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STRUCTURAL MECHANICS AND STRENGTH OF FLIGHT VEHICLES

Dynamic Stability of a Cylindrical Shell under Alternating Axial External Pressure

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Abstract—The dynamic stability of a cylindrical shell reinforced by ribs under the action of external pressure and axial compressive force is investigated. The dependences of the main instability region for three laws of pressure distribution are investigated.

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Keywords: dynamic stability, cylindrical shell, reinforcing elements, alternating axial external pressure, instability regions.

Combustion chamber is one of the most loaded elements of aircraft equipped with ramjets. Combustion chamber consists of two composite cylindrical shells. These shells are coaxially located and attached by means of pylons. A main frame with a set of cross holes is located on the surface of the inner shell. Hot air enters through these holes.

This paper is devoted to the study of the dynamic stability of the inner shell taking into account possible distribution of the external pressure in the gas path.

The problem of dynamic stability of thin-walled structures associated with the development of aeronautics and astronautics was solved in [1–4]. The stress strain state, stability and dynamic stability of composite shells were also investigated in [5–8]. Analytical solutions of shell deformation mechanics problems were considered in [9, 10]. The through-thickness stress distribution in the adhesive joint for the multilayer composite was studied in [11].

At present, in connection with development of essentially new types of aircraft and implementation of composite materials, we are confronted with a new class of relevant tasks, which are connected with supporting elements discretely located in structures [12–15].

We consider a shell reinforced by a set of annular ribs and loaded by the external pressure. This pressure consists of common constant, axially alternating, and time variable components (Fig. 1). The ends of the shell are assumed simply supported and loaded by the axial compressive force. Only radial components of the contact interaction between the shell and ribs are considered. Axial and tangential components of the inertial forces as well as structural damping in the motion equations are not taken into account.

Let us introduce the dimensionless cylindrical coordinates, all linear dimensions in which relate to the radius of the middle shell surface. Then the equation of shell motion can be represented in the following form [4]

$$\begin{gathered}\left\{a_3\nabla^8 + a_1 a_4 \frac{\partial^2}{\partial\alpha^4} + \frac{B}{R}\nabla^4\left[P_0 + P_{j\alpha}(\alpha) + P_1\cos\omega t\right]\left(\frac{\partial^2}{\partial\beta^2}+1\right)\right. \\ \left. + a_5\nabla^4\frac{\partial^2}{\partial\alpha^2} + a_7\nabla^4\frac{\partial^2}{\partial t^2}\right\}w + \nabla^4\sum_{i=1}^{M}\left[a_{8i}\left(\frac{\partial^2}{\partial\beta^2}+1\right)^2 + a_{9i}\frac{\partial^2}{\partial t^2}\right]w_i\delta(\alpha-\alpha_i) = 0,\end{gathered} \tag{1}$$

where

$$\nabla^8 = a_1 \frac{\partial^8}{\partial\alpha^8} + \left[a_4 + a_6\left(2a_1 - \nu_\beta\right)\right]\frac{\partial^8}{\partial\alpha^6\partial\beta^2} + \left\{a_1a_4 + 2\left[a_6\left(a_4 - a_6\nu_\beta\right) + a_1a_4\right]\left(\frac{\partial^2}{\partial\beta^2}+1\right)\frac{\partial^2}{\partial\beta^2}\right\}\frac{\partial^4}{\partial\alpha^4}$$

$$+a_4\left[\left(a_4 - a_6\nu_\beta\right)\left(\frac{\partial^2}{\partial\beta^2}+1\right)^2 + 2a_1a_6\frac{\partial^4}{\partial\beta^4}\right]\frac{\partial^4}{\partial\alpha^2\partial\beta^2} + a_1a_4^2\left(\frac{\partial^2}{\partial\beta^2}+1\right)^2\frac{\partial^4}{\partial\beta^4};$$

$$\nabla^4 = a_1\frac{\partial^4}{\partial\alpha^4} + \left(a_4 - a_6\nu_\beta\right)\frac{\partial^4}{\partial\alpha^2\partial\beta^2} + a_1a_4\frac{\partial^4}{\partial\beta^4};\; a_1 = \frac{G_{\alpha\beta}\left(1-\nu_\alpha\nu_\beta\right)}{E_\alpha h};\; a_2 = a_1 + \nu_\beta;\; a_3 = \frac{h^2}{12R^2};$$

$$a_4 = \frac{E_\alpha}{E_\beta};\; a_5 = \frac{BN_\alpha}{2\pi R^3};\; a_6 = a_1 + a_2;\; a_7 = B\rho_0 h;\; a_{8i} = \frac{BE_iI_i}{R^5};\; a_{9i} = \frac{BF_i\rho_i}{R};\; B = \frac{R^2\left(1-\nu_\alpha\nu_\beta\right)}{E_\alpha h}.$$

