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TECHNICAL NOTES

STRUCTURAL MECHANICS AND STRENGTH OF FLIGHT VEHICLES

Computational Experiment in the Problem of the Flutter of a Cylindrical Shell

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Abstract—The flutter of a cylindrical shell of finite length, a non-axisymmetric case, is under consideration. A modern algorithm without saturation is presented, specific calculations are considered that have demonstrated its high efficiency. It is demonstrated also that the critical flutter speed has a weak dependence on the number of circumferential waves.

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INTRODUCTION

Supersonic flutter of thin circular cylindrical shells is one of the important aspects in the structural design of aircraft such as rockets. The term "panel flutter" refers to the flutter of a thin plate, shell or membrane, typically where one surface is exposed to air flow and the other to air at rest. The panel is then subjected to elastic, inertial, and aerodynamic forces, which can lead to dynamic instability of the structure.

The first recorded fluttering for circular cylindrical projectiles appears to have taken place with a rocket V-2 (Germany). Since that time, the study of aeroelastic stability of cylindrical projectiles in axial flow is fundamental in the design of the fuselage skin of aerospace vehicles, high-performance aircraft and rockets.

The dynamic problem of shell theory is reduced to the corresponding static one, if in the static equations, the values of the components of the external load vector include the values of the inertia force components. In the dynamic problems, the components of the external load are equal to the corresponding components of the inertial forces, i.e. it is accepted:

$$X = -\rho\delta\frac{\partial^2 u}{\partial t^2}, \quad Y = -\rho\delta\frac{\partial^2 u}{\partial t^2}, \quad Z = -\rho\delta\frac{\partial^2 w}{\partial t^2}.$$

Since the dynamic rigidity of the shell in the direction of the middle surface is much higher than the rigidity in the direction of the normal to the surface, the tangential components of the inertial forces are usually neglected, i.e. it is believed that

$$X = 0; \quad Y = 0; \quad Z = -\rho\delta\frac{\partial^2 w}{\partial t^2}.$$

In this case, we are dealing with flexural type oscillations determined by normal displacements of points on the middle surface $w(\alpha, \beta)$.

The literature on cylindrical shell flutter is very extensive, see [1]. We also note the publication [2].

PROBLEM FORMULATION

Let a closed circular cylindrical shell have length l, radius R, and thickness δ. Let us assume that the shell in the direction of the generatric line (Fig. 1) is flown around by a supersonic flow of compressible gas at an undisturbed velocity U^0.

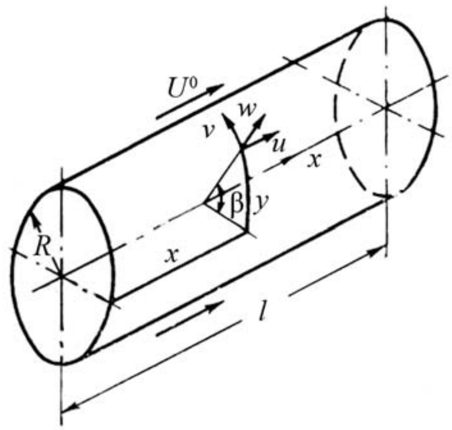

Fig. 1.

The movement of the shell can be described by a system of displacement equations

$$
\begin{aligned}
L_{11}u + L_{12}v + L_{13}w &= -\frac{\left(1-\mu^2\right)R^2}{E\delta}X;\\
L_{21}u + L_{22}v + L_{23}w &= -\frac{\left(1-\mu^2\right)R^2}{E\delta}Y;\\
L_{31}u + L_{32}v + L_{33}w &= \frac{\left(1-\mu^2\right)R^2}{E\delta}Z.
\end{aligned}
\tag{1}
$$

Here u, v, w are the longitudinal (axial), circumferential, and normal components of the points displacement on the middle surface of the shell; X, Y, Z are the load components per unit area of the middle surface; E is the Young modulus of elasticity; μ is the Poisson ratio.

For dimensionless coordinates $\alpha = x/R$, $\beta = y/R$, differential operators L_{ij} look like [1]

$$L_{11} = \frac{\partial^2}{\partial\alpha^2} + \frac{1-\mu}{2}\frac{\partial^2}{\partial\beta^2}; \quad L_{22} = \frac{1-\mu}{2}\frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\beta^2}; \quad L_{33} = c_*^2\nabla^2\nabla^2 + 1;$$

$$c_*^2 = \frac{\delta^2}{12R^2}; \quad \nabla^2\nabla^2 = \frac{\partial^4}{\partial\alpha^4} + 2\frac{\partial^4}{\partial\alpha^2\partial\beta^2} + \frac{\partial^4}{\partial\beta^4};$$

