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## FLIGHT DYNAMICS AND CONTROL OF FLIGHT VEHICLES

# The Influence of the Angular Dimension of Rectangular Cutouts on the Stress and Strain State of Sandwich Cylindrical Compartments

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**Abstract**—A model is presented for the layer-by-layer finite element stress and strain analysis of sandwich cylindrical shells. The influence of the angular dimension of rectangular cutouts on the stress and strain state of sandwich cylindrical compartments was studied.

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*Keywords*: sandwich cylindrical shells, rectangular cutouts, stress and strain state, layer-by-layer finite element analysis model.

## INTRODUCTION

Composite sandwich shells [1–7] are widely used in modern technology, especially, in rocket and aircraft manufacturing, and in shipbuilding [8–14].

Structures often have various kinds of cutouts, near which zones of increased stresses appear, and the largest of them can be several times higher than the stresses far from such zones [15–22].

The analysis of shells weakened by holes become the topic of a large body of research. However, only a few works are devoted to the study of the stress and strain state of sandwich cylindrical shells with cutouts rectangular in plan [23–28]. These are mostly the problem-setting works. A stress-strain analysis of sandwich shells weakened by rectangular holes is, as a rule, not given.

There are works on the analysis of sandwich spherical shells with circular, elliptical and curvilinear holes [29–33], as well as with rectangular cutouts [34, 35]. Some results of an experimental study of sandwich spherical shells with holes are given in [36].

Sandwich conical shells with rectangular cutouts were studied in [37–40].

A three-layer cylinder with circular holes was considered in [41].

An analysis of publications has shown that, with the accuracy necessary for practice, studying the stress and strain state of shells (especially of a sandwich structure) weakened by rectangular holes using analytical methods is associated with large, mostly insurmountable difficulties of a mathematical nature [16, 42] and it is impossible to solve this problem without the use of numerical methods [38, 43–47].

According to the review presented in [38], the influence of the angular dimension of rectangular cutouts on the stress and strain state of sandwich cylindrical shells, despite its relevance, has not been studied and the most suitable method for solving this problem is the finite element method (FEM) [48–51].

In modern technology and especially in rocket and aircraft manufacturing, and in shipbuilding, sandwich structures with thin and rigid facesheets and a relatively thick, but less rigid core are most used [52]. When analyzing shells of such a sandwich structure, models should take into account the bending state of the facesheets and the three-dimensional stress and strain state of the core [53–55].

This paper considers a model of layer-by-layer finite element stress-strain analysis of sandwich cylindrical shells, with the help of which the influence of the angular dimension of rectangular cutouts on the stress-strain state of sandwich cylindrical compartments is studied.

## A MODEL FOR CALCULATION

The layer-by-layer analysis model is constructed from two-dimensional finite elements (FEs) of momental facesheet layers and three-dimensional FEs of the core layer. These finite elements are cylindrical in shape. The coordinate systems are placed on the middle surfaces of the finite elements.

Let us introduce the numbering of layers of a sandwich cylindrical open shell of revolution—$i = 1, 2, 3$, starting from the inner surface of the shell. Additionally, we denote the facesheet layers with the subscript "*c*", and the core layer with the subscript "*f*". Let us denote the radii of the middle surfaces of the layers and their thicknesses with $R_i$ and $h_i$, respectively.

To study the stress-strain state in load-bearing layers, we consider the rectangular two-dimensional finite elements with nodes at corner points. When constructing a model of these finite elements, we use the classical general theory of cylindrical shells [56].

The nodal generalized displacements of the finite element for studying the stress and strain state in the facesheets will be the displacements of the points of the middle surface *u, v, w* and the angles of rotation of the normal $\vartheta_x$, $\vartheta_y$ to the middle surface relative to the *x* and *y* (φ) axes. Thus, the finite element will have four nodes with five degrees of freedom per node.

The rigid-body displacement functions of the finite element (taking them into account increases the rate of convergence of solutions) are determined by integrating the relations between the components of the generalized strain vector and the displacement vector [56] at zero values of deformation. When writing these obtained displacement functions of the finite element, six indefinite coefficients were used, which are the integration constants $\alpha_1,...,\alpha_6$.

The number of indefinite coefficients when recording the total displacements of the finite element is equal to the number of the FE's degrees of freedom. Taking into account the nature of the variation in stress and strain state parameters of the facesheet layers [57], we will write down the components of the generalized strain vector using the fourteen indefinite coefficients remaining after recording the rigid-body displacement functions. Let us represent the parameters of curvature change $æ_1$, $æ_2$ by polynomials of higher orders than that for the other parameters of the generalized strain vector $\varepsilon_1$, $\varepsilon_2$, $\gamma$, $\chi$ [57]. The functions for recording the generalized strain [57] should satisfy the equations of strain compatibility [58].

To obtain functions that approximate the displacements determined by the deformation of FEs, the integration of the relations between the components of the generalized strain vector and the displacement vector is carried out [56, 58] with the obtained approximations for the strain functions [59].