Here α, β are the dimensionless coordinates in the longitudinal and circumferential directions; w is the normal shell displacement; R, h are the radius and thickness of the shell; $E_\alpha, E_\beta, G_{\alpha\beta}$ are the elastic moduli in axial and circumferential directions and the shear modulus; ν_α, ν_β are Poisson's ratios; I_i, F_i are the moment of inertia and the area of the ith rib; E_i is the modulus of ith rib; ρ_0, ρ_i are the densities of the shell and rib materials; M is the number of ribs; N_α is the initial axial force; $P_0, P_{j\alpha}(\alpha), P_1$ are the common constant, variable, the amplitude ripple of external pressure, respectively; ω is the pulsation frequency; $\delta(\alpha)$ is the delta function.

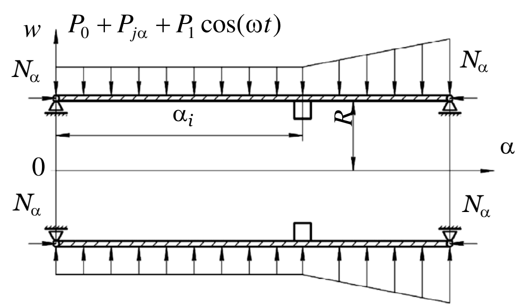

Fig. 1.

Depending on shell structural features, gas density or flow rate, the following pressure distribution along the variable axis can be taken, namely, constant $P_{1\alpha}$; linear $P_{2\alpha}$; quadratic $P_{3\alpha}$ (Fig. 2).

These relationships can be represented as

$$P_{1\alpha} = P_2\left[\sigma_0\left(\alpha - \alpha_1\right) - \sigma_0\left(\alpha - \alpha_0\right)\right];$$

$$P_{2\alpha} = \frac{P_2\left(\alpha - \alpha_1\right)}{\left(\alpha_0 - \alpha_1\right)}; \tag{2}$$

$$P_{3\alpha} = \frac{P_2\left(\alpha^2 - \alpha_1^{\,2}\right)}{\left(\alpha_0^{\,2} - \alpha_1^{\,2}\right)},$$

where $\alpha_0 = L/R$, L is the length of the shell; α_1 is the coordinate of the gas inlet; P_2 is the pressure on the rear shell of the section; $\sigma_0(\alpha)$ is the unit function equal to unity at $\alpha > 0$ and zero at $\alpha < 0$.

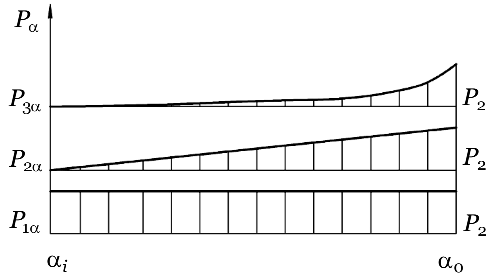

Fig. 2.

The solution of Eq. (1) will be sought as follows

$$w = \cos n\beta \sum_{m=1}^{\infty} f_m(t) \sin\gamma\alpha\,, \tag{3}$$

where $\gamma = m\pi/\alpha_0$; n is the number of waves in the circumferential direction; m is the number of half-waves in the axial direction; $f_m(t)$ is the unknown function of time (hereinafter, the argument t is omitted).

We substitute expression (3) into Eq. (1) and apply the Bubnov–Galerkin procedure. Each term of Eq. (1) is multiplied by $\sin\xi\alpha_0$ (where $\xi = k\pi/\alpha_0$) and integrated from 0 to α_0. As a result, we obtain an infinite system of differential equations of the non-uniform Mathieu–Hill type:

$$\frac{d^2 f_k}{dt^2} + b_1\cos\omega t f_k + b_2 f_k + \frac{2B\left(1-n^2\right)}{\alpha_0 a_7 R}\sum_{m=1}^{\infty} F_{mk}^{(j)} f_m + \sum_{m=1}^{\infty}\sum_{i=1}^{M}\theta_{mk}^{(i)}\left(b_{4i}\frac{d^2 f_m}{dt^2} + b_{3i} f_m\right) = 0, \tag{4}$$
$$k = 1,2,3\ldots,\; j = 1,2,3,$$

where

$$b_1 = BP_1\left(1-n^2\right)/a_7 R;\; b_2 = \frac{1}{a_7}\left[\frac{a_3\nabla_k^8}{\nabla_k^4} + \frac{a_1 a_4\xi^4}{\nabla_k^4} - a_5\xi^2 + \frac{BP_0\left(1-n^2\right)}{R}\right];$$

