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SURFACE WAVES IN COMPOSITE MATERIALS

# Dispersion and Attenuation Characteristics of Love-Type Waves in a Fiber-Reinforced Composite over a Viscoelastic Substrate

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**Abstract**—The problems concerned with the dispersion and attenuation of surface wave propagations due to imperfect elasticity are of great interest to seismologists. The present work reports the dispersion and attenuation characteristics of Love-type wave propagation in a fiber-reinforced layer laid on an inhomogeneous viscoelastic half-space. The inhomogeneity in the viscoelastic medium arises due to the hyperbolic trigonometric variation in depth. A complex frequency equation for the Love-type wave has been procured using the suitable boundary conditions. Thus, the dispersion and damping equations have been calculated to analyze the dispersion and attenuation peculiarities of the wave. Results for the uniform homogeneous isotropic media have been compared with existing solutions. Numerical computation and graphical sketches have been set forth for the relevant parametric variations.

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## 1. INTRODUCTION

The dynamical behavior of near-surface materials could not be explained on the basis of classical continuum mechanics due to their complex nature. The presence of many effective physical factors causes the materials to behave imperfect elastic to the propagation of waves through it. Hence, the problems of continuous media are not restricted to the mechanics of those elastic materials which are perfect elastic, rather the problems take a more general and realistic form when the media considered are imperfect elastic. The dynamical interaction between medium and seismic waves propagation through the imperfect elastic and reinforced medium have many important applications in geophysics, civil engineering and modern physical engineering.

Propagation of surface waves in fiber-reinforced composite materials is a widely known and prime feature of wave theory. The analysis of surface waves in reinforced medium plays a very important role in structural engineering to design earthquake resistant buildings. In 1983, Belfield et al. [1] introduced the continuous self-reinforcement at every point of an elastic solid. Othman and Abbas [2] examined the effect of rein-forcement on the total deformation of rotating body on the propagation of plane waves. Propagation of Love wave in a fiber-reinforced layered medium over an orthotropic half-space was investigated by Kundu et al. [3]. Abo-Dahab and Edfawy [4] investigated the analytic solution for the secular equation of surface waves in thermoelastic fiber-reinforced anisotropic solids. Alam et al. [5] considered corrugated irregularity with magneto-elasticity in the anisotropic (Monoclinic) medium for the study of SH-wave propagation.

The viscosity in a viscoelastic body arises due to its imperfect elasticity. The rocks are physically cold in lithosphere, whereas the materials in the asthenosphere are highly viscous, ductility deforming and mechanically weak. Although, both lithosphere and asthenosphere materials behave like viscoelastic. It has been observed that, the most dynamical Earth processes take place in these zones and are liable for the earthquake. In the last few years, the study of seismic wave propagation in the viscoelastic medium under different physical circumstances is the topic of prime interest for researchers [6–9].

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The problems related to the seismic response of elastic waves regulating through the bodies, those mechanical properties are functions of space (i.e. inhomogeneous bodies), are finding many applications in engineering and applied sciences. Considering the importance of seismic wave responses in inhomogeneous bodies, many researchers have assessed in large quantities under the different physical circumstances. For instance, Kundu et al. [10], Kumari et al. [11, 12], Alam et al. [13, 14], and Sahu et al. [15] considered trigonometric functions of space. Abd-Alla et al. [16] assumed exponential function of space for the orthotropic medium. In other papers, Kakar and Kakar [17] and Sahu et al. [18] took linear function of space for the surface wave study.

The aim of the present study is to find the dispersion and attenuation characteristics of Love-type waves in the presence of various affecting parameters (dissipation factor, attenuation coefficient, inhomogeneity, viscoelasticity and fiber-reinforcement) involved in the assumed model. The present Earth model consists in a fiber-reinforced layer lying on an inhomogeneous viscoelastic half-space for the Love-type wave regulation and the assumed inhomogeneity in the viscoelastic half-space is due to the hyperbolic cosine function of depth and inhomogeneity parameter. The viscoelastic materials have both elastic and viscous properties (i.e., linear anelastic) which may be fabricated as an infinite number of possible configurations of elastic springs and viscous dashpots (Kelvin–Voigt model). Mathematical analysis reveals that the damping nature of Love-type waves is exclusively due to the dissipation factor (viscosity) of the viscoelastic half-space. The damping occurrence can be perceived in the form of energy loss during viscous lubrication between moving particles of the viscoelastic medium.

## 2. FORMULATION OF THE PROBLEM

Consider a composite structure of single layered half-space media for the Love-type wave propagation, in which a inhomogeneous viscoelastic half-space ($M_2$) overloaded by a fiber-reinforced elastic layer ($M_1$). The geometry of the composite structure in a Cartesian coordinate system $o{-}xyz$ is shown in Fig. 1, where the wave is propagating along $x$-axis and $h$ is the thickness of layer.

