., [11] investigated the properties of 1D cylindrical and spherical DASWs in unmagnetized electron depleted dusty plasma. They showed that the behavior of non-extensive nonplanar DAW is quite different from that of planar. Cylindrical and spherical DASWs containing strongly correlated negatively charged dust grains in 1D geometry were investigated by Mamun et al. [12]. They showed that nonplanar geometry significantly modifies amplitude, width, and speed of DA solitary waves. They have shown that the amplitude of cylindrical solitary structures is larger than that of the planar solitary structures, but smaller than that of the spherical solitary structures. They also concluded that

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[1] The text was submitted by the authors in English.

strong correlation between the negatively charged dust grains leads to the formation of such solitary structures. Mamun et al. [13] also investigated DAW in unmagnetized dusty plasma system consisting of positively and negatively charged dust, with Boltzmann distributed electrons and ions. They examined the basic features of the planar and nonplanar DAW in 1D geometry and found that the amplitude of the cylindrical DAW is larger than that of planar, but smaller than that of the spherical, which confirmed the previous results [12]. DASWs in an unmagnetized dusty plasma, consisting of superthermal electrons, Boltzmann distributed ions, in the 1D cylindrical and spherical geometry were investigated by Ghosh et al. [14]. They found that both dip-like and hump-like solitary waves can exist, and the amplitude and width of these two different solitary waves are significantly affected by super thermal electrons. Rahim et al. [15] investigated dispersion relation of linear nonplanar DASWs in 1D geometry. They assumed that the plasma components were negatively charged dust grains and degenerate electrons and ions obeying the Thomas-Fermi distribution. They showed that consideration of a geometrical term in dispersion relation gives damping along the radial axis. Applying the reductive perturbation method, they derived the modified K-dV equation in 1D nonplanar geometry. They have shown that the numerical solution of nonplanar DAW is significantly different from planar geometry. They also found that in 1D geometry, the spherical DAW moves faster than the cylindrical and planar solitons. On the other hand, it was shown that in the laboratory and space plasma, the presence of fast ions trapping in the amplitude of wave potential may lead to the formation of phase space holes [16–18]. These trapped ions, which modify the propagation characteristics of DASWs, have velocity distribution that deviates from Maxwellian distribution. Particularly in most space plasma, these trapped ions obey vortex-like ion distribution.

Some authors have investigated the effects of the vortex-like ion distribution on the propagation of DASWs in 1D geometry related to dusty plasmas.

Paul et al. [19] showed that trapped ions, which form stronger nonlinearity (in comparison with Boltzmann ion distribution), modify the basic features of the DASWs. They have used the reductive perturbation method, which is valid for small but finite amplitude, to derive a modified K-dV equation. The effects of vortex-like ion and nonthermal electron distributions were investigated by Eslami et al. [20]. They have used reductive perturbation theory to derive a modified K-dV equation. They showed that the presence of vortex-like ions modify the nature of DASWs structures. Their results show that such dusty plasma model only supports negative potential.

As the reasons explained above it is of great significance to examine the effects of the vortex-like ion distribution and transverse perturbations on DASWs in higher-dimensional spherical geometry, which occur in laboratory and space plasma.

2. BASIC EQUATIONS

We study DAWs in unmagnetized two dimensions, nonplanar spherical geometry with vortex-like ion distribution and Boltzmann electron distribution. The normalized basic equations in 2D spherical geometry are given as follows:

$$\frac{\partial n_d}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 n_d u_d)}{\partial r} + \frac{1}{r}\frac{\partial (r^2 n_d v_d)}{\partial \theta} + \frac{n_d v_d}{r}\cot\theta = 0, \quad (1)$$

$$\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial r} + \frac{v_d}{r}\frac{\partial u_d}{\partial \theta} - \frac{v_d^2}{r} = \frac{\partial \phi}{\partial r}, \quad (2)$$