The approximating displacement functions of the finite element of facesheet layers, written in matrix form with the use of the vector of indefinite coefficients $\boldsymbol{\alpha}_i^c$, will have the form

$$\boldsymbol{\delta}_i^c = \mathbf{T}_i^c \boldsymbol{\alpha}_i^c, \tag{1}$$

where $\mathbf{T}_i^c(3 \times 20)$ is the matrix of displacement approximating functions of the sandwich open cylindrical shell of revolution facesheet FE:

**Table 1**

| | $u$ | $v$ | $w$ |
|---|---|---|---|
| $\alpha_1$ | 0 | $\sin\varphi$ | $-\cos\varphi$ |
| $\alpha_2$ | $R\cos\varphi$ | $x\sin\varphi$ | $-x\cos\varphi$ |
| $\alpha_3$ | 0 | $-\cos\varphi$ | $-\sin\varphi$ |
| $\alpha_4$ | $R\sin\varphi$ | $-x\cos\varphi$ | $-x\sin\varphi$ |
| $\alpha_5$ | 1 | 0 | 0 |
| $\alpha_6$ | 0 | 1 | 0 |
| $\alpha_7$ | $x$ | 0 | 0 |
| $\alpha_8$ | $x\varphi$ | $-x^2/(2R)$ | 0 |
| $\alpha_9$ | 0 | 0 | $R$ |
| $\alpha_{10}$ | 0 | 0 | $Rx$ |
| $\alpha_{11}$ | $R\varphi$ | 0 | 0 |
| $\alpha_{12}$ | 0 | 0 | $-x^2/2$ |
| $\alpha_{13}$ | 0 | 0 | $-x^3/6$ |
| $\alpha_{14}$ | 0 | 0 | $-x^2\varphi/2$ |
| $\alpha_{15}$ | 0 | 0 | $-x^3\varphi/6$ |
| $\alpha_{16}$ | 0 | $R^2\varphi$ | $-R^2$ |
| $\alpha_{17}$ | $-R^3\varphi^2/2$ | $R^2x\varphi$ | $-R^2x$ |
| $\alpha_{18}$ | 0 | $R^2\varphi^2$ | $-R^2\varphi$ |
| $\alpha_{19}$ | $R^3\varphi\left(-\varphi^2/6+1\right)$ | $R^2x\left(\varphi^2/2-1\right)$ | $-R^2x\varphi$ |
| $\alpha_{20}$ | $-R^2\varphi$ | $Rx$ | 0 |

Taking into account formula (1) and writing the matrix $\left(\mathbf{T}_i^c\right)^{\mathrm{T}}$, we obtain expressions for the rotation angles of the normal relative to the axial and circumferential coordinate axes $x$, $y$ ($\varphi$) [60]

$$\vartheta_x = \frac{\partial w}{R\partial\varphi} - \frac{v}{R} = -\alpha_6\frac{1}{R} + \alpha_8\frac{x^2}{2R^2} - \alpha_{14}\frac{x^2}{2R} - \alpha_{15}\frac{x^3}{6R} - \alpha_{16}R\varphi - \alpha_{17}Rx\varphi$$
$$-\alpha_{18}R\left(1+\frac{\varphi^2}{2}\right) - \alpha_{19}Rx\frac{\varphi^2}{2} - \alpha_{20}x;$$

$$\vartheta_y = -\frac{\partial w}{\partial x} = \alpha_2\cos\varphi + \alpha_4\sin\varphi - \alpha_{10}R + \alpha_{12}x + \alpha_{13}\frac{x^2}{2} + \alpha_{14}x\varphi + \alpha_{15}\frac{x^2}{2}\varphi + \alpha_{17}R^2 + \alpha_{19}R^2\varphi.$$

It is shown in [61] that the approximating displacement functions considered are highly efficient, since they lead to a high rate of convergence of numerical solutions.

Layer-by-layer analysis models are characterized by a large number of degrees of freedom [62], which makes it difficult to carry out the finite element analysis of sandwich open shells with the required accuracy. Therefore, the construction and application of effective approximating displacement functions, leading to a decrease in the number of FEs in the calculation, is an urgent scientific problem [63], which has important applied significance [64].

Let us apply the considered approximating displacement functions (ADFs) of the finite elements of the facesheets to construct the ADFs of three-dimensional shell FEs of the core. We write the displacement vector in these FEs $\delta^f = \{u, v, w\}^{\mathrm{T}}$ in terms of the displacement vectors on the inner $\delta^1$ and outer $\delta_2$ cylindrical surfaces of the finite elements of the core:

$$\delta^f = \delta^1\varphi_i^1 + \delta^2\varphi_i^2, \tag{2}$$

where $\varphi_i^1 = \frac{1}{2}\left(1 - 2\frac{z}{h}\right)$, $\varphi_i^2 = 1 - \varphi_i^1$, $z$ is the coordinate along the normal to the FE's middle surface; $h$ is the thickness of the three-dimensional FE of the core.

In expression (2) and further, the superscripts "1" and "2" correspond to the inner and outer cylindrical surfaces of the finite element of the core.

We will reduce the approximating functions of the facesheet FE displacements to the corresponding cylindrical interface with the finite elements of the core using transition matrices, similarly to what was done in [65].

When modeling the core of a sandwich shell with one three-dimensional FE along the thickness, its cylindrical surfaces will be the interface with the facesheets.

Let us write down the displacement vectors of the facesheet FE on the cylindrical interface surfaces with the core FEs

$$\overline{\delta}_1^c = \left\{\overline{u}_1^c, \overline{v}_1^c, \overline{w}_1^c\right\}^T; \quad \overline{\delta}_3^c = \left\{\overline{u}_3^c, \overline{v}_3^c, \overline{w}_3^c\right\}^T, \tag{3}$$

where $\overline{u}_1^c = u_1^c + \vartheta_{y1}^c h_1^c/2$, $\overline{v}_1^c = v_1^c - \vartheta_{x1}^c h_1^c/2$, $\overline{w}_1^c = w_1^c$, $\overline{u}_3^c = u_3^c - \vartheta_{y3}^c h_3^c/2$, $\overline{v}_3^c = v_3^c + \vartheta_{x3}^c h_3^c/2$, $\overline{w}_3^c = w_3^c$; $h_1^c$, $h_3^c$ are the thicknesses of the inner and outer facesheets of the sandwich open cylindrical shell of revolution.

