ISSN 1068-7998, Russian Aeronautics (Iz.VUZ), 2018, Vol. 61, No. 3, pp. 371–378. © Allerton Press, Inc., 2018.
Original Russian Text © V.V. Erokhin, 2018, published in Izvestiya Vysshikh Uchebnykh Zavedenii, Aviatsionnaya Tekhnika, 2018, No. 3, pp. 49–56.

FLIGHT DYNAMICS AND CONTROL OF FLIGHT VEHICLES

Aircraft Trajectory Control at the Motion on the Predetermined Route Based on the Global Navigation Satellite System

V. V. Erokhin

Irkutsk Branch of Moscow State Technical University of Civil Aviation, ul. Kommunarov 3, Irkutsk, 664047 Russia
e-mail: Ww_erohin@mail.ru

Received June 9, 2017

Abstract—An algorithm to control the aircraft trajectory is proposed. This algorithm is based on the dynamic stochastic systems optimal control theory. The optimal control implementation is shown to reduce the deviation of the controlled trajectory from the predetermined one. The optimal control is based on estimating phase coordinates with the high accuracy by the global navigation satellite system.

DOI: 10.3103/S106879981803008X

Keywords: free routing, trajectory, navigation, optimal control, Kalman filter.

INTRODUCTION

According to the 2013–2028 Global Air Navigation Plan developed by the International Civil Aviation Organization (ICAO), one of the trends to enhance the air traffic management is the CNS/ATM concept (Communications, Navigation, Surveillance/Air Traffic Management) [1]. As per the stated development strategy, elaborating new principles for Air Traffic Control (ATC) is among the ICAO priorities. The up-to-date requirements for the ATC are the following ones: increasing the ATC capacity; operating along optimal trajectories; minimizing the deviation of aircraft from the planned trajectories; ensuring a high guaranteed level of air safety, etc. [1].

To implement these requirements, the free routing concept was elaborated [1], the primal goal of which is to optimize the aircraft trajectory, while moving in the allocated spatial-temporal region of the airspace, and to arrive exactly at the predetermined destination point (DP) in due time by providing high-precision determination of the trajectory parameters [1].

The concept puts forward hard requirements for the quality of the navigational-temporal provision of aircraft (including unmanned) interacting in the common airspace [1]. According to the existing concept, a high-precision determination of navigational-temporal parameters in any point of the near-Earth space is accomplished by using the GLONASS and GPS Global Navigation Satellite Systems (GNSS).

The algorithms to solve problems of programming four-dimension trajectories are provided in [2–4]. However, in actual practice, the flight trajectory is affected by various destabilizing factors. Therefore, implementing the program algorithms in actual practice faces numerous difficulties. The latter include lateral wind, effect of disturbances and noise on the precision of determining the aircraft coordinates, as well as the limitations related to the prohibited sectors, climb and landing profiles, etc. [1].

At the route navigation method, the trajectories are not strictly determined. As long as the aircraft deviates from the initial trajectory, the automatic control system does not return it into this trajectory, but only maintains the farther flight toward the chosen DP. The trajectory greatest bending, in that case, occurs at the lateral wind. In [2–4], the authors proposed some algorithms in the determined statement to optimize trajectories, taking into account the wind velocity and direction. Thus, it is necessary to solve the trajectory control problem minimizing the deviation from the predetermined trajectory, taking into account the effect of disturbances and noise on the aircraft coordinate determination precision. Developing an algorithm to control the trajectory in space is based on the methods of the optimal control theory [5, 6], the part of which is the optimal filtration theory [7].

The purpose of this study is to synthesize a control algorithm for an aircraft trajectory at a planned-rout flight, based on the secondary processing of the navigation information by the GNSS airborne equipment.

DYNAMIC MODEL FOR AN AIRCRAFT

To develop an algorithm to control the trajectory, we apply the model that imitates the behavior of the aircraft during a controlled flight under the conditions of a real atmosphere [8–11]. The aircraft is regarded as a mass point, the state of which features the following vector [11]:

$$\mathbf{X} = \left| x, y, h, V, \psi, m \right|^T, \tag{1}$$

where x, y are the aircraft coordinates in the horizontal plane; h is the geopotential height; V is the true air velocity; ψ is the aircraft course; m is the mass.