$$\frac{\partial v_d}{\partial t} + u_d\frac{\partial v_d}{\partial r} + \frac{v_d}{r}\frac{\partial v_d}{\partial \theta} + \frac{u_d v_d}{r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta}, \quad (3)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial \phi}{\partial \theta} \quad (4)$$
$$= n_d + n_e - n_i,$$

where n_d is the dust number density that is normalized to unperturbed equilibrium dust number density n_{d0}, ϕ is the electrostatic potential that is normalized to T_i/e, where e is the electron charge. The parameters r, θ are the radial and angle coordinates, respectively; u_d, v_d are the dust fluid velocity in r, θ directions, respectively. These velocities are normalized to effective dust acoustic velocity $C_d = \sqrt{Z_{d0} T_i/m_d}$, where m_d is the mass of negatively charged dust particles and Z_{d0} is number of charges residing on the dust grain.

The time t is normalized to the dust plasma frequency $\omega_{pd}^{-1} = \sqrt{m_d/4\pi n_{do}Z_{d0}^2 e^2}$, and the parameter r is normalized to Debye radius $\lambda_{Dd} = \sqrt{T_i/4\pi n_{do}Z_{d0}e^2}$; n_e is the Boltzmann electron distribution and is defined as

$$n_e = \frac{\mu}{1-\mu}e^{\delta\phi}, \quad (5)$$

where $\mu = n_{e0}/n_{i0}$, $\delta = T_i/T_e$, n_{e0}, n_{i0} are unperturbed equilibrium electron and ion density, respectively. The two parameters T_e, T_i are electron and ion temperature, respectively.

To study the effects of non-Maxwellian ions on the nonlinear DAWs, we employ the vortex-like ion distribution function of Schamel [21, 22] that solves the ion Vlasov equation:

$$f_{if} = \frac{1}{\sqrt{2\pi}}e^{-(v^2+2\phi)/2}, \quad |v| > \sqrt{-2\phi}, \quad (6)$$

$$f_{it} = \frac{1}{\sqrt{2\pi}} e^{-\alpha(v^2+2\phi)/2}, \quad |v| \le \sqrt{-2\phi}. \tag{7}$$

The above distribution functions are continuous in velocity space and satisfy the regularity requirements for an admissible BGK solution [22]. The parameter α is the ratio of the free-ion temperature T_{if} to the trapped-ion temperature T_{it}. The condition $\alpha < 0$ represents a vortex-like excavated trapped ion distribution, and $\alpha = 0$ represents the plateau of Maxwellian ion distribution. The condition $-\alpha < 0$ was considered in our calculations. Subscripts f and t indicate the free and trapped ion contributions, respectively. Integrating the ion distribution functions over the velocity space, we obtain the ion number density n_i as

$$n_i = \frac{1}{1-\mu}\left[1 - \phi - \frac{4}{3}a(-\phi)^{3/2} + \frac{1}{2}\phi^2\right], \tag{8}$$

where $a = (1-\alpha)/\sqrt{\pi}$.

To investigate the propagation of spherical DASWs in dusty plasmas, the standard reductive perturbation method was employed to obtain 2D spherical modified K-dV equation [23]. According to this method, an appropriate coordinate frame is required. To find this frame, we need to introduce the following stretched coordinates:

$$\xi = \varepsilon^{1/4}(r - v_0 t), \tag{9}$$

$$\eta = \varepsilon^{-1/4}\theta, \tag{10}$$

$$\tau = \varepsilon^{3/4}t, \tag{11}$$

where ε $(0 < \varepsilon < 1)$ is a dimensionless small parameter measuring the weakness of nonlinearity and dispersion and v_0 is wave phase speed. The variables can be expanded as

$$n_d = 1 + \varepsilon n_{d1} + \varepsilon^{3/2}n_{d2} + ..., \tag{12}$$

$$u_d = \varepsilon u_{d1} + \varepsilon^{3/2}u_{d2} + ..., \tag{13}$$

$$v_d = \varepsilon^{5/4}v_{d1} + \varepsilon^{3/2}v_{d2} + ..., \tag{14}$$

$$\phi = \varepsilon\phi_1 + \varepsilon^{3/2}\phi_2 + ... \tag{15}$$

Now, substituting Eqs. (9)−(11) and (12)−(15) into (1)−(4), we obtain the lowest order in ε:

$$n_{d1} = -\frac{1}{v_0^2}\phi_1, \tag{16}$$

$$u_{d1} = -\frac{1}{v_0}\phi_1, \tag{17}$$

$$v_0 = \sqrt{\frac{1-\mu}{1+\mu\delta}}. \tag{18}$$

Equation (18) describes the phase speed of the perturbation mode for the low-frequency DASWs regard-

ing the dusty plasma under consideration. This equation does not depend on the α parameter. For $\mu \to 0$, i.e., electron depleted plasma, it is in good agreement with obtained by Mamun [24]:

$$\frac{\partial v_{d1}}{\partial \xi} = -\frac{1}{v_0^2\tau}\frac{\partial \phi_1}{\partial \eta}. \tag{19}$$

For the next higher order in ε, we obtain

$$\frac{\partial n_{d1}}{\partial \tau} - v_0\frac{\partial n_{d2}}{\partial \xi} + \frac{1}{v_0\tau}\frac{\partial v_{d1}}{\partial \eta} + \frac{v_{d1}}{v_0\tau\eta} + \frac{\partial u_{d2}}{\partial \xi} + \frac{2}{v_0\tau}u_{d1} = 0, \tag{20}$$

$$\frac{\partial u_{d1}}{\partial \tau} - v_0\frac{\partial u_{d2}}{\partial \xi} = \frac{\partial \phi_2}{\partial \xi}, \tag{21}$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = n_{d2} + \frac{1}{1-\mu}(1+\mu\delta)\phi_2 + \frac{4}{3(1-\mu)}a(-\phi_1)^{3/2}. \tag{22}$$

Now, using Eqs. (16)−(22) and eliminating n_{d2}, u_{d2}, and ϕ_2 from Eqs. (20)−(22), we obtain the following equation:

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi_1}{\partial \tau} + B\frac{\partial^3 \phi_1}{\partial \xi^3} + A\sqrt{-\phi_1}\frac{\partial \phi_1}{\partial \xi} + \frac{\phi_1}{\tau}\right] + \frac{1}{2v_0\tau^2}\left(\frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial \phi_1}{\partial \eta}\right) = 0, \tag{23}$$

where nonlinear term A and dispersive term B are defined as:

$$A = \frac{v_0}{1+\mu\delta}a, \tag{24}$$

$$B = v_0^3/2. \tag{25}$$

From Eq. (24), one can say that the nonlinear term A is always positive as a and other parameters (μ, δ, v_0) are positive. From Eq. (23), it is clear that as the value of τ decreases, the effect of spherical geometry which appears in ϕ_1/τ becomes important in comparison with the case of 1D planar modified K-dV equation. Also under circumstance of $\tau \to \infty$, the 2D spherical geometry solution approaches the solution of 1D planar geometry. For the cylindrical geometry, it is observed that the term ϕ_1/τ decreased by half ($\phi_1/2\tau$). Therefore, the characteristic properties of solitary waves (such as phase shift, amplitude) in spherical geometry are different from those in cylindrical geometry. This scenario was predicted by Mamun et al. [12, 13] and Rahim et al. [15].

It is clear that the term $\frac{1}{2v_0\tau^2}\left(\frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial \phi_1}{\partial \eta}\right)$ in Eq. (23) indicates the effect of the transverse perturbations. If the wave propagates without the transverse perturbation, this mentioned term in Eq. (23) disappears, and Eq. (23) reduces to the planar modified K-dV equation.