We use the approximating functions of the facesheet FE displacement fields as the ADFs of the core FEs on the inner and outer cylindrical surfaces. Then we will get

$$\delta^1 = \overline{\delta}_1^c; \quad \delta^2 = \overline{\delta}_3^c. \tag{4}$$

Taking into account expressions (1) – (4) and knowing the ADFs of the facesheet finite elements of a sandwich cylindrical open shell of revolution, we obtain the approximating displacement functions of a three-dimensional core FE, written using forty indefinite coefficients $\alpha_1, \ldots, \alpha_{40}$:

$$\begin{aligned}
u = u^1\varphi_i^1 + u^2\varphi_i^2 = \Big(&\alpha_2\left(R_1 + h_1^c/2\right)\cos\varphi + \alpha_4\left(R_1 + h_1^c/2\right)\sin\varphi + \alpha_5 + \alpha_7 x \\
&+\alpha_8 x\varphi - \alpha_{10} R_1 h_1^c/2 + \alpha_{11} R_1\varphi + \alpha_{12} h_1^c x/2 + \alpha_{13} h_1^c x^2/4 + \alpha_{14} h_1^c x\varphi/2 \\
&+\alpha_{15} h_1^c x^2\varphi/4 + \alpha_{17} R_1^2\left(h_1^c - R_1\varphi^2\right)/2 + \alpha_{19} R_1^2\varphi\left(h_1^c/2 + R_1\left(1 - \varphi^2/6\right)\right) - \alpha_{20} R_1^2\varphi\Big)\varphi_i^1 \\
&+\Big(\alpha_{22}\left(R_3 - h_3^c/2\right)\cos\varphi + \alpha_{24}\left(R_3 - h_3^c/2\right)\sin\varphi + \alpha_{25} + \alpha_{27} x \\
&+\alpha_{28} x\varphi + \alpha_{30} R_3 h_3^c/2 + \alpha_{31} R_3\varphi - \alpha_{32} h_3^c x/2 - \alpha_{33} h_3^c x^2/4 - \alpha_{34} h_3^c x\varphi/2 \\
&-\alpha_{35} h_3^c x^2\varphi/4 - \alpha_{37} R_3^2\left(h_3^c + R_3\varphi^2\right)/2 - \alpha_{39} R_3^2\varphi\left(h_3^c/2 - R_3\left(1 - \varphi^2/6\right) - \alpha_{40} R_3^2\varphi\right)\Big)\varphi_i^2;
\end{aligned}$$

$$\begin{aligned}
v = v^1\varphi_i^1 + v^2\varphi_i^2 = \Big(&\left(\alpha_1 + \alpha_2 x\right)\sin\varphi - \left(\alpha_3 + \alpha_4 x\right)\cos\varphi + \alpha_6\left(1 + h_1^c/2R_1\right) \\
&-\alpha_8\left(1 + h_1^c/2R_1\right)x^2/2R_1 + \alpha_{14} h_1^c x^2/4R_1 + \alpha_{15} h_1^c x^3/12R_1 \\
&+\alpha_{16} R_1\left(R_1 + h_1^c/2\right)\varphi + \alpha_{17} R_1\left(R_1 + h_1^c/2\right)x\varphi + \alpha_{18} R_1\left(\left(R_1 + h_1^c/2\right)\varphi^2 + h_1^c\right)/2 \\
&+\alpha_{19} R_1\left(\left(R_1 + h_1^c/2\right)\varphi^2/2 - R_1\right)x + \alpha_{20}\left(R_1 + h_1^c/2\right)x\Big)\varphi_i^1
\end{aligned}$$

$$+\left(\left(\alpha_{21}+\alpha_{22}x\right)\sin\varphi-\left(\alpha_{23}+\alpha_{24}x\right)\cos\varphi+\alpha_{26}\left(1+h_3^c/2R_3\right)\right.$$
$$+\alpha_{28}\left(h_3^c/2R_3-1\right)x^2/2R_3-\alpha_{34}h_3^c\,x^2/4R_3-\alpha_{35}h_3^c\,x^3/12R_3$$
$$+\alpha_{36}R_3\left(R_3-h_3^c/2\right)\varphi+\alpha_{37}R_3\left(R_3-h_3^c/2\right)x\varphi+\alpha_{38}R_3\left(\left(R_3-h_3^c/2\right)\varphi^2-h_3^c\right)/2$$
$$\left.+\alpha_{39}R_3\left(\left(R_3-h_3^c/2\right)\varphi^2/2-R_3\right)x+\alpha_{40}R_3\left(1-h_3^c/2\right)x\right)\varphi_i^2;$$

$$w=w^1\varphi_i^1+w^2\varphi_i^2=\left(-\left(\alpha_1+\alpha_2x\right)\cos\varphi-\left(\alpha_3+\alpha_4x\right)\sin\varphi+\alpha_9R_1+\alpha_{10}R_1x\right.$$
$$\left.-\alpha_{12}\,x^2/2-\alpha_{13}\,x^3/6-\alpha_{14}\,x^2\varphi/2-\alpha_{15}\,x^3\varphi/6-\alpha_{16}R_1^2-\alpha_{17}R_1^2x-\alpha_{18}R_1^2\varphi-\alpha_{19}R_1^2x\varphi\right)\varphi_i^1$$
$$+\left(-\left(\alpha_{21}+\alpha_{22}x\right)\cos\varphi-\left(\alpha_{23}+\alpha_{24}x\right)\sin\varphi+\alpha_{29}R_3+\alpha_{30}R_3x-\alpha_{32}\,x^2/2\right.$$
$$\left.-\alpha_{33}\,x^3/6-\alpha_{34}\,x^2\varphi/2-\alpha_{35}\,x^3\varphi/6-\alpha_{36}R_3^2-\alpha_{37}R_3^2x-\alpha_{38}R_3^2\varphi-\alpha_{39}R_3^2x\varphi\right)\varphi_i^2.$$

Knowing the ADFs of the finite elements of the open cylindrical shell of revolution facesheets and core and following the algorithms from [40], we calculate the FE stiffness matrices. Further solution of the problem is carried out using the procedures of the FEM displacement method [48–50].