Love-type wave propagation is characterized by

$$\begin{aligned} &u_i = 0, \quad v_i = v_i(x,z,t), \quad w_i = 0, \\ &\frac{\partial}{\partial y} \equiv 0, \quad i = 1, 2, \end{aligned} \tag{1}$$

where $(u_i, v_i, w_i)$ is the displacement vector, $i = 1$ for the layer ($M_1$) and $i = 2$ for half-space ($M_2$).

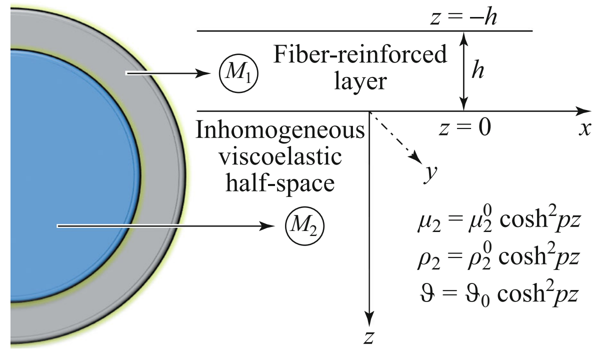


**Fig. 1.** (Color online) Considered Earth model of the problem.

## 3. DYNAMICS OF THE FIBER-REINFORCED LAYER ($M_1$)

The formative equations for fiber-reinforced linearly anisotropic elastic medium with respect to preferred direction are given by [1]

$$\begin{aligned} \Omega_{ij} &= 2\mu_{\mathrm{T}} e_{ij} + \lambda e_{kk}\delta_{ij} \\ &\quad + \alpha(a_m a_k e_{km}\delta_{ij} + a_i a_j e_{kk}) \\ &\quad - 2(\mu_{\mathrm{T}} - \mu_{\mathrm{L}})(a_k a_i e_{kj} + a_k a_j e_{ki}) \\ &\quad + \beta(a_j a_i a_m a_k e_{km}), \quad k, m, i, j = 1, 2, 3, \end{aligned} \tag{2}$$

where $\Omega_{ij}$ are the components of the stress vector; $e_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)$ are the components of infinitesimal strain, $\mu_{\mathrm{T}}$ and $\mu_{\mathrm{L}}$ are longitudinal and transverse elastic shear moduli, respectively; $\delta_{ij}$ is Kronecker delta; $\alpha$, $\beta$, $\lambda$ are elastic constants with dimension of stress; $a_i$ are components of $\mathbf{a}$ in the rectangular Cartesian coordinates $xyz$, and $\mathbf{a} = (a_1, a_2, a_3)$ is the preferred directions of reinforcement such that $a_1^2 + a_2^2 + a_3^2 = 1$. Therefore, the characteristic of the Love-type wave allows us to take the direction of reinforcement as $(a_1, 0, a_3)$.

Substituting (1) in (2), we get the following nonzero stress components:

$$\begin{aligned} \Omega_{xy} &= a_1(\mu_{\mathrm{L}} - \mu_{\mathrm{T}})\left(a_1 \frac{\partial v_1}{\partial x} + a_3 \frac{\partial v_1}{\partial z}\right) + \mu_{\mathrm{T}} \frac{\partial v_1}{\partial x}, \\ \Omega_{yz} &= a_3(\mu_{\mathrm{L}} - \mu_{\mathrm{T}})\left(a_1 \frac{\partial v_1}{\partial x} + a_3 \frac{\partial v_1}{\partial z}\right) + \mu_{\mathrm{T}} \frac{\partial v_1}{\partial z}. \end{aligned} \tag{3}$$

We have the only non-vanishing dynamical equation of motion for the Love-type waves propagation along the $x$-direction in the absence of any body force [19]:

$$\frac{\partial \Omega_{xy}}{\partial x} + \frac{\partial \Omega_{yz}}{\partial z} = \rho_1 \frac{\partial^2 v_1}{\partial t^2}. \tag{4}$$

Equations (4) together with Eq. (3) gives

$$\left[1+\left(\frac{\mu_{\mathrm{L}}}{\mu_{\mathrm{T}}}-1\right)a_1^2\right]\frac{\partial^2 v_1}{\partial x^2}+2\left[\left(\frac{\mu_{\mathrm{L}}}{\mu_{\mathrm{T}}}-1\right)a_1a_3\right]\frac{\partial^2 v_1}{\partial x\,\partial z}+\left[1+\left(\frac{\mu_{\mathrm{L}}}{\mu_{\mathrm{T}}}-1\right)a_3^2\right]\frac{\partial^2 v_1}{\partial z^2}=\frac{\rho_1}{\mu_{\mathrm{T}}}\frac{\partial^2 v_1}{\partial t^2}. \tag{5}$$

Taking $v_1 = \Psi_1(z)\exp[i(\omega t - kx)]$ as a harmonic wave solution for (5), we have

$$\frac{d^2\Psi_1}{dz^2}+\alpha\frac{d\Psi_1}{dz}+\beta\Psi_1=0, \tag{6}$$

where $\alpha = -\dfrac{2ik[(\mu_{\mathrm{L}}/\mu_{\mathrm{T}}-1)a_1a_3]}{1+(\mu_{\mathrm{L}}/\mu_{\mathrm{T}}-1)a_3^2}$ and $\beta = \dfrac{\omega^2(\rho_1/\mu_{\mathrm{T}})-k^2[1+(\mu_{\mathrm{L}}/\mu_{\mathrm{T}}-1)a_1^2]}{1+(\mu_{\mathrm{L}}/\mu_{\mathrm{T}}-1)a_3^2}$.