Dynamics of the state vector (1) is described through the following system [11]:

$$\frac{d\mathbf{X}}{dt} \equiv \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{h} \\ \dot{V} \\ \dot{\psi} \\ \dot{m} \end{pmatrix} = \begin{pmatrix} V\cos\psi\cos\gamma + w_x \\ V\sin\psi\cos\gamma + w_y \\ V\sin\gamma + w_h \\ (T-D)/m - g_0\sin\gamma \\ L\sin\varphi/(mV) \\ -\eta T \end{pmatrix}, \tag{2}$$

where γ is the aircraft flight-path angle; g_0 is the acceleration of gravity at the mean sea level; φ is the bank angle; η is the thrust specific fuel consumption (the corresponding calculation technique is in [12]); T is the engine thrust; D is the drag force; L is the aerodynamic lift; $w = \left| w_x, w_y, w_h \right|$ is the wind velocity vector.

In model (2), one assumes independence of the true air velocity V from the wind velocity w. The L and D forces are determined as functions of the true air velocity [12]:

$$L = \frac{C_L S_\rho}{2} V^2; \qquad D = \frac{C_D S_\rho}{2} V^2,$$

where C_L is the lift coefficient; C_D is the drag coefficient; S is the aircraft rated wing area; ρ is the air density.

The procedure and formulas to calculate C_L and C_D are provided in [12]. The limiting values of parameters for the dynamic model h, V, m, T, γ, φ are specified taking into account the limitations on the aircraft parameters. The control actions are the engine thrust T, the aircraft flight-path angle γ, and the bank angle φ.

PARAMETERS FOR ROUTE FLIGHT

The flight route, profile, and operations are chosen depending on the task with taking into account the particular meteorological and navigation situation. The flight plan is formed based on the current navigation information and routines as well as taking into account the typical flight profiles stored in the aircraft navigation processor [13]. In route flight operations, the GNSS airborne equipment provides air navigation, where the aircraft is guided at a DP [13]. Herewith, one calculates the next DP bearing that represents the aircraft course:

$$\psi = \arctan\left[\frac{y_i - y}{x_i - x}\cos\frac{x}{r}\right], \tag{3}$$

where x_i, y_i are the coordinates of the ith DP; x, y are the aircraft current coordinates; $\cos^2(x/r)$ is the correction to convert the arc length $(y_i - y)$ into the distance along the orthodromic parallel with the coordinate x; r is the Earth radius.

The aircraft coordinates are estimated as a result of navigation measurements from the GNSS. The GNSS airborne equipment performs the following functions: introducing the DP coordinates and calculating the navigation parameters [14, 15]. Unlike the classical approach, when implementing the free-routing flight concept, the trajectory current parameters (in particular, the actual course) are not determined by radio bearings (due to the DP radio visibility absence), but are calculated in the GNSS airborne equipment. Those calculations are based on the coordinates determined with the Δx and Δy errors. Obviously, the navigation determination errors will lead to errors in determining the deviation of the controlled flight trajectory from the predetermined one. Therefore, one of the control efficiency requirements is a high accuracy of estimating the trajectory motion parameters from the GNSS [15].

PROBLEM STATEMENT

The control problem is to transfer the controlled aircraft from an initial state (departure DP) to a final state (terminal DP) along a predetermined (planned, programmed) trajectory. In implementing the route flight, the set parameter is the specified phase coordinate vector $\mathbf{x}_P$. To maintain the aircraft flight along the predetermined trajectory, one should continuously or discretely control its motion. In the trajectory control problem, we assume that there is an object to control, a predetermined controlled dynamic system mapped in its state space by the controlled phase coordinate vector $\mathbf{x}_C$.

An important case (from the practical viewpoint) is the control problem, in which the aim is to guide the aircraft along the predetermined trajectory at a final instant t_N into the predetermined region of the space referred to as terminal set N (Fig. 1).

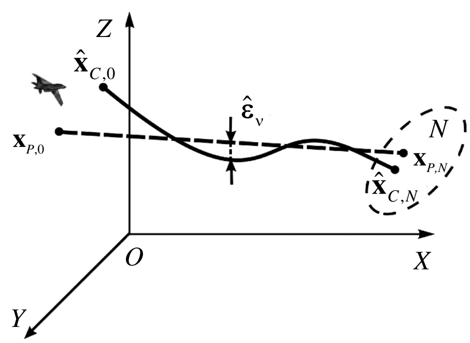


Fig. 1. Geometric interpretation of the aircraft flight following the predetermined route.