## THE INFLUENCE OF THE ANGULAR DIMENSION OF RECTANGULAR CUTOUTS ON THE STRESS AND STRAIN STATE OF SANDWICH CYLINDRICAL COMPARTMENTS

The stress-strain state of a three-layer circular cylindrical compartment with two diametrically opposite rectangular cutouts located symmetrically relative to the ends of the shell was studied. The shell was loaded with an axial compressive load $p$ (resulting axial compressive force $P$) uniformly distributed at the ends (Fig. 1).

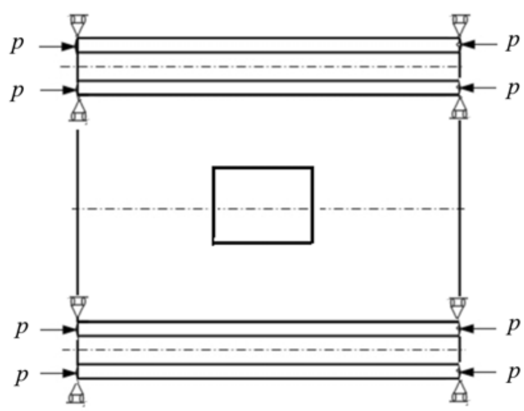


**Fig. 1.**

The boundary conditions at the ends corresponded to the hinged support of the facesheets.

The shell has the following geometric parameters: the compartment length is $L$ = 80 cm; the radius of the middle surface of the core is $R_2$ = 17.65 cm; the thicknesses of the facesheets and core are $h_1 = h_3$ = 0.2 cm and $h$ = 0.7 cm; the length of cutouts is $l_c$ = 20 cm.

We considered a sandwich compartment with facesheets made of composite materials: ($E_1 = 2.1\times10^5$ kg/cm$^2$, $E_2 = 1.9\times10^5$ kg/cm$^2$ are the axial and circumferential moduli of elasticity; $G_{12} = 0.36\times10^5$ kg/cm$^2$ is the shear modulus; $\mu_1$ = 0.1 is Poisson's ratio) and a lightweight core ($E_3$ = 240 kg/cm$^2$ is the transverse modulus of elasticity; $G_{13} = G_{23}$ = 100 kg/cm$^2$ are the transverse shear moduli).

The range of angular dimensions of the cutouts was $30 \le \beta_c \le 50\,\text{deg}$. A 1/8 symmetrical part of the compartment was considered with a variation in φ from 0 to 90 deg and in $x$ from 0 to $L/2$. The mesh size was refined in the area of the cutout.

As a result of the calculations carried out, the stresses in the facesheets and the core of a sandwich circular cylindrical compartment were determined.

The highest stresses in the facesheets are axial compressive membrane stresses. The change in these stresses in dimensionless form $\left(\bar{\sigma} = \sigma 2\pi\left(R_1 h_1 + R_3 h_3\right)/P\right)$ in the internal facesheet of a sandwich compartment along a section passing near the curvilinear edge of the cutout for the rectangular cutout angular dimensions of 30, 40, and 50 deg is shown in Fig. 2.

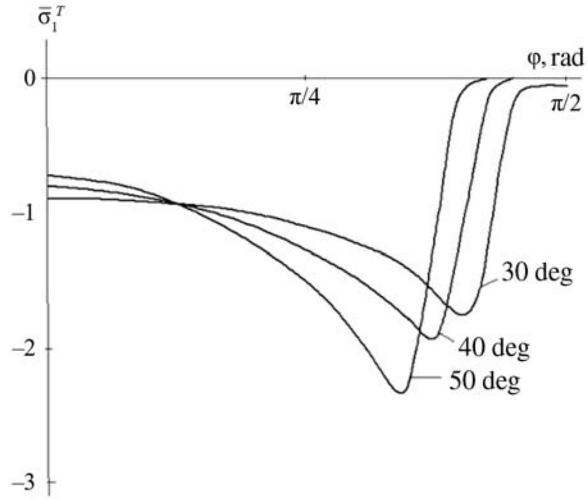


**Fig. 2.**

Near the corner point of the cutout, the axial bending stresses increase significantly in magnitude, reaching their highest values, while being in the negative region, then decrease and stabilize towards the middle of the curved edge of the cutout (Fig. 3).