In Eq. (23), two terms, i.e. ϕ_1/τ and $\frac{1}{2v_0\tau^2}\left(\frac{\partial^2\phi_1}{\partial\eta^2} + \frac{1}{\eta}\frac{\partial\phi_1}{\partial\eta}\right)$, can be canceled if we assume $\zeta = \xi - v_0\eta^2\tau/2$, $\tau = \tau$, and $\phi_1 = \phi_1(\zeta, \tau)$. In such condition, Eq. (23) is reduced to the spherical modified K-dV equation, which is given below:

$$\frac{\partial\phi_1}{\partial\tau} + A\sqrt{-\phi_1}\frac{\partial\phi_1}{\partial\zeta} + B\frac{\partial^3\phi_1}{\partial\zeta^3} = 0. \quad (26)$$

The modified K-dV Eq. (26) is a third-order differential equation that contains a nonlinear term. This is in contrast to the usual K-dV equation, which deals with the dynamics of nonlinear ion-acoustic waves in an electron−ion plasma without the dust component [23, 24]. A stationary solitary wave solution of Eq. (26) can be obtained by performing two steps: (i) introducing new variables $\chi = \zeta - u_0\tau$ and $\tau = \tau$, and using them instead of ζ, τ, where u_0 is a constant solitary wave velocity normalized by the dust acoustic speed C_d, and (ii) imposing the boundary conditions for localized disturbances, i.e. $\phi \to 0$, $d\phi/d\chi \to 0$, $d^2\phi/d^2\chi \to 0$ as $\chi \to \pm\infty$.

Such condition results in the following equations:

$$\phi_1 = -\phi_m \sec h^4\left[\frac{\xi - \left(u_0 + v_0\eta^2/2\right)\tau}{\sigma}\right], \quad (27)$$

where the amplitude ϕ_m and the width σ are given by $\phi_m = (15u_0/8A)^2$ and $\sigma = \sqrt{16B/u_0}$, respectively.

This amplitude of the DA solitary waves indicates that it does not depend on the sign of nonlinear parameter A. This is due of the vortex-like ion distribution.

3. NUMERICAL RESULTS AND DISCUSSIONS

Equation (27) shows that by considering transverse perturbation, spherical solitary wave with a small amplitude can be formed. Equation (27) can be rewritten versus n_{d1}, if we replace ϕ_1 by $-v_0^2 n_{d1}$. Equation (27) indicates that under these conditions, negative potential or compressive DASWs (DAW with density hump) can be formed. As u_0 increases, the amplitude ϕ_m increases while the width σ decreases. The width is proportional to B while amplitude ϕ_m inversely proportional to A and a. The value of α does not have significant effect on width σ but can influence the parameter ϕ_m. At the condition of $\alpha < 0$ (a vortex-like excavated trapped ion distribution), the absolute value

of $|\alpha|$ is important so that when $|\alpha|$ increases when the amplitude ϕ_m decreases.

Our calculations show that by keeping all parameters constant but increasing μ (the ratio of the unperturbed electron density n_{e0} to unperturbed ion density n_{i0}) or increasing δ (the ratio of the ion temperature T_i to the electron temperature T_e), the amplitude increases whereas the width decreases. Furthermore, if the transverse perturbation is neglected, i.e. $\eta \to 0$, the stationary solitary solution of 2D spherical DASWs reduces to modified K-dV stationary solitary solution in 1D spherical geometry, and the solitary waves would have standard-shaped and propagation of the DASWs becomes stable. Therefore, 2D spherical DASWs differ from 1D spherical solitary waves in dusty plasmas. Also for $\mu \to 0$, the values of nonlinear coefficient A and dispersion coefficient B are in good agreement with the results of Mamun et al. [24].

We have numerically shown how the potential ϕ_1 of the DASWs and dust density n_{d1} of the spherical DASWs change with τ, α and positions of coordinates ξ, η. The dusty plasma parameters (μ, δ, α) we have used [9, 21, 25] in our numerical analysis are relevant to laboratory and astrophysical objects. This is shown in Figs. 1−4.