Due to disturbances and noise, the precise implementation of this motion is, as a rule, impossible. Therefore, the actual motion differs from the predetermined (programmed) one. Let us introduce the deviation of the controlled trajectory from the predetermined one in the form of $\varepsilon_\nu = \mathbf{x}_{P,\nu} - \mathbf{x}_{C,\nu}$. The synthesis purpose in the control problem is generating such controls $\mathbf{u}_\nu$, for which the controlled trajectory $\mathbf{x}_{C,\nu}$ reproduces (traces) the predetermined trajectory $\mathbf{x}_{P,\nu}$ in the best (optimal) way.

The control optimality is understood in sense of minimizing this or that quality criterion. In practice, the flight trajectory optimization criteria are stated like minimizing the deviation of the controlled flight trajectory from the predetermined one. In solving practical problems of the aircraft trajectory control, the losses depend not on absolute values but on their difference or error. Herewith, the quality index is the control error generalized quadratic functional [5, 16–19] that, with reference to the addressed case, we present like

$$J = \min_{\mathbf{u}_1^{N-1}\in\mathbf{U}} M\left[\sum_{\nu=1}^{N}\left\{\left(\mathbf{x}_{P,\nu}-\mathbf{x}_{C,\nu}\right)^T\mathbf{Q}_\nu\left(\mathbf{x}_{P,\nu}-\mathbf{x}_{C,\nu}\right)+\mathbf{u}_\nu^T\mathbf{P}_\nu\mathbf{u}_\nu\right\}\right] = \min_{\mathbf{u}_1^{N-1}\in\mathbf{U}} M\left[\sum_{\nu=1}^{N}c_\nu\left(\mathbf{x}_{P,\nu},\mathbf{x}_{C,\nu},\mathbf{u}_\nu\right)\right], \tag{4}$$

where $\nu=\overline{0,N-1}$ is the time coefficient; N is the number of counts, $\mathbf{Q}_\nu$ is the non-negatively determined matrix of penalties for errors of the state vector parameters (the matrix characterizes the relevance degree of this or that trajectory component); $\mathbf{P}_\nu$ is the control cost account matrix; $\mathbf{u}_\nu$ is the vector of control effects; $\mathbf{u}_\nu\in\mathbf{U}$ are sets of admissible control values; $c_\nu\left(\mathbf{x}_{P,\nu},\mathbf{x}_{C,\nu},\mathbf{u}_\nu\right)$ is the function of current losses that increases with a growth in the deviation of the controlled trajectory from the predetermined one, and with a growth in the control costs.

To solve the optimal control problem in real systems, one, most often, uses the following local optimization criterion:

$$J_\nu = M\left\{c_\nu\left(\mathbf{x}_{P,\nu},\mathbf{x}_{C,\nu},\mathbf{u}_\nu\right)\right\}. \tag{5}$$

Thus, we need to synthesize an optimal algorithm for controlling the aircraft trajectory in flight along the predetermined route in accordance with the selected criterion.

ALGORITHM SYNTHESIS

In order to formalize the subsequent calculations to a greater extent with using the standard set of the optimal control theory, one introduces the augmented state vector $\mathbf{x}=(\mathbf{x}_P,\mathbf{x}_C)^T$, for which a difference equation [15] should be written as

$$\mathbf{x}_{\nu+1} = \mathbf{\Phi}_{\nu/\nu+1}\mathbf{x}_\nu + \mathbf{b}_\nu\mathbf{u}_\nu + \mathbf{G}_{\nu/\nu+1}\mathbf{n}_{x,\nu}, \tag{6}$$

where $\mathbf{\Phi}_\nu$ is the system dynamics matrix; $\mathbf{G}_\nu$ is the matrix of limitations on the system noise; $\mathbf{n}_{x,\nu}$ is the discrete white Gaussian noise vector with a zero mathematical expectation and with the dispersion matrix $\mathbf{\Psi}$; $\mathbf{b}_\nu$ is the vector of the system control effect coefficients;

$$\mathbf{\Phi}=\begin{vmatrix}\mathbf{\Phi}_P & 0\\ 0 & \mathbf{\Phi}_C\end{vmatrix},\quad \mathbf{G}=\begin{vmatrix}0 & 0\\ 0 & \mathbf{G}_C\end{vmatrix},\quad \mathbf{b}=\begin{vmatrix}0\\ \mathbf{b}_C\end{vmatrix},\quad \mathbf{n}=\begin{vmatrix}0\\ \mathbf{n}_C\end{vmatrix}.$$

The quality index is written like [16]

$$J=\min_{\mathbf{U}_1^{N-1}} M\left[\sum_{\nu=1}^{N}\mathbf{x}_\nu^T\tilde{\mathbf{Q}}_\nu\mathbf{x}_\nu+\mathbf{u}_\nu^T\mathbf{P}_\nu\mathbf{u}_\nu\right]=\min_{\mathbf{U}_1^{N-1}} M\left[\sum_{\nu=1}^{N}c_\nu(\mathbf{x}_\nu,\mathbf{u}_\nu)\right],$$

where $\tilde{\mathbf{Q}}_\nu=\begin{vmatrix}\mathbf{Q}_\nu & -\mathbf{Q}_\nu\\ -\mathbf{Q}_\nu & \mathbf{Q}_\nu\end{vmatrix}$.