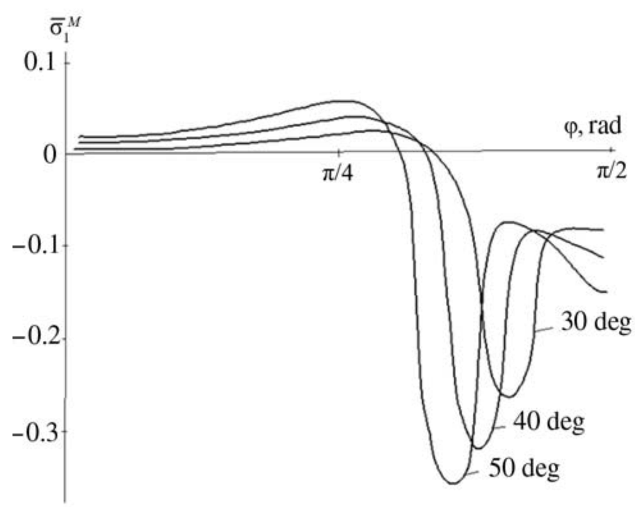


**Fig. 3.**

Bending stresses $\bar{\sigma}_1^M$ are significantly lower than the membrane stresses $\bar{\sigma}_1^T$ that is observed for other stresses in the facesheets of the sandwich compartment.

The difference between the stress values in the outer and the inner facesheets is insignificant.

The change in the maximum stresses in the core of a sandwich compartment, which are the transverse shear stresses $\tau_{23}$, in dimensionless form $\left(\bar{\tau}_{23} = \tau_{23} 2\pi R_2 h_2 / P\right)$ in a section passing near the straight edge of the cutout is shown in Fig. 4.

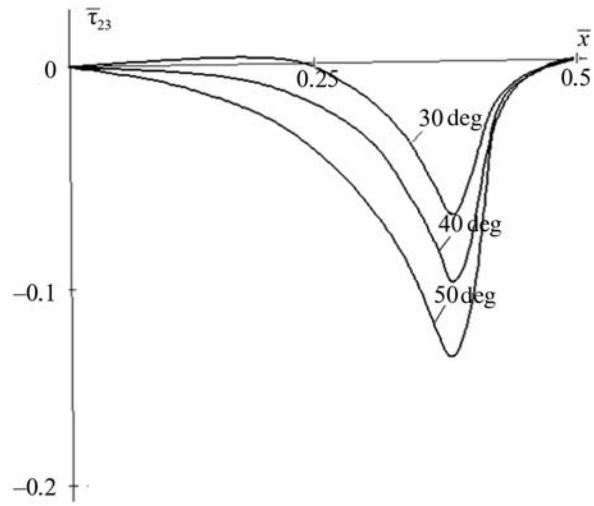


**Fig. 4.**

As evident from Fig. 4, the transverse shear stresses $\bar{\tau}_{23}$ in the core of the sandwich compartment are almost completely in the negative region, reaching maximum values in the area of the corner point of the cutout.

The maximum stress values $\tau_{13}$ in the core of a sandwich compartment are almost 2.5 times lower than the maximum stress values $\tau_{23}$, and the maximum values of radial stresses $\tau_{33}$ are more than three times less than the maximum stress values $\tau_{13}$.

Thus, for the first time, the important scientific and applied research was carried out on the effect of the angular dimension of rectangular cutouts on the stresses in the layers of a sandwich circular cylindrical compartment.

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## CONFLICT OF INTEREST

The author of this work declares that he has no conflict of interest.

## REFERENCES

1. Vasil'ev, V.V., *Mekhanika konstruktsii iz kompozitsionnykh materialov* (Mechanics of Composite Structures), Moscow: Mashinostroenie, 1988.