Therefore, the displacement $v_1$ of layer $(M_1)$ is found as

$$v_1=\exp\left(-\frac{\alpha z}{2}\right)\left[A_1\cos(\chi_1 z)+A_2\sin(\chi_1 z)\right]\exp\left[i(\omega t-kx)\right]. \tag{7}$$

where $\chi_1=\sqrt{\beta-\alpha^2/4}$ and $A_1$, $A_2$ are arbitrary constants.

## 4. DYNAMICS OF THE INHOMOGENEOUS VISCOELASTIC HALF-SPACE $(M_2)$

The only non-vanishing dynamical equation of motion for a viscoelastic medium (without any body forces) is given by [19]

$$\frac{\partial\Theta_{xy}}{\partial x}+\frac{\partial\Theta_{yz}}{\partial z}=\rho_2\frac{\partial^2 v_1}{\partial t^2}, \tag{8}$$

where $\Theta_{xy}$ and $\Theta_{yz}$ are the stress components of the viscoelastic layer and given by

$$\Theta_{xy}=\left(\mu_2+\vartheta\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial x},\qquad \Theta_{yz}=\left(\mu_2+\vartheta\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial z}, \tag{9}$$

where $\mu_2$ is elastic modulus, $\rho_2$ is density, and $\vartheta$ is viscosity of viscoelastic layer. If $\vartheta = 0$, then the viscoelastic medium becomes elastic solid.

We consider the following transformations for these material constants:

$$\mu_2=\mu_2^0\cosh^2 pz,\quad \rho_2=\rho_2^0\cosh^2 pz,\quad \vartheta=\vartheta_0\cosh^2 pz, \tag{10}$$

where $p$ is inhomogeneity parameter having dimension $L^{-1}$.

Substituting (9) into (8), we get

$$\left(\mu_2+\vartheta\frac{\partial}{\partial t}\right)\frac{\partial^2 v_2}{\partial x^2}+\frac{\partial}{\partial z}\left[\left(\mu_2+\vartheta\frac{\partial}{\partial t}\right)\frac{\partial v_2}{\partial z}\right]=\rho_2\frac{\partial^2 v_2}{\partial t^2}. \tag{11}$$

We are assuming $v_2 = \Psi_2(z)\exp[i(\omega t - kx)]$ as a solution of Eq. (11). Therefore, Eq. (11) leads to

$$\frac{d^2}{dz^2}[\Psi_2(z)]+\frac{1}{\overline{\mu_2}}\frac{d\overline{\mu_2}}{dz}\frac{d}{dz}[\Psi_2(z)]-\left[k^2-\frac{\omega^2\rho_2}{\overline{\mu_2}}\right]\Psi_2(z)=0. \tag{12}$$

We simplify Eq. (12) taking a substitution $\Psi_2(z)=\Phi(z)/\sqrt{\mu_2}$:

$$\frac{d^2\Phi(z)}{dz^2}+\left\{\frac{1}{4\overline{\mu_2}^2}\left[\left(\frac{d\overline{\mu_2}}{dz}\right)^2-2\,\overline{\mu_2}\,\frac{d^2\overline{\mu_2}}{dz^2}\right]+\left(\frac{\omega^2\rho_2}{\overline{\mu_2}}\right)-k^2\right\}\Phi(z)=0. \tag{13}$$

where $\overline{\mu_2}=\mu_2+i\omega\vartheta$.

Substituting relations from (10) into (13), we get

$$d^2\Phi(z)/dz^2-\chi_2^2\,\Phi(z)=0, \tag{14}$$

where $\chi_2=\sqrt{p^2+k^2-\omega^2\rho_2^0/\overline{\mu_2^0}}$ and $\overline{\mu_2^0}=\mu_2^0+i\omega\vartheta_0$.

So, the solution of Eq. (14) is

$$\Phi(z) = B_1 \exp(-\chi_2 z) + B_2 \exp(\chi_2 z), \tag{15}$$

where $B_1$ and $B_2$ are arbitrary constants.