When synthesizing the optimal control algorithm, we consider that the state vector is estimated based on the observation processing in the GNSS airborne equipment. The GNSS observation model represents the measurement of pseudo-ranges to navigation satellites (NSs). Therefore, one should introduce the parameters characterizing the GNSS airborne equipment functioning into the state vector: the timescale bias $\delta\tau_{GNSS}$ and the time standard and frequency instability δf_{GNSS}. For the m-dimension observation vector $\xi_\nu=\left[\xi_{1,\nu},...\xi_{m,\nu}\right]^T$, we present the measurement equation in the form of [15]:

$$\xi_\nu = \mathbf{H}_\nu \mathbf{x}_\nu + \mathbf{n}_\nu , \tag{7}$$

where $\mathbf{n}_\nu$ is the *m*-dimensional vector of discrete white Gauss noise with zero mathematical expectations and with a dispersion matrix $\mathbf{V}$ of the $(m \times m)$ dimensionality, assuming that the noise $\mathbf{n}_{x,\nu}$ and $\mathbf{n}_\nu$ are independent; the matrix of direction cosines for the aircraft–NS line-of-sight $\mathbf{H}_\nu$ has the following form:

$$\mathbf{H}_\nu = \begin{bmatrix} -\cos(\alpha_1) & -\cos(\beta_1) & -\cos(\gamma_1) & 1 \\ -\cos(\alpha_2) & -\cos(\beta_2) & -\cos(\gamma_2) & 1 \\ \cdots & \cdots & \cdots & \cdots \\ -\cos(\alpha_m) & -\cos(\beta_m) & -\cos(\gamma_m) & 1 \end{bmatrix},$$

where $\cos(\alpha_i) = \dfrac{x_{i,\nu} - x_\nu}{d_{i,\nu}}$, $\cos(\beta_i) = \dfrac{y_{i,\nu} - y_\nu}{d_{i,\nu}}$, $\cos(\gamma_i) = \dfrac{z_{i,\nu} - z_\nu}{d_{i,\nu}}$ are the direction cosines of the aircraft–the *i*th NS line-of-sight; x_i, y_i, z_i are the rectangular geocentric coordinates of the *i*th NS; x, y, z are the aircraft rectangular geocentric coordinates; $d_{i,\nu} = \sqrt{\left(x_{i,\nu} - x_\nu\right)^2 + \left(y_{i,\nu} - y_\nu\right)^2 + \left(z_{i,\nu} - z_\nu\right)^2}$ is the range to the *i*th NS.

Since the control should meet the physical feasibility requirement at each instant t_ν, then $\mathbf{u}_\nu$ may depend only on the observations available at a given instant, i.e. $\mathbf{u}_\nu = f\left(\xi_{\nu-1}\right)$. One should determine the control law $\mathbf{u}_\nu = \mathbf{u}_\nu\left(\xi_1^{\nu-1}\right)$ optimal by the local quality criterion:

$$\mathbf{u}_\nu = \underset{\mathbf{u}_\nu \in \mathbf{U}}{\arg\min}\, J_\nu = \underset{\mathbf{u}_\nu \in \mathbf{U}}{\arg\min} \left\{ \int_{\mathbf{x}} c_\nu\left(\mathbf{x}_\nu, \mathbf{u}_\nu\left(\xi_1^{\nu-1}\right)\right) p\left(\mathbf{x}_\nu \middle| \xi_1^{\nu-1}\right) d\mathbf{x}_\nu \right\} = \underset{\mathbf{u}_\nu \in \mathbf{U}}{\arg\min}\, M\left\{ c_\nu\left(\mathbf{x}_\nu, \mathbf{u}_\nu\right)\Big|_{\xi_1^{\nu-1}} \right\}. \tag{8}$$