2. Paimushin, V.N. and Gazizullin, R.K., Refined Analytical Solutions of the Coupled Problems on Free and Forced Vibrations of a Rectangular Composite Plate Surrounded by Acoustic Media, *Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki*, 2020, vol. 162, no. 2, pp. 160–179.
3. Dudchenko, A.A., *Prochnost' i proektirovanie aviatsionnykh konstruktsii iz kompozitsonnogo materiala* (Strength and Design of Aircraft Composite Structures), Moscow: MAI, 2007.
4. Turkin, I.K., *Elementy konstruktsii LA s primeneniem kompozitsionnykh materialov* (Use of Composite Materials in Structural Aircraft Elements), Moscow: MAI, 1997.
5. Bakulin, V.N., Obraztsov, I.F., and Potopakhin, V.A., *Dinamicheskie zadachi nelineinoi teorii mnogosloinykh obolochek; deistvie intensivnykh termosilovykh nagruzok kontsentrirovannykh potokov energii* (Dynamic Problems of the Nonlinear Theory of Multilayer Shells: The Action of Intense Thermopower Loading of Concentrated Energy Fluxes), Moscow: Fizmatlit, 1998.
6. Smerdov A.A. and Shon F.T., Comparative Analysis of Optimal Composite Multiwalled and Sandwich Shells for Modules of Launch Vehicles and Upper Stages, *Konstruktsii iz Kompozitnykh Materialov*, 2016, no. 3 (143), pp. 58–65.
7. Bakulin, V.N. and Ostrik, A.V., *Kompleksnoe deistvie izluchenii i chastits na tonkostennye konstruktsii s geterogennymi pokrytiyami* (Complex Effect of Radiation and Particles on Thin-Walled Structures with Heterogeneous Coatings, Moscow: Nauka, FIzmatlit, 2015.
8. Panin, V.F. and Gladkov, Yu.A., *Konstruktsii s zapolnitelem* (Structures with a Filler), Moscow: Mashinostroenie, 1991.
9. Solomonov, Yu.S., Georgievskii, V.P., Nedbai, A.Ya., and Andryushin, V.A., *Prikladnye zadachi mekhaniki kompozitnykh tsilindricheskikh obolochek* (Applied Problems of Mechanics of Composite Cylindrical Shells), Moscow: Fizmatlit, 2014.
10. Bakulin, V.N., Three-Layer Shells—Effective Elements of Modern Aircraft Structures. Strength Analysis Models, *Materialy 14oi mezhdunarodnoi konferentsii po prikladnoi matematike i mekhanike v aerokosmicheskoi otrasli* (Proc. 14th Int. Conf. on Applied Mathematics and Mechanics in the Aerospace Industry), Moscow: MAI, 2022, pp. 270–272.
11. Prokhorov, B.F. and Kobelev, V.N., *Trekhsloinye konstruktsii v sudostroenii* (Three-Layer Structures in Shipbuilding), Leningrad: Sudostroenie, 1972.
12. Endogur, A.I., Vainberg, M.V., and Ierusalimskii, K.M., *Sotovye konstruktsii. Vybor parametrov i proektirovanie* (Honeycomb Structures. Choice of Parameters and Design), Moscow: Mashinostroenie, 1986.
13. Avdeev, V. S., *Tsentral'nyi nauchno-issledovatel'skii institut spetsial'nogo mashinostroeniya. Put' dlinoi v polveka* (Central Research Institute for Special Machinery. A Journey of Half a Century), Khotkovo, 2015.
14. Bakulin, V.N., Investigation of the Influence of the Cutout Dimensions on the Stress-Strain State of Three-Layer Shells with Load-Bearing Layers of Composite Materials, *Journal of Physics: Conference Series: Materials Science and Engineering,* 2020, vol. 714, Article no. 012002.
15. Guz', A.N., Chernyshenko, I.S., Chekhov, Val.N., Chekhov, Vik.N., and Shnerenko, K.I., *Metody rascheta obolochek* (Techniques of Shell Analysis), 5 vols., vol. 1: *Teoriya tonkikh obolochek oslablennykh otverstiyami* (Theory of Thin Shells Weakened by Openings), Kiev: Naukova Dumka, 1980.
16. Pelekh, B.L. and Syas'kii, A.A., *Raspredelenie napryazhenii okolo otverstii v podatlivykh na sdvig anizotropnykh obolochkakh,* (Distribution near Openings in Shear-Compliant Anisotropic Shells), Kiev: Naukova Dumka, 1975.
17. Bakulin, V.N. and Revenko, V.P., Analytical and Numerical Method of Finite Bodies for Calculation of Cylindrical Orthotropic Shell with Rectangular Hole, *Izv. Vuz. Matematika*, 2016, vol. 60, no. 6, pp. 3–14 [Russian Mathematics (Engl. Transl.), 2016, vol. 60, no. 6, pp. 1–11].
18. Savin, G.N., *Raspredelenie napryazhenii okolo otverstii* (Distribution of Stresses Around Holes), Kiev: Naukova Dumka, 1968, 891 p.
19. Salo, V.A., *Kraevye zadachi statiki obolochek s otverstiyami* (Boundary Value Problems of Static Shells with Holes), Kharkiv: NTU KhPI, 2003.
20. Vorobei, V.V., Deformability of an Impregnated Fiberglass Shells Reinforced Around a Hole, *Prikladnaya Mekhanika*, 1979, vol. 15, no. 1, pp. 82–85 [Soviet Applied Mechanics (Engl. Transl.), vol. 15, no. 1, pp. 63–66].