In Fig. 1, variations in the potential ϕ_1 of the spherical DASWs versus position coordinates ξ and η at four different times $\tau = 0, 5, 10, 15$ are shown. The results in this figure are based on the solution of the spherical modified K-dV Eq. (26). This figure shows that negative potential solitary waves are formed and because of the spherical geometry and transvers perturbation, by increasing time τ, these solitary waves transform from a line soliton (plotted in $\tau = 0$) to C-like soliton (plotted in times $\tau = 5, 10, 15$). Figure 2 shows the variation in the potential of spherical DAWs ϕ_1 with respect to the position coordinates ξ and η. It can be seen that the peak amplitude is negative which means that the negative spherical DAW is formed. It is obvious from this figure how the solitary structure changes to C-like solitary owing to the 2D spherical geometry. It is clear from Figs. 1 and 2 that as the value of time increases, the solitary structure deviates in the radial direction, and in the angular direction a transverse perturbation appeares. Obviously, this deviation or transverse perturbation cannot appear in 1D spherical geometry (or 1D planner). Therefore, the properties of the DASWs in a 2D spherical geometry differ from 1D spherical geometry. Figure 3 represents the variation in the dust density of the spherical DASWs (by replacing ϕ_1 by $-v_0^2 n_{d1}$) with the position of coordinates ξ and η for two different time $\tau = 0$ (Fig. 3a) and $\tau = 10$ (Fig. 3b). As the figure shows, the peak amplitude of dust den-
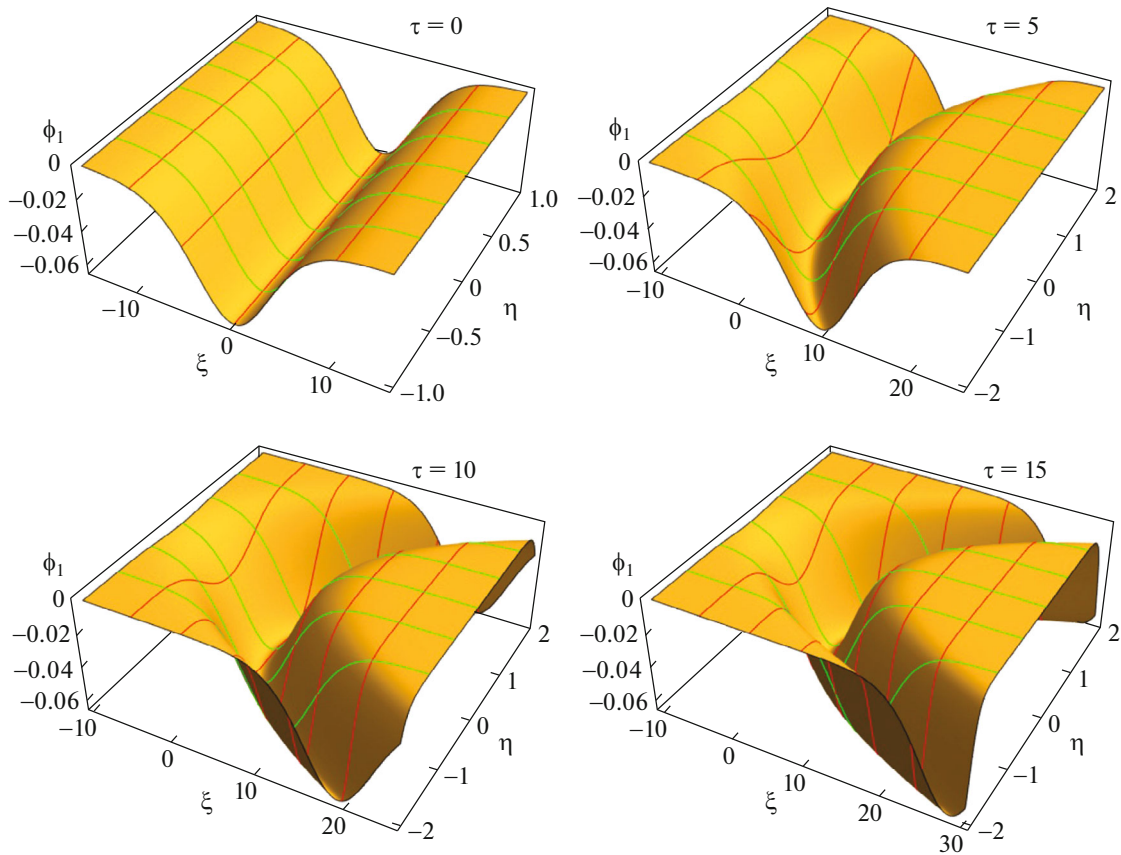
Fig. 1. (Color online) Spherical DASWs versus position coordinates ξ and η at different times $\tau = 0, 5, 10, 15$. The other parameters used in the calculations are $u_0 = 0.1$, $\delta = 0.1$, $\mu = 0.2$, $\alpha = -0.5$.