So, the displacement $v_2$ of medium ($M_2$) is obtained as

$$v_2 = \frac{B_1 \operatorname{sech} pz}{\sqrt{\mu_2^0}} \exp(-\chi_2 z) \exp\big[i(\omega t - kx)\big]. \tag{16}$$

## 5. BOUNDARY CONDITIONS

**5.1.** The continuity of stresses and displacement components at the common interface gives (i.e., at $z=0$)

**(a)** $a_3(\mu_\mathrm{L} - \mu_\mathrm{T})\left(a_1 \frac{\partial v_1}{\partial x} + a_3 \frac{\partial v_1}{\partial z}\right) + \mu_\mathrm{T} \frac{\partial v_1}{\partial z} = \left(\mu_2 + \vartheta \frac{\partial}{\partial t}\right) \frac{\partial v_2}{\partial z},$

**(b)** $v_1 = v_2.$

**5.2.** Stress-free surface of the layer gives (i.e., at $z=-h$)

**(a)** $a_3(\mu_\mathrm{L} - \mu_\mathrm{T})\left(a_1 \frac{\partial v_1}{\partial x} + a_3 \frac{\partial v_1}{\partial z}\right) + \mu_\mathrm{T} \frac{\partial v_1}{\partial z} == 0.$

## 6. DISPERSION AND DAMPING EQUATIONS

Using solutions (7) and (16) in the above boundary conditions, we obtain the following homogeneous system of equations:

$$ik\, \mu_\mathrm{T} a_1 a_3 \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}} - 1\right) A_1 - \mu_\mathrm{T} \chi_1 \left[1 + \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}} - 1\right) a_3^2\right] A_2 - \chi_2 \sqrt{\mu_2^0}\, B_1 = 0, \tag{17}$$

$$\sqrt{\mu_2^0}\, A_1 - B_1 = 0, \tag{18}$$

$$\begin{aligned}&\left\{\chi_1 \left[1 + \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}} - 1\right) a_3^2\right] \sin(\chi_1 h) - ik\, a_1 a_3 \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}} - 1\right) \cos(\chi_1 h)\right\} A_1 \\ &+ \left\{\chi_1 \left[1 + \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}} - 1\right) a_3^2\right] \cos(\chi_1 h) + ik\, a_1 a_3 \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}} - 1\right) \sin(\chi_1 h)\right\} A_2 = 0.\end{aligned} \tag{19}$$

For the nontrivial solution of above system of equations, the determinant of the coefficient matrix should be zero. Therefore, the following equation will satisfy for a non-trivial solution of $A_1$, $A_2$, and $B_1$:

$$f(k,c) = \tan(\chi_1 h) - \left\{\frac{\overline{\mu_2^0}\, \chi_1 \chi_2 \left[1 + \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}} - 1\right) a_3^2\right]}{\mu_\mathrm{T}\, \chi_1^2 \left[1 + \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}} - 1\right) a_3^2\right]^2 - k a_1 a_3 \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}} - 1\right)\left[k \mu_\mathrm{T}\, a_1 a_3 \left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}} - 1\right) + i \chi_2 \overline{\mu_2^0}\right]}\right\} = 0. \tag{20}$$

The above expression of $f(k,c)$ gives the wave velocity profile of Love-type waves in a fiber-reinforced layer resting over an inhomogeneous viscoelastic half-space. Considering the wave number $k$ as a complex number

$$k = k_1(1 + i\delta), \tag{21}$$

where $\delta = k_2/k_1$ is the attenuation coefficient which is dimensionless and $k_1$, $k_2$ are real. Thus, the velocity $c$ of the wave can be evaluated by the relation

$$\omega = ck_1 \tag{22}$$

The dimensionless dissipation factor (inverse of quality factor $Q_0 = \mu_2^0/\omega\vartheta_0$) is given by

$$Q_0^{-1} = \omega\vartheta_0/\mu_2^0. \tag{23}$$

The real part of the frequency equation $\mathrm{Re}[f(k,c)]$ characterizes the dispersion occurrence, whereas the imaginary part $\mathrm{Im}[f(k,c)]$ describes damping of the wave. Hence, the velocity associated with the dispersion is defined as the 'phase velocity' ($V_\mathrm{p}=c/c_1$) of wave, and the velocity associated with the damping is termed as the 'damped velocity' ($V_\mathrm{d}=c/c_1$). To find out the dispersion and damping equations, we are considering

$$\chi_1 = x_1 + iy_1 = \sqrt{r_1 \exp(i\theta_1)}, \tag{24}$$

$$\chi_2 = x_2 + iy_2 = \sqrt{r_2 \exp(i\theta_2)}, \tag{25}$$

where

$$x_1 = \sqrt{r_1}\cos(\theta_1/2), \quad y_1 = \sqrt{r_1}\sin(\theta_1/2), \quad x_2 = \sqrt{r_2}\cos(\theta_2/2), \quad y_2 = \sqrt{r_2}\sin(\theta_2/2),$$

$$r_1\cos\theta_1 = \frac{k_1^2}{1+\left(\dfrac{\mu_\mathrm{L}}{\mu_\mathrm{T}}-1\right)a_3^2}\left\{\frac{c^2}{c_1^2} + (1-\delta^2)\left[\frac{a_1^2a_3^2\left(\dfrac{\mu_\mathrm{L}}{\mu_\mathrm{T}}-1\right)^2}{1+\left(\dfrac{\mu_\mathrm{L}}{\mu_\mathrm{T}}-1\right)a_3^2} - \left[1+\left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}}-1\right)a_1^2\right]\right]\right\},$$