$$b_{3i} = 2a_{8i}\left(1-n^2\right)^2/\alpha_0 a_7;\; b_{4i} = 2a_{9i}/\alpha_0 a_7;\; \nabla_k^4 = a_1\xi^4 + \left(a_4 - a_6\nu_\beta\right)\xi^2 n^2 + a_1 a_4 n^4;$$

$$\nabla_k^8 = a_1\xi^8 + \left[a_4 + a_6\left(2a_1 - \nu_\beta\right)\right]\xi^6 n^2 + \left\{a_1 a_4 + 2\left[a_6\left(a_4 - a_6\nu_\beta\right) + a_1 a_4\right]\left(n^2-1\right)n^2\right\}\xi^4$$

$$+a_4\left[\left(a_4 - a_6\nu_\beta\right)\left(n^2-1\right)^2 + 2a_1 a_6 n^4\right]\xi^2 n^2 + a_1 a_4^2\left(n^2-1\right)^2 n^4;\; \theta_{mk}^{(i)} = \sin\gamma\alpha_i\sin\xi\alpha_i;\; F_{mk}^{(1)} = \begin{Bmatrix}\psi_{10} \text{ at } m = k\\ \psi_{11} \text{ at } m \neq k\end{Bmatrix};$$

$$\psi_{10} = \left(\alpha_0 - \alpha_1\right)/2 + \left(\sin 2\xi\alpha_1\right)/4\xi;\; \psi_{11} = \frac{1}{\left(\gamma^2 - \xi^2\right)}\left(\xi\sin\gamma\alpha_1\cos\xi\alpha_1 - \gamma\cos\gamma\alpha_1\sin\xi\alpha_1\right);$$

$$F_{mk}^{(2)} = \begin{Bmatrix}\psi_{20} \text{ at } m = k\\ \psi_{21} \text{ at } m \neq k\end{Bmatrix};\; \psi_{20} = \frac{1}{\alpha_0 - \alpha_1}\left[\frac{\alpha_0^2 - \alpha_1^2}{4} - \frac{1}{2}\left(\frac{1}{4\xi^2} - \frac{\alpha_1}{2\xi}\sin 2\xi\alpha_1 - \frac{1}{4\xi^2}\cos 2\xi\alpha_1\right) - \alpha_1\psi_{10}\right];$$

$$\psi_{21} = \frac{1}{\alpha_0 - \alpha_1}\left\{\frac{(-1)^{m+k}2\gamma\xi}{\left(\gamma^2 - \xi^2\right)^2} + \frac{1}{2\left(\gamma+\xi\right)^2}\left[\cos\left(\gamma+\xi\right)\alpha_1 + \left(\gamma+\xi\right)\alpha_1\sin\left(\gamma+\xi\right)\alpha_1\right]\right.$$

$$\left.-\frac{1}{2\left(\gamma-\xi\right)^2}\left[\cos\left(\gamma-\xi\right)\alpha_1 + \left(\gamma-\xi\right)\alpha_1\sin\left(\gamma-\xi\right)\alpha_1\right] - \alpha_1\psi_{11}\right\};\; F_{mk}^{(3)} = \begin{Bmatrix}\psi_{30} \text{ at } m = k\\ \psi_{31} \text{ at } m \neq k\end{Bmatrix};$$

$$\psi_{30}=\frac{1}{\alpha_0^2-\alpha_1^2}\left[\frac{\alpha_0^3-\alpha_1^3}{6}-\frac{\alpha_0}{4\xi^2}+\frac{1}{4\xi}\left(\alpha_1-\frac{2}{4\xi^2}\right)\sin 2\xi\alpha_1+\frac{\alpha_1}{4\xi^2}\cos 2\xi\alpha_1-\alpha_1^2\psi_{10}\right];$$

$$\psi_{31}=\frac{1}{\alpha_0^2-\alpha_1^2}\left\{\frac{(-1)^{m+k}4\gamma\xi}{\left(\gamma^2-\xi^2\right)^2}-\frac{1}{2(\gamma-\xi)}\left[\alpha_1^2-\frac{2}{(\gamma-\xi)^2}\right]\sin\left[\alpha_1^2-\frac{2}{(\gamma-\xi)^2}\right]\sin(\gamma-\xi)\alpha_1\right.$$

$$\left.-\frac{\alpha_1}{(\gamma-\xi)^2}\cos(\gamma-\xi)\alpha_1+\frac{1}{2(\gamma+\xi)}\left[\alpha_1^2-\frac{2}{(\gamma+\xi)^2}\right]\sin(\gamma+\xi)\alpha_1+\frac{\alpha_1}{(\gamma+\xi)^2}\cos(\gamma+\xi)\alpha_1-\alpha_1^2\psi_{11}\right\}.$$

The solution of Eq. (4) will be sought in the following form

$$f_k=\sum_{S=1,3,\ldots}^{\infty}\left(A_{sk}\sin\frac{s\omega t}{2}+B_{sk}\cos\frac{s\omega t}{2}\right), \tag{5}$$

where A_{sk}, B_{sk} are the constant coefficients.