21. Bakulin, V.N., Investigation of the Stress-Strain State of the Composite Cylindrical Shell with Rectangular Cutouts, *IOP Conference Series: Materials Science and Engineering,* 2020, vol. 927, Article no. 012066.
22. Ushakov, A.E., Choice of Rational Reinforcement of Structural Notches in Carbon Plastic Panels. Part 2. Results of the Theoretical and Experimental Investigation of the Effectiveness of Methods of Reinforcement, *Mechanics of Composite Materials*, 1990, vol. 26, no. 1, pp. 83–87.
23. Aksentyan, K.B. and Krasnobaev, I.A., Calculation of a Circular Three-Layer Cylindrical Shell with a Large Rectangular Cutout, *Izv. Vuz. Stroitel'stvo i Arkhitektura,* 1973, no. 2, pp. 45–51.
24. Aksentyan, K.B. and Krasnobaev, I.A., The Basic Equations of Bending and the Method for Calculating a Circular Three-Layer Cylindrical Shell with a Large Rectangular Cutout, in *Teoriya Obolochek i Plastin* (Theory of Shells and Plates), Moscow: Nauka, 1973, pp. 601–605.
25. Bakulin, V.N., Analysis of the Effect of the Physicomechanical Properties of Composite Materials of Carrier Layers on the Stress State of Sandwich Shells with Rectangular Cutouts, *Journal of Physics: Conference Series*, 2020, vol. 1705, Article no. 012022.
26. Sakharov, A.S., Gondlyakh, A.V., Mel'nikov, S.L., and Snitko, A.N., Numerical Modeling of Progressive Crack Formation in Multilayer Shells, in *Problemy chislennogo modelirovaniya i avtomatizatsii proektirovaniya inzhenernykh konstruktsii* (Problems of Numerical Modeling and Automation of Design of Engineering Structures), Leningrad: LIIZhT, 1987, pp. 9–14.
27. Bakulin, V.N., An Efficient Model for Layer-by-Layer Analysis of Sandwich Irregular Cylindrical Shells of Revolution, *Doklady Akademii Nauk*, 2018, vol. 478, no. 2, pp. 148–152 [Doklady Physics (Engl. Transl.), vol. 63, no. 1, pp. 37–41].
28. Bakulin, V.N., Model for Analysis of the Stress-Strain State of Three-Layer Cylindrical Shells with Rectangular Cutouts, *Izv. RAN Mekhanika Tverdogo Tela,* 2022, vol. 57, no. 1, pp. 122–132 [Mechanics of Solids (Engl. Transl.), 2022, vol. 57, no. 1, pp. 102–110].
29. Van Fo Fy, G.A. Stress Concentration Close to Holes in Three-Layer Shells, *Prikladnaya Mekhanika*, 1969, vol. 5, no. 2, pp. 51–61 [Soviet Applied Mechanics (Engl.Transl.), vol. 5, no. 2, pp. 147–155].
30. Van Fo Fy, G.A. and Savichenko, A.A., Stress State Around a Circular Cutout in a Spherical Sandwich Shell, *Soviet Applied Mechanics*, 1970, vol. 6, no. 8, pp. 897–900.
31. Van Fo Fy, G.A. and Zhalilo, A.I., Equilibrium of a Three-Layer Spherical Shell with an Oval Cutout, in *Raschet i konstruirovanie izdelii iz stekloplastikov* (Calculation and Design of Fiberglass Products), Kiev: Naukova Dumka, 1970, pp. 79–106.
32. Zhalilo, A.I., Stress-Strain State Near the Elliptical Cutout in a Three-Layer Spherical Shell, in *Ustoichivost' i deformativnost' elementov konstruktsii iz kompozitsionnykh materialov* (Stability and Deformability of Structural Elements Made of Composite Materials), Kiev: Naukova Dumka, 1972, pp. 55–62.
33. Vanin, G.A. and Savichenko, A.A., Interference of Two Holes on the Stressed State in a Three-Layered Spherical Shell, *Russian Applied Mechanics*, 1975, vol. 11, no. 12, pp. 1260–1264.
34. Bakulin, V.N., Block Finite-Element Model of Layer-by-Layer Analysis of the Stress-Strain State of Three-Layer Generally Irregular Shells of Double-Curvature Revolution, *Doklady Akademii Nauk*, 2019, vol. 484, no. 1, pp. 35–40 [Doklady Physics (Engl. Transl.), vol. 64, no. 1, pp. 9–13].
35. Bakulin, V.N., Model for Layer-by-Layer Analysis of the Stress-Strain State of Three-Layer Irregular Shells of Revolution of Double Curvature, *Izv. RAN Mekhanika Tverdogo Tela,* 2020, vol. 55, no. 2, pp. 112–123 [Mechanics of Solids (Engl. Transl.), 2020, vol. 55, no. 2, pp. 248–257].
36. Kotel'nikov, V.U. and Tarasenko, V.G., Some Results of an Experimental Study of Three-Layer Spherical Shells with Holes, *Izv.Vuz. Av. Tekhnika*, 1985, vol. 28, no. 2, pp. 81–83 [Soviet Aeronautics (Engl. Transl.), vol. 28, no. 2, pp. 98–100].
37. Bakulin, V.N., Block Based Finite Element Model for Layer Analysis of Stress Strain State of Three-Layered Shells with Irregular Structure, *Izv. RAN Mekhanika Tverdogo Tela,* 2018, vol. 53, no. 4, pp. 66–73 [Mechanics of Solids (Engl. Transl.), 2018, vol. 53, no. 4, pp. 411–417].
38. Bakulin, V.N., Layer-by-Layer Analysis of the Stress-Strain State of Three-Layer Shells with Cutouts, *Izv. RAN Mekhanika Tverdogo Tela,* 2019, vol. 54, no. 2, pp. 111–125 [Mechanics of Solids (Engl. Transl.), 2019, vol. 54, no. 2, pp. 448–460].
39. Bakulin, V.N., A Model for Refined Calculation of the Stress-Strain State of Sandwich Conical Irregular Shells, *Mechanics of Solids*, 2019, vol. 54, no. 5, pp. 786–796.