For nonlinear systems, the separation theorem (statistical equivalence theorem) is approximately valid. According to this theorem one can separately synthesize the system of the object parameter estimate and the optimal control system [5, 17]. The basis for that is the fact that, when synthesizing algorithms for optimal estimate in the aircraft radio-electronic suites, one achieves good estimate convergence to the true phase coordinates [16]. As applied to Eqs. (6) and (7), the state vector extrapolated value probability density that is a part of Eq. (8) is normal at each step: $p\left(\mathbf{x}_\nu \middle| \xi_1^{\nu-1}\right) = N\left\{\tilde{\mathbf{x}}_\nu, \tilde{\mathbf{R}}_\nu\right\}$. Parameters of this probability density are determined based on the Kalman filter [7, 15, 16]:

$$\tilde{\mathbf{x}}_\nu = \Phi_{\nu/\nu-1}\hat{\mathbf{x}}_{\nu-1} + \mathbf{B}_\nu \mathbf{u}_\nu, \tag{9}$$

$$\hat{\mathbf{x}}_\nu = \tilde{\mathbf{x}}_\nu + \mathbf{K}_\nu\left[\xi_\nu - \mathbf{H}_\nu \tilde{\mathbf{x}}_\nu\right], \tag{10}$$

$$\mathbf{K}_\nu = \mathbf{R}_\nu \mathbf{H}_\nu^T \mathbf{V}_\nu^{-1}, \tag{11}$$

$$\tilde{\mathbf{R}}_\nu = \Phi_{\nu/\nu-1}\mathbf{R}_\nu \Phi_{\nu/\nu-1}^T + \mathbf{G}_{\nu/\nu-1}\mathbf{Q}_\nu \mathbf{G}_{\nu/\nu-1}^T, \qquad \mathbf{R}_\nu^{-1} = \tilde{\mathbf{R}}_\nu^{-1} + \mathbf{H}_\nu^T \mathbf{V}_\nu^{-1}\mathbf{H}_\nu. \tag{12}$$

By using the above ratios, we obtain [16–19]:

$$J_\nu = M\left\{c_\nu\left(\mathbf{x}_\nu, \mathbf{u}_\nu\right)\middle| \xi_1^{\nu-1}\right\} = \int_x c_\nu\left(\mathbf{x}_\nu, \mathbf{u}_\nu\left(\xi_1^{\nu-1}\right)\right) p\left(\mathbf{x}_\nu \middle| \xi_1^{\nu-1}\right) d\mathbf{x}_\nu = \int_x c_\nu\left(\mathbf{x}_\nu, \mathbf{u}_\nu\left(\xi_1^{\nu-1}\right)\right) N\left(\tilde{\mathbf{x}}_\nu \middle| \tilde{\mathbf{R}}_\nu\right) d\mathbf{x}_{C,\nu}$$

$$= \tilde{\mathbf{x}}_\nu^T \mathbf{Q}_\nu \tilde{\mathbf{x}}_\nu + \mathrm{tr}\left\{\tilde{\mathbf{Q}}_\nu \tilde{\mathbf{R}}_\nu\right\} + \mathbf{u}_\nu^T \mathbf{P}_\nu \mathbf{u}_\nu = \left(\Phi_{\nu/\nu-1}\tilde{\mathbf{x}}_{\nu-1} + \mathbf{b}_\nu \mathbf{u}_\nu\right)^T \tilde{\mathbf{Q}}_\nu \left(\Phi_{\nu/\nu-1}\tilde{\mathbf{x}}_{\nu-1} + \mathbf{b}_\nu \mathbf{u}_\nu\right) + \mathrm{tr}\left\{\tilde{\mathbf{Q}}_\nu \tilde{\mathbf{R}}_\nu\right\} + \mathbf{u}_\nu^T \mathbf{P}_\nu \mathbf{u}_\nu,$$

where “tr” is a mathematical operation to find a matrix trace.

The optimal control minimizing this criterion is found as follows. Since $\mathrm{tr}\left\{\tilde{\mathbf{Q}}_\nu \tilde{\mathbf{R}}_\nu\right\} = \mathrm{const}\left(\mathbf{u}_\nu\right)$ does not depend on control, then [16–19]:

$$\frac{\partial J_\nu}{\mathbf{u}_\nu} = 2\mathbf{b}_\nu^T \tilde{\mathbf{Q}}_\nu \left(\Phi_{\nu/\nu-1}\hat{\mathbf{x}}_{\nu-1} + \mathbf{b}_\nu \mathbf{u}_\nu\right) + 2\mathbf{P}_\nu \mathbf{u}_\nu = 0. \tag{13}$$