sity waves (ϕ_m / v_0^2) is positive (density hump), and dust density is deformed from a line soliton (plotted in Fig. 3a) to a C-like soliton (plotted in Fig. 3b). Figure 4 shows
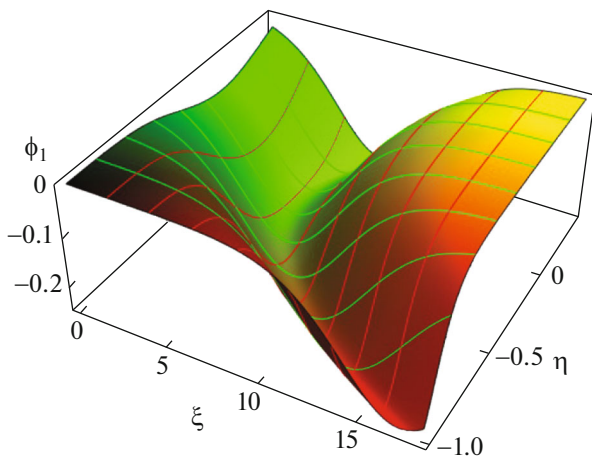

Fig. 2. (Color online) Variation of the amplitude of the spherical DASWs position coordinates versus ξ and η. The other parameters used in the calculations are $u_0 = 0.2$, $\delta = 0.01$, $\mu = 0.1$, $\alpha = -0.4$, $\tau = 25$.

the potential ϕ_1 as a function of χ at different values of α. It is obvious from this figure that the amplitude of the nonlinear structures potential wave increases as the trapping parameter $|\alpha|$ decreases. We note that in this figure, DASWs exist with the negative potential ($\phi_1 < 0$), but not with a positive potential ($\phi_1 > 0$). It may be mentioned that instead of a Boltzmann ion density distribution when we consider a trapped (vortex-like) ion distribution, we obtain spherical DASWs only with negative potential ($\phi_1 < 0$). It may be stressed here that, there are some differences between vortex-like ion distribution and the Boltzmann ion distribution [24]. For example, the solitary wave solutions (and their amplitude) of the vortex-like ion distribution are different from those of Boltzmann ion distribution. In trapped ion distribution case, the nonlinearity is much stronger than a Boltzmann ion distribution case [25]. Finally the solutions for vortex-like ion distribution and Boltzmann ion distribution have been, respectively, under the form of $\phi_1 \sim \sec h^4(\chi)$ and $\phi_1 \sim \sec h^2(\chi)$. Therefore, it can be concluded that the results for vortex-like ion distribution are com-

(a)