Substituting the first sum from expression (5) into the system of inhomogeneous differential equations (4) and equating coefficients with equal $\sin(s\omega t)/2$, we get an infinite system of algebraic equations. In the following we will restrict ourselves to the first term of the series that defines the main area of instability. It is sufficient for practical calculations according to [1].

As a result, we get

$$d_k A_{1k}+\sum_{m=1}^{\infty}C_{km}A_{1m}=0,(k=1,2,3,\ldots), \tag{6}$$

where $d_k=\left(\left(4b_2\pm 2b_1-\omega^2\right)\right)/4$; $C_{km}=\sum_{i=1}^{M}\left(\left(4b_{3i}-b_{4i}\omega^2\right)/4\right)\theta_{mk}^{(i)}+\left(B\left(1-n^2\right)P_2/a_7R\right)F_{mk}^{(i)}$.

We obtain the characteristic equation for determining the critical frequencies by means of reducing the system (6) to the number of members that provide the required accuracy and equating to zero the determinant of the truncated matrix.

We get the characteristic equation (6) if the second sum from expression (5) is substituted into the system of inhomogeneous differential equations (4). It will be correct if the coefficients A_{1k} be replaced by B_{1k} and the coefficient d_k takes the plus sign.

As an example, the shell reinforced by a rib is considered. The basic parameters of the shell, ribs, and the load are the following ones:

$$\alpha_0=L/R=8;$$

$$h/R=0.02;$$

$$\left(E_\beta,E_i\right)/E_\alpha=1.53;\ G_{\alpha\beta}/E_\alpha=0.17;\ \nu_\alpha=0.15;\ \nu_\beta=0.23;$$

$$\rho_i/\rho_0=1;\ F_i/R^2=1.5\times10^{-3};\ I/FR^2=2\times10^{-4};\ \alpha_i=5;\ P_0/E_\alpha=5.5\times10^{-6}.$$

Three variants of the law of distribution for the variable component of external pressure are investigated. All variants have the same force integral along the length. If this condition takes place, the pressure P_2 (excluding common constant)) will have the following values:

—for the constant law $P_2=2P_0$;

—for linearly increasing law $P_2=4P_0$;

—for quadratic law $P_2=6P_0\left(\alpha_0+\alpha_1\right)/\left(\alpha_0+2\alpha_1\right)$.

It was taken that $P_0 = 0.7P^*$ for all calculations, where P^* is the critical buckling pressure of the unsupported shell.

Figure 3 presents the regions of shell instability (this part is shaded) for constant (*1*), linear increasing (2), and quadratic increasing (*3*) laws of pressure distribution.

Here the ordinate $(Y = \omega/\omega_0)$ is the ratio of the critical pulsation frequency and the natural frequency of the unsupported shell. The value $(X = P_1/P_0)$ is the ratio of pulse component of external pressure and the common constant.

Figure 4 presents the similar dependences for a shell loaded by the axial compressive force $N_\alpha = 0.4N_\alpha^*$, where N_α^* is the critical buckling force for the unsupported shell.

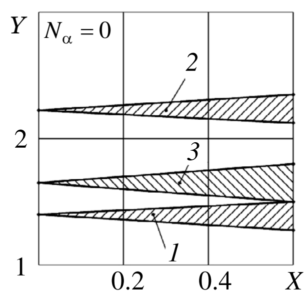

Fig. 3.

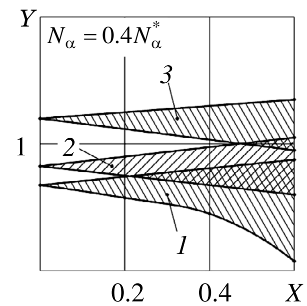

Fig. 4.

CONCLUSIONS

The following conclusions can be made:

—the lowest boundaries of the instability region are obtained when the variable component of external pressure law is constant;

—the area of the instability regions increases twice when the axial compressive force is 40 % of critical force;

—the instability region boundaries at a linear increasing law are 1.3 times higher than in the case of quadratic law for the shell unstressed by the axial force. These boundaries are 1.3 times lower taking into account the action of the axial force;

—the instability region boundaries at a constant law are in 1.4 times lower than in the case of the linear increasing or quadratic law of the variable component of the external pressure. It indicates a need of taking into account these factors;

—variability of the pressure along the axial coordinate is essential when $\alpha_1 < 0.5\alpha_0$.

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