$$r_1\sin\theta_1 = \frac{k_1^2}{1+\left(\dfrac{\mu_\mathrm{L}}{\mu_\mathrm{T}}-1\right)a_3^2}\left\{2\delta\left[\frac{a_1^2a_3^2\left(\dfrac{\mu_\mathrm{L}}{\mu_\mathrm{T}}-1\right)^2}{1+\left(\dfrac{\mu_\mathrm{L}}{\mu_\mathrm{T}}-1\right)a_3^2} - \left[1+!\left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}}-1\right)a_1^2\right]\right]\right\},$$

$$r_2\cos\theta_2 = p^2 + k_1^2(1-\delta^2) - \frac{k_1^2c^2/c_2^2}{1+(Q_0^{-1})^2},$$

$$r_2\sin\theta_2 = k_1^2\left[2\delta + \frac{Q_0^{-1}c^2/c_2^2}{1+(Q_0^{-1})^2}\right], \quad c_1 = \sqrt{\frac{\mu_\mathrm{T}}{\rho_1}}, \quad c_2 = \sqrt{\frac{\mu_{02}}{\rho_{02}}}.$$

The real and imaginary parts of Eq. (20) generate two equations

$$\mathrm{Re}\,[f(k,c)] = \frac{\sin(2x_1h)}{\cos(2x_1h)+\cosh(2y_1h)} - \frac{D_1N_1+D_2N_2}{D_1^2+D_2^2} = 0, \tag{26}$$

$$\mathrm{Im}\,[f(k,c)] = \frac{\sinh(2y_1h)}{\cos(2x_1h)+\cosh(2y_1h)} - \frac{D_1N_2-D_2N_1}{D_1^2+D_2^2} = 0, \tag{27}$$

where

$$N_1 = (x_1x_2 - y_1y_2)\,\mu_2^0T_1, \qquad N_2 = (x_2y_1 + x_1y_2)\,\mu_2^0T_1,$$
$$D_1 = \mu_\mathrm{T}\,k_1^2T_2^2(\delta^2-1) + \mu_\mathrm{T}\,T_1^2(x_1^2-y_1^2) + k_1T_2\mu_2^0(\delta x_2 + y_2),$$
$$D_2 = 2x_1y_1\mu_\mathrm{T}\,T_1^2 + k_1T_2\left[(\delta y_2 + x_2)\mu_2^0 - 2k_1T_2\mu_\mathrm{T}\delta\right],$$
$$T_1 = \left[1+\left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}}-1\right)a_3^2\right], \qquad T_2 = a_1a_3\left(\frac{\mu_\mathrm{L}}{\mu_\mathrm{T}}-1\right).$$

## 7. PARTICULAR CASE

If the layer is free from reinforcement and the half-space is homogeneous elastic, then Eq. (26) becomes the classical dispersion equation of Love wave [20, 21]:

$$\tan\left(k_1h\sqrt{\frac{c^2}{c_1^2}-1}\right) = \frac{\mu_2^0}{\mu_3}\frac{\sqrt{1-c^2/c_2^2}}{\sqrt{c^2/c_1^2-1}}, \tag{28}$$

where $c_1=\sqrt{\mu_1/\rho_1}$, $c_2=\sqrt{\mu_{02}/\rho_{02}}$, $\mu_\mathrm{L}=\mu_\mathrm{T}=\mu_1$ and Eq. (27) vanishes to zero.

## 8. NUMERICAL COMPUTATION AND DISCUSSION

To inspect the dispersion and attenuation characteristics of Love-type waves in the presence of various affecting parameters, we are considering some numerical examples of material constants for medium ($M_1$) and medium ($M_2$) as given in Table 1 [22].

In this section, the variational impacts of dimensionless parameters—such as fiber-reinforcement parameters ($a_1^2, a_3^2$), dissipation factor ($Q_0^{-1}$), attenuation coefficient ($\delta$) and inhomogeneity parameter

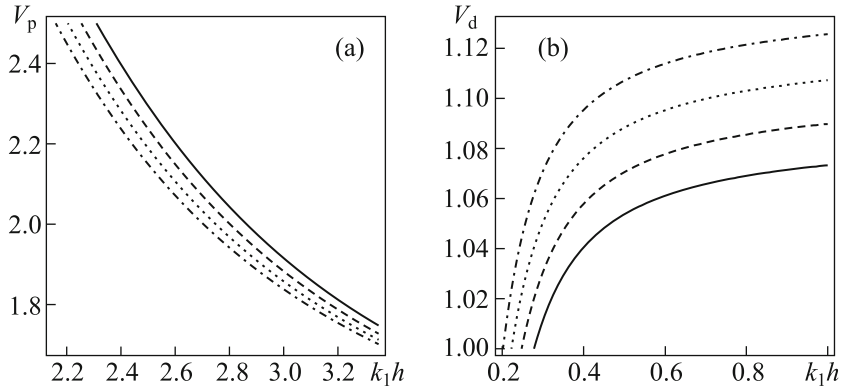


**Fig. 2.** Variation in reinforcement parameters $(a_1^2, a_3^2)$ for velocity profile versus real wave number $(k_1h)$: (a, b) $a_1^2 = 0.30$, $a_3^2 = 0.70$ (solid curve), 0.35, 0.65 (dashed curve), 0.40, 0.60 (dotted curve), and 0.45, 0.65 (dash-dotted curve).