(b)
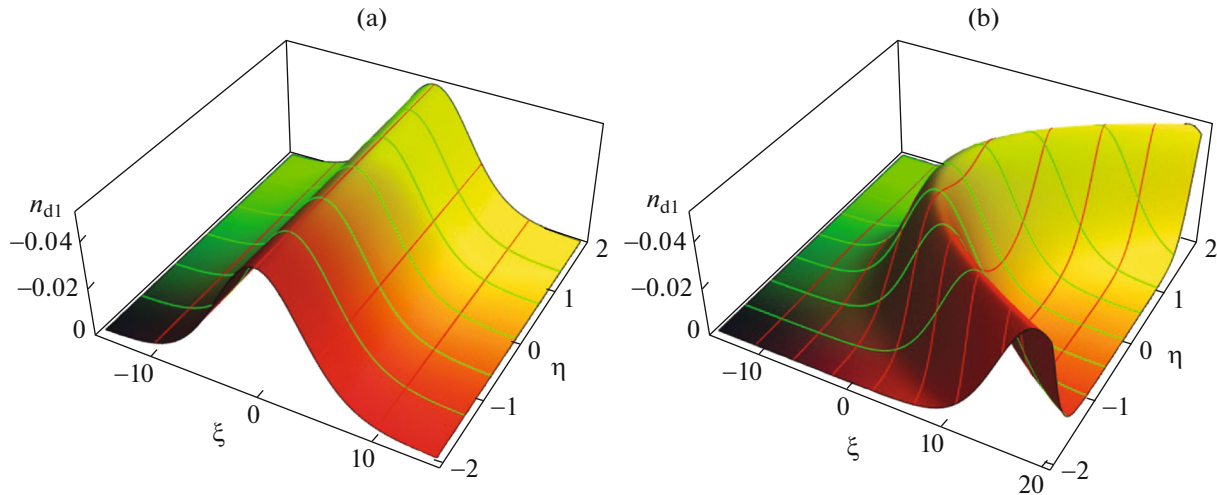

Fig. 3. (Color online) Variation in the dust density of the spherical DASWs versus the position coordinates ξ and η. Other parameters are $u_0 = 0.1$, $\delta = 0.1$, $\mu = 0.2$, $\alpha = -0.9$; (a) $\tau = 0$ and (b) $\tau = 10$.
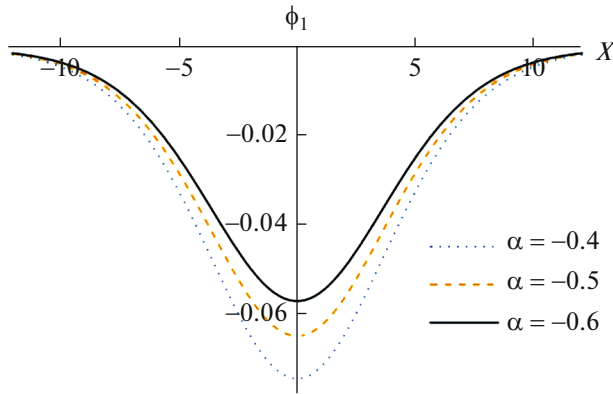

Fig. 4. (Color online) Variation of ϕ_1 as a function of χ for three different values of α. Other parameters are $u_0 = 0.1$, $\delta = 0.1$, $\mu = 0.2$.

pletely different from those of Boltzmann ion distribution in dusty plasma.

4. CONCLUSIONS

2D spherical dust acoustic solitary waves in an unmagnetized dusty plasma containing negatively charged dust fluid, vortex-like ion distribution and Boltzmann electron distribution was investigated. For the small-amplitude solitary waves formation, we have employed the reductive perturbation method, and modified K-dV spherical equation in higher spherical geometry has derived. From the results, it was concluded that only negative potential (compressive DASWs) can be formed in such dusty plasma. It is seen that by increasing the value of trapping parameter $|\alpha|$, the amplitude of DA solitary structures decreases while its width remains unchanged. The time history

of DAWs in 2D spherical geometry shows that with time, the trajectory of solitary structures changes from a linear to a C-like shape due to higher dimensional system, which introduces transverse perturbation into the system. For a comparatively larger value of τ ($\tau \rightarrow \infty$), the 2D spherical DASWs approaches 1D planar structures. In the absence of angular direction ($\eta \rightarrow 0$), 2D spherical DASWs reduces to 1D spherical structures and transverse perturbations have vanished.

We hope that the results of this investigation may help in understanding the nonlinear features of electrostatic disturbances in laboratory and space plasmas where dusty plasma is immersed in trapped ions.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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