The solution of Eq. (13) represents an algorithm for locally-optimal control, and looks like [16–19]:

$$\mathbf{u}_\nu = \left(\mathbf{b}_\nu^T \tilde{\mathbf{Q}}_\nu \mathbf{b}_\nu + \mathbf{P}_\nu\right)^{-1} \mathbf{b}_\nu^T \tilde{\mathbf{Q}}_\nu \left[\Phi_{P,\nu/\nu-1} \hat{\mathbf{x}}_{P,\nu-1} - \Phi_{C,\nu/\nu-1} \hat{\mathbf{x}}_{C,\nu-1}\right] = \mathbf{L}_\nu \left[\hat{\mathbf{x}}_{P,\nu-1} - \hat{\mathbf{x}}_{C,\nu-1}\right], \tag{14}$$

in which the matrix gain coefficient $\mathbf{L}_\nu$ is determined by the expression

$$\mathbf{L}_\nu = \left(\mathbf{b}_\nu^T \tilde{\mathbf{Q}}_\nu \mathbf{b}_\nu + \mathbf{P}_\nu\right)^{-1} \mathbf{b}_\nu^T \tilde{\mathbf{Q}}_\nu \Phi_{\nu/\nu-1}.$$

To obtain the optimal control algorithm, the reference ratios provided in [16–19] were used.

In such problems of trajectory control, there is no estimate contour $\mathbf{x}_P$, because this value is known. Herewith, there is a system of tracking $\mathbf{x}_P$ and system of forming the controlled trajectory $\hat{\mathbf{x}}_C$. Then, we should rewrite Eq. (14) like

$$\mathbf{u}_\nu = -\left(\mathbf{b}_\nu^T \mathbf{Q}_\nu \mathbf{b}_\nu + \mathbf{P}_\nu\right)^{-1} \mathbf{b}_\nu^T \mathbf{Q}_\nu \Phi_{\nu/\nu-1} \hat{\varepsilon}_{\nu-1} = -\mathbf{L}_\nu \left(\mathbf{x}_{P,\nu-1} - \hat{\mathbf{x}}_{C,\nu-1}\right), \tag{15}$$

where $\mathbf{x}_P$ is the determined function of time, and the estimate $\hat{\mathbf{x}}_C$ is obtained based on algorithm (9)–(12).

With reference to the trajectory control at a route flight, the vector of the specified phase coordinates will look like $\mathbf{x}_P = \psi_P$, the vector of the controlled phase coordinates being $\mathbf{x}_C = \psi_C$. Let us define Eq. (15) and present it as

$$u_\nu = l_\nu \left(\psi_{P,\nu-1} - \hat{\psi}_{C,\nu-1}\right).$$

To produce a control effect, one can optimally control the dynamic object affected by random disturbances only instantly, by using both a priori and current information provided by the measurement system. The optimal system represents a structure with negative feedbacks over all the controlled variable states (Fig. 2), which testifies to its high stability. The control signal is determined not by the system state but by its current control error $\varepsilon_\nu = \mathbf{x}_{P,\nu} - \mathbf{x}_{C,\nu}$.

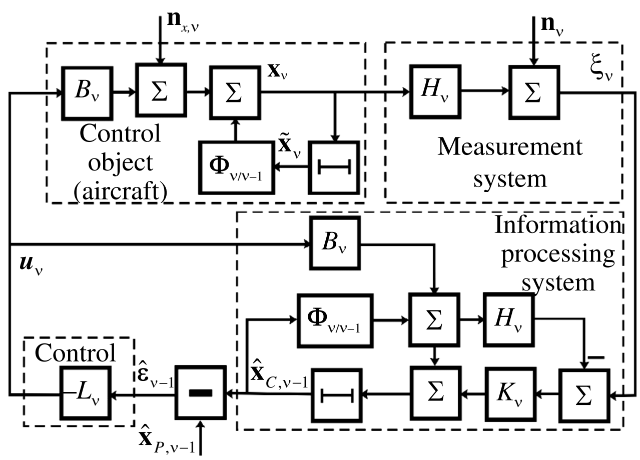
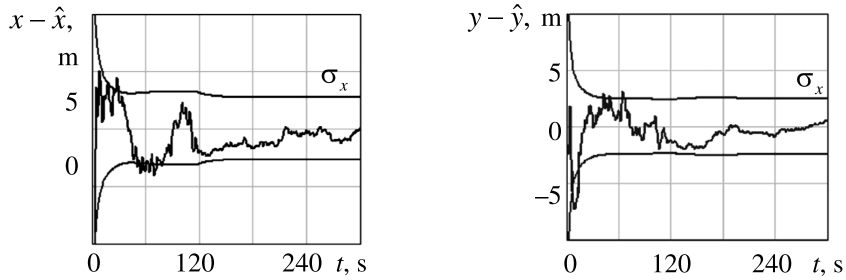


Fig. 2. Structure of the trajectory control stochastic system with a state estimate.