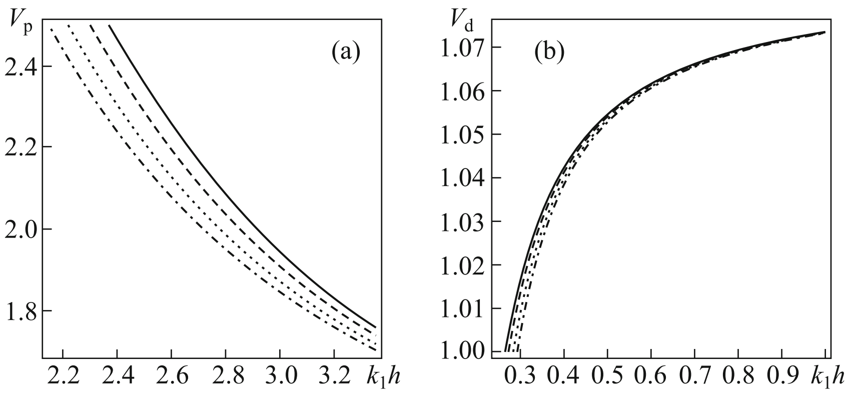


**Fig. 3.** Variation in dissipation factor $(Q_0^{-1})$ for velocity profile versus real wave number $(k_1h)$: (a, b) $Q_0^{-1} = 0.05$ (solid curve), 0.35 (dashed curve), 0.65 (dotted curve), and 0.95 (dash-dotted curve).

$(p/k_1)$—on the phase velocity $(V_p)$ and damping velocity $(V_d)$ with respect to the real wave number $(k_1h)$ of Love-type wave are shown by several curves in Figs. 2–8. The numerical fixed values of dimensionless parameters in these figures are listed in Table 2. Brief observation of all figures allows one to conclude that as the wave number $(k_1h)$ decreases, the phase velocity $(V_p)$ increases, whereas damped velocity $(V_d)$ increases with the wave number $(k_1h)$.

Figure 2 exhibits the effect of reinforce parameters $(a_1^2, a_3^2)$ associated with the fiber-reinforced medium on the propagation of Love-type wave. The curves of Fig. 2(a) show the variation in phase velocity $(V_p)$ and the curves of Fig. 2(b) show the variation in damped velocity $(V_d)$ of the wave. It is seen in the figures that as the value of $a_1^2$ increases and $a_3^2$ decreases, the damped velocity of the wave increases, whereas the phase velocity of the wave decreases.

Figure 3 is plotted to depict the impact of dissipation factor $Q_0^{-1}$ affined with the viscoelastic half-space on the propagating wave. The shifting of curves in Fig. 3(a) shows the phase velocity $(V_p)$ variation and the shifting of curves in Fig. 3(b) shows damped velocity $(V_d)$ variation of the wave. Careful consideration of figures points out that as the value of $Q_0^{-1}$ increases, the phase velocity $(V_p)$ and damped velocity $(V_d)$ of the wave also increases.

**Table 1.** Material constants

| Medium | Rigidity, $\times 10^9$ N m$^{-2}$ | Density, kg m$^{-3}$ |
|---|---|---|
| $(M_1)$ | $\mu_L = 7.07$, $\mu_T = 3.5$ | $\rho_1 = 1600$ |
| $(M_2)$ | $\mu_{02} = 219.7$ | $\rho_{02} = 5563$ |

**Table 2.** Values of parameters

| Parameter | Fig. 2 | Figs. 3–6 | Figs. 4–7 | Figs. 5–8 |
|---|---|---|---|---|
| $a_1^2$ | — | 0.3 | 0.3 | 0.3 |
| $a_3^2$ | — | 0.7 | 0.7 | 0.7 |
| $Q_0^{-1}$ | 0.5 | — | 0.5 | 0.5 |
| $\delta$ | 0.05 | 0.05 | — | 0.05 |
| $p/k_1$ | 0.1 | 0.1 | 0.1 | — |

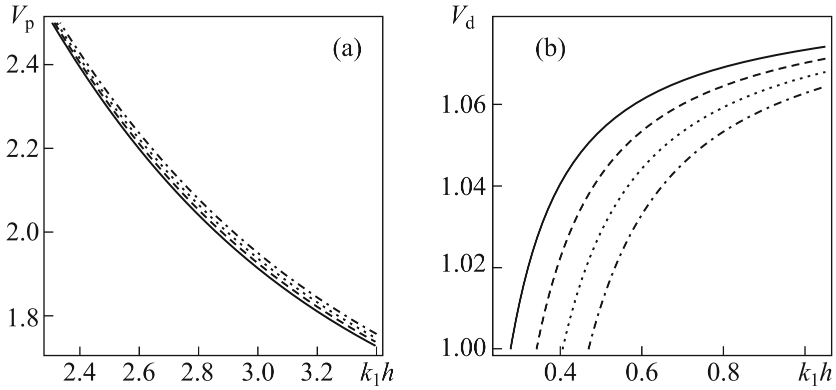


**Fig. 4.** Variation in attenuation coefficient ($\delta$) for velocity profile versus real wave number ($k_1h$): (a, b) $\delta = 0.05$ (solid curve), 0.06 (dashed curve), 0.07 (dotted curve), and 0.08 (dash-dotted curve).