$$L_{12} = L_{21} = \frac{1+\mu}{2}\frac{\partial^2}{\partial\alpha\,\partial\beta}; \quad L_{13} = L_{31} = \mu\frac{\partial}{\partial\alpha}; \quad L_{23} = L_{32} = \frac{\partial}{\partial\beta};$$

$$X = 0; \quad Y = 0; \quad Z = -\rho\delta\frac{\partial^2 w}{\partial t^2} - \Delta p; \quad \Delta p = \frac{\kappa p_0}{c_0}\left(U^\circ\frac{\partial w}{\partial\alpha} + \frac{\partial w}{\partial t}\right),$$

where p_0, c_0 are the pressure and sound velocity in the undisturbed flow; ρ is the plate material density; κ is the polytropic coefficient (~1.4 for air).

Let us look for a solution of problem (1) in the form:

$$u = U(\alpha)\cos n\beta e^{\omega t}; \quad v = V(\alpha)\sin n\beta e^{\omega t}; \quad w = W(\alpha)\cos n\beta e^{\omega t},$$

where ω is the complex frequency;

Then:

$$L_{11}u = \left\{ U''(\alpha)\cos n\beta - n^2 \frac{1-\mu}{2} U(\alpha)\cos n\beta \right\} e^{\omega t};$$

$$L_{12}v = \left\{ n\frac{1+\mu}{2} V'(\alpha)\cos n\beta \right\} e^{\omega t};$$

$$L_{13}w = \left\{ \mu W'(\alpha)\cos n\beta \right\} e^{\omega t}.$$

As a result, we obtain the first equation:

$$U''(\alpha) - n^2 \frac{1-\mu}{2} U(\alpha) + n\frac{1+\mu}{2} V'(\alpha) + \mu W'(\alpha) = 0. \tag{2}$$

By the next, we have the following:

$$L_{21}u = \left\{ -n\frac{1+\mu}{2} U'(\alpha)\sin n\beta \right\} e^{i\omega t};$$

$$L_{22}v = \left\{ \frac{1-\mu}{2} V''(\alpha)\sin n\beta - n^2 V(\alpha)\sin n\beta \right\} e^{i\omega t};$$

$$L_{23}w = \left\{ -nW(\alpha)\sin n\beta \right\} e^{i\omega t}.$$

As a result, we obtain the second equation:

$$-n\frac{1+\mu}{2} U'(\alpha) + \frac{1-\mu}{2} V''(\alpha) - n^2 V(\alpha) - nW(\alpha) = 0; \tag{3}$$

$$L_{31}u = \left\{ \mu U'(\alpha)\cos n\beta \right\} e^{i\omega t};$$

$$L_{32}v = \left\{ nV(\alpha)\cos n\beta \right\} e^{i\omega t};$$

$$L_{33}w = \left\{ c_*^2 \left[W^{(IV)}(\alpha)\cos n\beta - 2n^2 W''(\alpha)\cos n\beta + n^4 W(\alpha)\cos n\beta \right] + W(\alpha)\cos n\beta \right\} e^{\omega t}.$$

As a result, we obtain the third equation:

$$\begin{gathered} \mu U'(\alpha) + nV(\alpha) + c_*^2 \left[W^{(IV)}(\alpha) - 2n^2 W''(\alpha) + n^4 W(\alpha) \right] + W(\alpha) = \gamma\lambda W(\alpha); \\ \gamma = \frac{(1-\mu^2)R^2}{E\delta}; \quad \lambda = -\rho\delta\omega^2 - \frac{\kappa p_0}{c_0}\omega. \end{gathered} \tag{4}$$

Boundary conditions should be added to equations (2)–(4). Let us denote $b = l/R$, then the pinching conditions at the ends of the shell are

$$U(0) = U(b) = 0; \quad V(0) = V(b) = 0; \quad W(0) = W(b) = 0; \quad W'(0) = W'(b) = 0. \tag{5}$$

Let us introduce dimensionless parameters (with prime marks): $E = E' p_0$, $\delta = \delta' R$, $\rho = \rho' p_0 / c_0^2$, $\omega = \omega' c_0 / R$, $U = U' R$, $W = W' R$, $x = \alpha R$, $y = \beta R$. Thereafter, we skip the prime marks for dimensionless quantities, then equations (5) keep their form, but

$$\gamma = \frac{(1-\mu^2)}{E\delta}; \quad \lambda = -\rho\delta\omega^2 - \kappa\omega; \quad \frac{\kappa p_0}{c_0} \to \kappa.$$

Thus, we have a non-self-adjoint eigenvalue problem (2)–(5).