40. Bakulin, V.N., Layer-by-Layer Study of the Stress and Strain State of Sandwich Conical Aircraft Compartments with Rectangular Cutouts, *Izv. Vuz. Av. Tekhnika*, 2022, vol. 65, no. 4, pp. 37–43 [Russian Aeronautics (Engl. Transl.), vol. 65, no. 4, pp. 668–676].
41. Sun, F., Wang, P., Li, W., Fan, H., and Fang, D., Effects of Circular Cutouts on Mechanical Behaviors of Carbon Fiber Reinforced Lattice-Core Sandwich Cylinder, *Composites Part A: Applied Science and Manufacturing*, 2017, vol. 100, pp. 313–323.
42. Pelekh, B.L. and Lun', E.I., Stress Concentration Near Holes in Transversely Isotropic Shells, *Mekhanika Polimerov*, 1976, no. 6, pp. 1076–1081.
43. Bakulin, V.N., Block-Layer Approach for the Analysis of the Stress-Strain State of Three-Layer Irregular Cylindrical Shells of Rotation, *Mechanics of Solids*, 2021, vol. 56, no. 7, pp. 1451–1460.
44. Dmitriev, V.G., Egorova, O.V., Zhavoronok, S.I., and Rabinskii, L.N., Investigation of Buckling Behavior for Thin-Walled Bearing Aircraft Structural Elements with Cutouts by Means of Numerical Simulation, *Izv. Vuz. Av. Tekhnika*, 2018, vol. 61, no. 2, pp. 18–26 [Soviet Aeronautics (Engl. Transl.), vol. 61, no. 2, pp. 165–174].
45. Dlugach, M.I. and Koval'chuk, N.V., Investigation of Stressed State of Ribbed Cylindrical Shells with Rectangular Holes Using the Finite Element Method, *Prikladnaya Mekhanika*, 1974, vol. 10, no. 10, pp. 22–30 [Soviet Applied Mechanics (Engl. Transl.), vol. 10, no. 10, pp. 1056–1061].
46. Dlugach M.I. and Gavrilenko G.D., Grid Calculations of Ribbed Cylindrical Shells with Large Rectangular Holes, *Prikladnaya Mekhanika*, 1975, vol. 11, no. 12, pp. 22–30 [Soviet Applied Mechanics (Engl. Transl.), vol. 11, no. 12, pp. 1265–1271].
47. Bakulin, V.N. and Snesarev, S.L., Natural Vibrations of Cylindrical Shells with Rectangular Cutout, *Izv. Vuz. Av. Tekhnika*, 1988, vol. 31, no. 4, pp. 3–6 [Soviet Aeronautics (Engl.Transl.), vol. 31, no. 4, pp. 1–5].
48. Obraztsov, I.F., Savel'ev, L.M., and Khazanov, Kh.S., *Metod konechnykh elementov v zadachakh stroitel'noi mekhaniki letatel'nykh apparatov* (The Finite Element Method in Aircraft Structural Mechanics), Moscow: Vysshaya Shkola, 1985.
49. Postnov, V.A. and Kharkhum, I.Ya., *Metod konechnykh elementov v raschetakh sudovykh konstruktsii* (The Finite Element Method in Calculating Ship Structures), Leningrad: Sudostroenie, 1974.
50. Bakulin, V.N. and Rassokha, A.A., *Metod konechnykh elementov i gologràficheskaya interferometriya v mekhanike kompozitov* (The Finite Element Method and Holographic Interferometry in the Mechanics of Composites), Moscow: Mashinostroenie, 1987.
51. Zienkiewiez, O.C., *The Finite Element Method in Engineering Science*, London: McGraw-Hill, 1971.
52. Bakulin, V.N., Features of Building the Shell Models for a Layer-By-Layer Stress-Strain Analysis of Sandwich Shells with Rectangular Cuts, *Materialy 23-oi mezhdunarodnoi konferentsii po vychislitel'noi mekhanike i sovremennym prikladnym programmnym sistemam* (Proc. 23rd Int. Conf. on Computational Mechanics and Moderm Applied Software Systems (CMMASS'23), Moscow: MAI, 2023, pp. 171–173.
53. Bolotin, V.V. and Novichkov, Yu.N., *Mekhanika mnogosloinykh konstruktsii* (Mechanics of Multilayer Structures), Moscow: Mashinostroenie, 1980.
54. Bakulin, V.N., A Three-Dimensional Shell Model for Layer-by-Layer Study of the Stress and Strain State of Irregular Conical Sandwich Shells, *Doklady Physics*, 2023, vol. 68, no. 10, pp. 334–339.
55. Bakulin, V.N., The Influence of Elasticity of the Filler Material on Stresses in the Layers of Three-Layered Shells of Rotation under the Action of Local Loads, *Materials Physics and Mechanics*, 2016, vol. 26, no.1, pp. 33–37.
56. Obraztsov, I.F., Bulychev, L.A., and Vasil'ev, V.V., *Stroitel'naya mekhanika letatel'nykh apparatov* (Structural Mechanics of Aircraft), Moscow: Mashinostroenie, 1986.
57. Bakulin, V.N., An Efficient Model for Layer-by-Layer Analysis of Sandwich Irregular Cylindrical Shells of Revolution, *Doklady Physics*, 2018, vol. 63, no. 1, pp. 23–27.
58. Balabukh, L.I., Kolesnikov, K.S., Zarubin, V.S., et al., *Osnovy stroitel'noi mekhaniki raket* (Foundations of Structural Mechanics of Rockets), Moscow: Vysshaya Shkola, 1969.
59. Bakulin, V.N., Model for Analysis of the Stress-Strain State of Three-Layer Cylindrical Shells with Rectangular Cutouts, *Mechanics of Solids*, 2022, vol. 57, no. 1, pp. 102–110.
60. Novozhilov, V.V., *Teoriya tonkikh obolochek* (Theory of Thin Shells), Leningrad: Sudostroenie, 1962.

61. Bakulin, V.N., Effective Model of Load-Bearing Layers for Layer-by-Layer Analysis of the Stress-Strain State of Three-Layer Cylindrical Irregular Shells of Revolution, *Mechanics of Solids*, 2020, vol. 55, no. 3, pp. 557–565.
62. Bakulin, V.N., Block Finite-Element Approach to Building Refined Models of Layer-by-Layer Analysis of the Stress-Strain State of Three-Layer Irregular Shells, *Journal of Physics: Conference Series*, 2019, vol. 1392, Article no. 012065.
63. Strang, G. and Fix, G.J., *An Analysis of the Finite Element Method,* Englewood Cliffs, NJ: Prentice-Hall, 1973.
64. Obraztsov, I.F., Problems of Creating Effective Models and Methods for Calculating Complex Spatial Structures, in *Mekhanika i nauchno-tekhnicheskii progress* (Mechanics and Scientific–Technological Progress), in 4 vols, Moscow: Nauka, 1988, vol. 3, *Mekhanika deformiruemogo tela* (Solid Mechanics), pp. 7–22.
65. Bakulin, V.N., Krivtsov, V.S., and Rassokha, A.A., Algorithm for Obtaining Anisotropic Shell Finite-Element Stiffness Matrix, *Izv. Vuz. Av. Tekhnika*, 1983, vol. 26, no. 4, pp. 14–18 [Soviet Aeronautics (Engl. Transl.), vol. 26, no. 4, pp. 11–14].