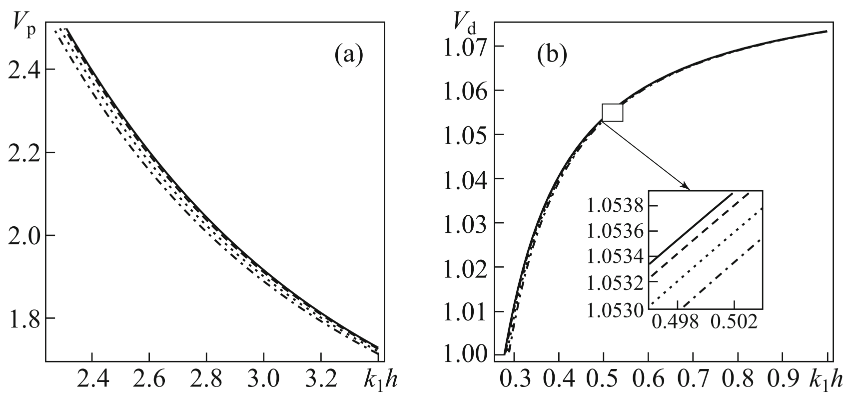


**Fig. 5.** Variation in inhomogeneity parameter ($p/k_1$) for velocity profile versus real wave number ($k_1h$): (a, b) $p/k_1 = 0.05$ (solid curve), 0.35 (dashed curve), 0.65 (dotted curve), and 0.95 (dash-dotted curve).

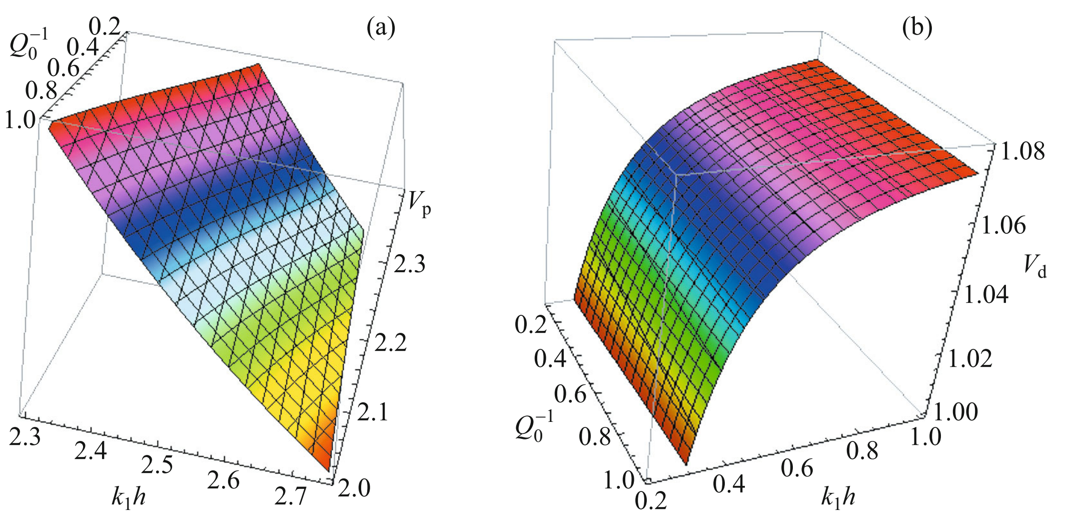


**Fig. 6.** (Color online) Surface plot of dissipation factor ($Q_0^{-1}$) versus real wave number ($k_1h$): (a) $V_p$ and (b) $V_d$.

In Fig. 4, curves are drawn to show the impact of attenuation coefficient $\delta$. The curves of Fig. 4(a) show the variation in phase velocity ($V_p$) and the curves of Fig. 4(b) show the variation in damped velocity ($V_d$) of the wave. It is clear from both figures that as the value of $\delta$ increases, the phase velocity of Love-type wave also increases, while damped velocity of the wave decreases.