The oscillations of the plate will be stable or not depending on whether $\mathrm{Re}\omega > 0$ or $\mathrm{Re}\omega < 0$. If $\lambda_1 = \alpha_1 + \beta_1 i$ is the eigenvalue, then the written inequalities correspond to $F(\alpha_1, \beta_1) > 0$ or $F(\alpha_1, \beta_1) < 0$, where $F(\alpha_1, \beta_1) = \alpha_1 k^2 - \rho h \beta_1^2$. Let λ_1 be the first of the eigenvalues satisfying the condition $F = 0$; we are therefore talking about finding the zeros of the function $F\left(\alpha_1\left(U^0\right), \beta_1\left(U^0\right)\right)$ for a given direction of the flow velocity vector.

So, the oscillations of the plate will be stable or not depend on whether the eigenvalues λ of the spectral problem under consideration are inside the parabola $y^2 = \frac{\kappa^2}{\rho h} x$ or not. Specific calculations were carried out for the data: $\mu = 0.35$; $\rho_s = 8902.19$ kg/m^3; $E = 110316145388.42$ Pa; $R = 0.09652$ m; $l = 0.39116$ m; $\delta = 0.0001016$; $M_\infty = 3$. The rest of the parameters are the same as in [1]: $p_0 = 1.0133\times10^5$ Pa; $\rho_0 = 1.2928$ kg/m^3; $\kappa = 1.4$. Dimensionless density is $\rho' = \kappa\rho/\rho_0$, so the parabola parameter is $\kappa\rho_0/\rho\delta$.

CALCULATION RESULTS

Calculations were carried out for the parameters of Olson's experiments (1966) [3]. Equations (2) and (3) could be represented as a block matrix 2×2 regarding variables U and V (values of the corresponding displacements at grid nodes). Inverting this block matrix using the Frobenius formula [4], we express U and V in terms of W. Then we substitute the result into equation (4) and obtain a non-self-adjoint eigenvalue problem for a matrix of size $m\times m$, whose eigenvalues are calculated using a standard program (a freely redistributable package *EISPACK* is used). The calculation results are presented in Table 1.

Table 1

n	V_{CR}	n	V_{CR}
0	V = 0.298887 IN = 1	10	V = 0.421207 IN = 1
1	V = 0.253175 IN = 4	11	V = 0.326958 IN = 5
2	V = 0.369852 IN = 4	12	V = 0.407252 IN = 4
3	V = 0.280300 IN = 3	13	V = 0.422564 IN = 1
4	V = 0.417215 IN = 1	14	V = 0.422769 IN = 1
5	V = 0.226167 IN = 3	15	V = 0.422677 IN = 1
6	V = 0.261352 IN = 5	16	V = 0.345211 IN = 4
7	V = 0.325629 IN = 5	17	V = 0.419488 IN = 1
8	V = 0.382811 IN = 5	18	V = 0.409492 IN = 1
9	V = 0.420629 IN = 1	19	V = 0.389287 IN = 1

Calculations were made for $n = 0$ on grids of 25 and 50 nodes, for $n > 0$—on grids of 10 and 20 nodes. Both grids had demonstrated close values. The table contains values on the fine grid. As we can see from examining the table, the values for different n are close. Within the limits of experimental error, they could be considered equal. Thus, the statement of Voss [5] is confirmed, that in the study of the critical speed, it is sufficient to consider the axisymmetric case.

In conclusion, let us present a form for the critical eigenvalue, axisymmetric case (Fig. 2).

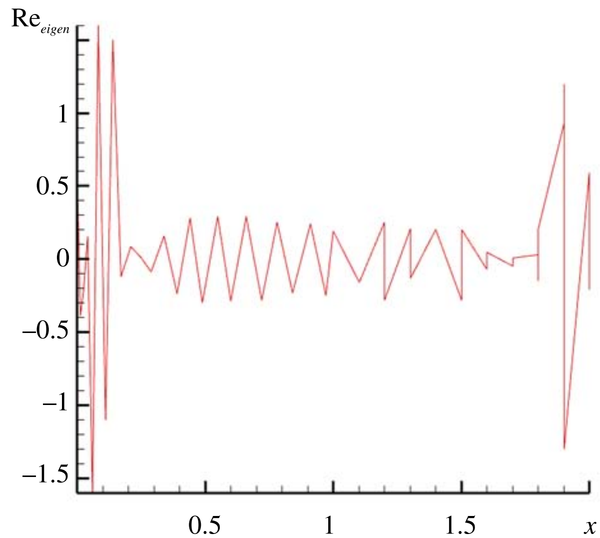

Fig. 2.

Specific calculations were carried out for experimental data [3], taking into account the experimental error. The obtained values of the critical speed for different numbers of circumferential waves can be considered approximately equal, and to calculate the critical speed, it is sufficient to consider the axisymmetric case.

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CONFLICT OF INTEREST

The author of this work declares that he has no conflict of interest.

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