MODELING AND DISCUSSION

The trajectory motion parameters and the precision characteristics were studied based on the models for motion of the aircraft and the NS within the GNSS orbital group. The situation of solving the navigation problem for the aircraft operating along Route A937 westward at the flight level H = 10600 m was modeled. During the flight, the coordinates were determined as a result of navigational and temporal definitions by four GNSS NSs. As the model input data, we used the parameters characterizing the GNSS functioning, provided in [16].

To investigate the characteristics for the synthesized algorithms, we address the solution of the navigation problem for the case of the trajectory course control. The control of the trajectory turn angle was implemented relative to the initial direction of the velocity vector. The main limitation at the course control is the limitation on the course angle change rate at a time unit. Figure 3 presents the implementations of errors in estimating the aircraft coordinates $\varepsilon_x = x - \hat{x}$ and $\varepsilon_y = y - \hat{y}$, respectively, and plots for the mean-square deviation of the estimates σ_x and σ_y. The analysis of the results shows a high precision in determining the aircraft coordinates ($\sigma \approx 2$–3 m) due to implementing the algorithm for secondary processing of the navigation information in the GNSS airborne equipment.



(a) (b)

Fig. 3. Error in estimating the coordinates x (a) and y (b) along the controlled flight trajectory.

Figure 4 presents the plots for the aircraft course dynamics at a route flight: curve *1* is the aircraft predetermined course ψ_P, curve *2* is the aircraft course when implementing the control algorithm, curve *3* is the estimated value of the aircraft course $\hat{\psi}_C$ obtained based on the Kalman filter (9)–(12) and expression (3). The analysis of the results shows that, for the modeled situation, the error in determining the aircraft course is $\varepsilon_\psi \approx 0.018$ deg.

Figure 5 provides the plots for the modeled aircraft flight trajectories: curve *1* is the predetermined flight route; curve *2* is the controlled trajectory; curve *3* is a flight trajectory by DPs. From the presented plots, one can see that the controlled flight trajectory enables to reduce the flight distance and time as compared with the classical route flight by DPs. For the modeled situation, applying the flight free routing reduced the distance by approximately 50 km and the time by 3.3 minutes.

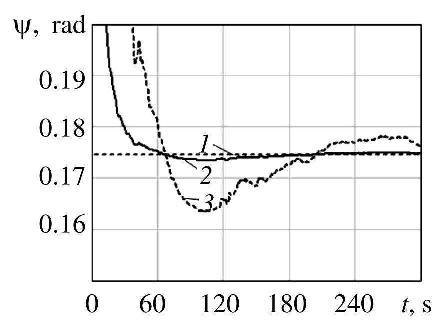


Fig. 4. Modeled values of the aircraft course.

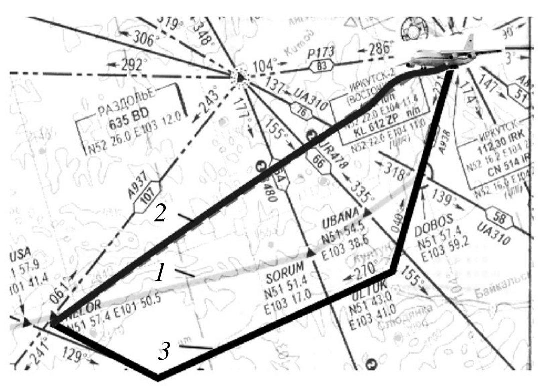


Fig. 5. Modeled aircraft flight trajectories.

Thus, we synthesized the optimal algorithm enabling to reduce the deviation of the controlled trajectory from the predetermined one at a high precision of determining the trajectory motion parameters from the GNSS. The free routing concept based on determining the trajectory motion parameters in the GNSS airborne equipment enables to reduce the flight distance and time as compared with the classical route flight by radio navigation points.