Figure 5 exhibits the effect of heterogeneity parameter ($p/k_1$) associated with the half-space. The

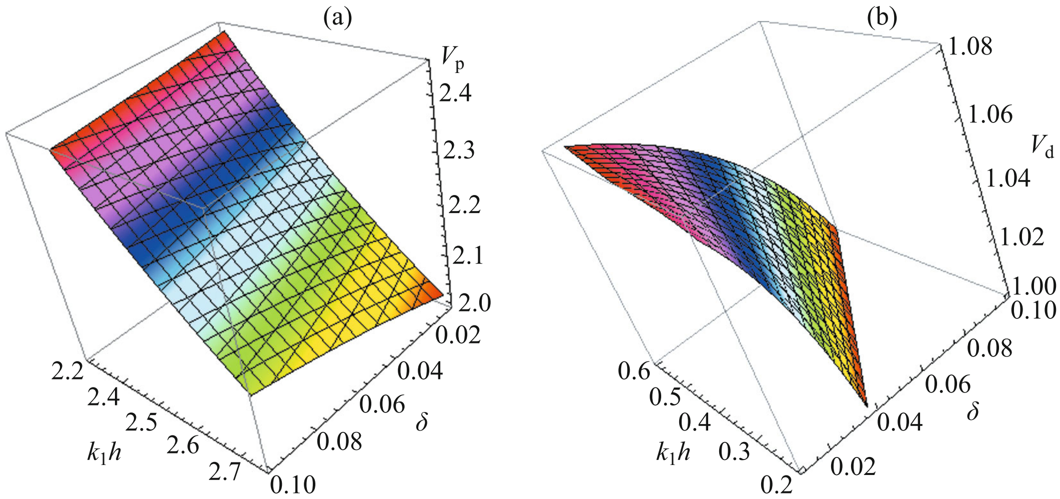


**Fig. 7.** (Color online) Surface plot of attenuation coefficient ($\delta$) versus real wave number ($k_1h$): (a) $V_p$ and (b) $V_d$.

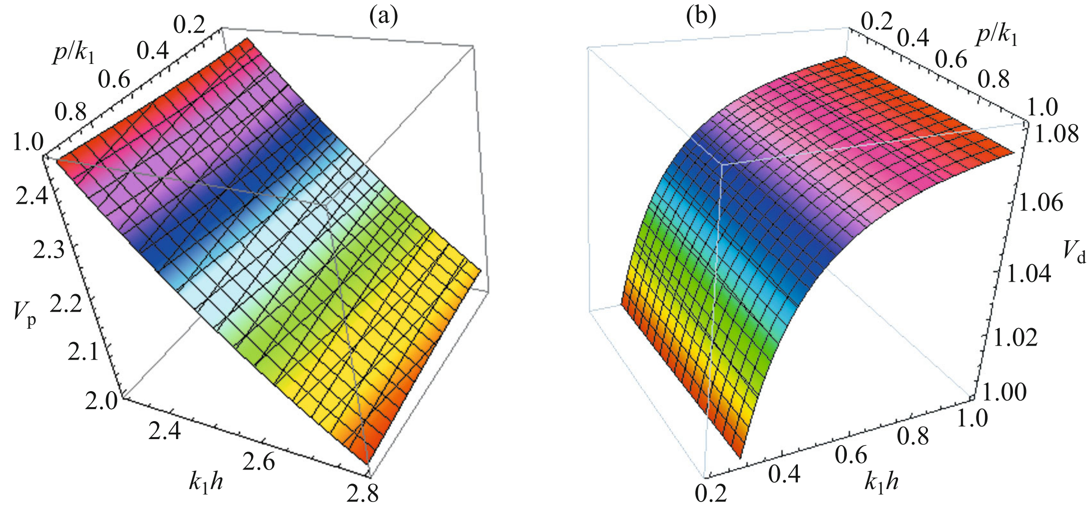


**Fig. 8.** (Color online) Surface plot of inhomogeneity parameter ($p/k_1$) versus real wave number ($k_1h$): (a) $V_p$ and (b) $V_d$.

curves of Fig. 5(a) show the variation in phase velocity ($V_p$) and the curves of Fig. 5(b) show the variation in damped velocity ($V_d$) of the wave. Brief observation of the figure reveals that as the value of $p/k_1$ increases, both damped and phase velocities of the wave also increases.

Keeping in mind the dependency of phase velocity ($V_p$) and damped velocity ($V_d$) on the wave number ($k_1h$), surface plots of phase and damped velocities against wave number, dissipation factor, attenuation coefficient and heterogeneity parameter are shown in Figs. 6–8 by 3D plot.

## 9. CONCLUSIONS

The present layered Earth model is proposed to study the Love-type waves propagating in a fiber-reinforced strip placed over an inhomogeneous viscoelastic half-space. The model has been extended to include the influence of dissipation factor, attenuation coefficient, and inhomogeneity present in the viscoelastic medium. It also includes the effect fiber-reinforcement of the strip on the phase as well as the damped velocity of the wave, which is usually ignored in theoretical studies. In this problem, a closed form frequency relation has been obtained and then dispersion and damping equations are derived. Effects of involved parameters on the considered waves have been studied and demonstrated graphically. As a whole, we conclude by this study that as wave number increases, damped velocity also increases, while the phase velocity decreases with an increment in the wave number; which is the feature of Love-type waves. The effect of heterogeneity on phase velocity has been found to be less pronounced than that of the reinforcement and dissipation factor. On the other hand, the effect of heterogeneity and dissipation factor on damped velocity is found less effective than reinforcement and attenuation coefficient. The obtained results of this study may provide a better prediction to the nature of Love-type waves in bedded structure and may also find applications in the geophysical explorations.

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