REFERENCES

1. The 2013-2028 Global Air Navigation Plan. Doc 9750-AN/963, Canada, Montréal: ICAO, 2013.
2. Maolaaisha, A., Free-Flight Trajectory Optimization by Mixed Integer Programming, *Master of Science Thesis*, Hamburg, Helmut-Schmidt-Universität / Universität der Bundeswehr Hamburg, 2015.
3. Rubén, Antón Guijarro, Commercial Aircraft Trajectory Optimization Using Optimal Control, *Bachelor Thesis*, Madrid, Universidad Carlos III de Madrid, 2015.
4. Toratani, D., Study on Simultaneous Optimization Method for Trajectory and Sequence of Air Traffic Management, *Doctoral Thesis,* Yokohama: Yokohama National University, 2016.
5. Sage, A.P. and White, C.C., *Optimum Systems Control*, London: Prentice-Hall, 1977.
6. Stratonovich, R.L., *Uslovnye markovskie protsessy i ikh primenenie k teorii optimal'nogo upravleniya* (Conventional Markov Processes and Their Application to the Optimal Control Theory), Moscow: MGU, 1966.
7. Tikhonov, V.I. and Kharisov, V.N., *Statisticheskii analiz i sintez radoitekhnicheskikh ustroistv i system* (Statistical Analysis and Synthesis of Radio Devices and Systems), Moscow: Radio i Svyaz', 1991.
8. Averkiev, N.F., Vlasov, S.A., Salov, V.V., and Kiselev, V.V., Route Optimization of the Aircraft Flight, *Izv. Vuz. Av. Tekh*., 2016, vol. 59, no. 4, pp. 33–37 [Russian Aeronautics (Engl. Transl.), vol. 59, no. 4, pp. 474–479].
9. Kudryavtsev, D.Yu., Aminev, D.A., and Sviridov, A.S., Computationally Effective Mathematical Model of Airplane Flight, *Izv. Vuz. Av. Tekh*., 2016, vol. 59, no. 3, pp. 45–51 [Russian Aeronautics (Engl. Transl.), vol. 59, no. 3, pp. 344–350].
10. Andreev, K.V., Khoroshen'kikh, S.N., and Moiseev, G.V., Flight Path Optimization for an Electronic Intelligence Unmanned Aerial Vehicle, *Izv. Vuz. Av. Tekh*., 2015, vol. 58, no. 1, pp. 14–18 [Russian Aeronautics (Engl. Transl.), vol. 58, no. 1, pp. 15–20].
11. Kiselev, V.Yu. and Monakov, A.A., Aircraft Trajectory Prediction in Air Traffic Control Systems, *Informatsionno-Upravlyashchie Sistemy*, 2015, no. 4(77), pp. 33–40.
12. Nuic, A., User Manual for the Base of Aircraft Data (BADA). Revision 3.12, *Eurocontrol Experimental Centre*, EEC Note no. 10/04, 2014.
13. Yarlykov, M.S., *Statisticheskaya teoriya radionavigatsii* (Statistical Radio Navigation Theory), Moscow: Radio i Svyaz', 1985.
14. Solov'ev, Yu.A., *Sistemy sputnikovoi navigatsii* (Airborne Satellite Navigation Equipment), Moscow: Eko-Trendz, 2000.
15. *GLONASS. Printsipy postroeniya i funktsionirovaniya* (GLONASS. Construction and Function Principles), Perov, A.I. and Kharisov, V.N., Eds., Moscow: Radiotekhnika, 2010.
16. Yarlykov, M.S., Bogachev, A.S., Merkulov, V.I., and Drogalin, V.V., *Radioelektronnye kompleksy navigatsii, pritselivaniya i upravleniya vooruzheniem letatel'nykh apparatov* (Radioelectronic Suites for Navigation, Targeting and Control of Aircraft), Moscow: Radiotekhnika, 2012, vol. 1. Fundamental theory.
17. Bukov, V.N., *Adaptivnye prognoziruyushchie sistemy upravleniya poletom* (Adaptive Predicting Flight Control Systems), Moscow: Nauka, 1987.
18. Degtyarev, G.L. and Sirazetdinov, T.K., *Teoreticheskie osnovy optimal'nogo upravleniya uprugimi kosmicheskimi apparatami* (Fundamental Theory to Optimal Control of Elastic Spacecraft), Moscow: Mashinostroenie, 1986.
19. Degtyarev, G.L., and Rizaev, I.S., *Sintez lokal'no-optimal'nykh algoritmov upravleniya letatel'nymi apparatami* (Synthesis of Local-Optimal Algorithms for Flight Vehicle Control), Moscow: Mashinostroenie, 1991.