2SSN 1068-7998, Russian Aeronautics (Iz.VUZ), 2015, Vol. 58, No. 2, pp. 145–151. © Allerton Press, Inc., 2015.
Original Russian Text © S.S. Kurennov, A.G. Koshevoi, A.G. Polyakov, 2015, published in Izvestiya VUZ. Aviatsionnaya Tekhnika, 2015, No. 2, pp. 10–15.

STRUCTURAL MECHANICS AND STRENGTH OF FLIGHT VEHICLES

Through-Thickness Stress Distribution in the Adhesive Joint for the Multilayer Composite Material

S. S. Kurennov, A. G. Koshevoi, and A. G. Polyakov

N.E. Zhukovskii National Aerospace University "Kharkiv Avaition Institute", ul. Chakalova 17, Kharkiv, 61070 Ukraine

e-mail: ss_kurennov@mail.ru

Received February 19, 2014

Abstract—The adhesive joint Goland and Reissner model was generalized for the arbitrary number of the adherend layers. This model was used for the stress state analysis in the layered composite rod with metal edging.

DOI: 10.3103/S1068799815020026

Keywords: adhesive joint, Goland and Reissner model, multilayer rod.

Several features of composite materials such as low crumpling strength, structural inhomogeneity, sensibility to irregularity of the geometry of parts, etc., promote the necessity to apply the surface adhesive and combined joints [1, 2]. The development of high-strength glues and improvement of adhesion technology and adhesion quality control are also conducive to their application. Compared to the usual types of joints, the adhesive joints have a number of advantages in the aerospace engineering, such as tightness, vibration isolation and vibration damping, high aerodynamic efficiency, manufacturability, low joint weight, relatively low cost of process equipment, and so on. However, in operation the access to the joint is usually complicated, and it is impossible to detect the fracture initiation by sight. This fact leads to increase of requirements to design of joints and techniques of stress state analysis.

The finite element method (FEM) is used to analyze the stress state of joints. In the case of the complex joint geometry or nonlinear behavior of joint elements, this technique seems more preferable or uniquely possible. However, the analytical models and computation techniques allow us to find out in detail the effect of mechanical and geometrical joint parameters on the joint stress state, explain the fracture mechanisms, conduct the parametric investigation, create the design and optimization techniques, and find out the ways for increasing the bearing capacity of joints.

Most of contemporary analytical techniques of analyzing the stress state of adhesive joints are based on the Goland and Reissner joint model [3, 4]. One of the lines in developing the theory of overlapped joints is the study of the through-thickness stress distribution in adherends. The solutions of the given problem [5, 6] are available that were obtained based on the hypotheses of structural mechanics. In [7], the solution was obtained using the hypothesis on the absence of lateral strains in the composite part. The analytical solution [8] of the two-dimensional problem of the elasticity theory involves a fair number of difficulties. A disadvantage of the techniques mentioned above is that the materials of bearing layers are assumed to be homogeneous and isotropic. Experience in the operation shows that extension of an interlayer crack and extraction of the layer adjacent to the glue line is one of the fracture behavior type in overlapped joints of the layered composite materials [9, 10]. Therefore, in determining the bearing capacity of the joint, the layered structure of the jointed parts should be taken into account.

A disadvantage of the numerical techniques based on the structural discretization [1, 11] is that the composite material is modeled by a set of parallel rods working in tension–compression only, and interlayer connections work in shear only. Moreover, in this case, the normal stresses between the layers are not taken into account. In the Rzhanitsin model [0] that is similar in formulation, the bearing layers are considered as Bernoulli beams, however, the lateral connections between them are also supposed being ideal rigid, owing to this the normal interlayer stresses are absent. Therefore, these models can not quite adequately describe the joint stress state.

The purpose of this work is to generalize the adhesive joint Goland and Reissner mathematical model for an arbitrary number of layers, and to apply the results obtained for determining the stress state of the adhesive joint in the layered composite rod with metal edge.

MODEL OF A COMPOSITE BEAM

Let us consider a multilayered rod, which consists of *m* separate rods considered as Bernoulli's beams. They are connected by joint layers, working in shear only and in tension–compression in the transversal direction. Under deformation of this system, the tangent and normal stresses appear in the joint layers; these stresses are supposed to be proportional to the differences of the longitudinal and lateral shifts of the rod sides, adjoining to the joint layer. The forces acting on the rod element are shown in Fig. 1.

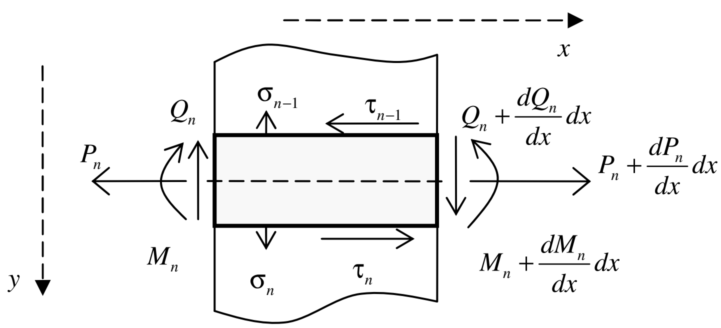

Fig. 1. Equilibrium of the rod differential element.

Let us consider a package consisting of *m* rods. The outer limits of rods numbered as 1 and *m* are supposed being free of the distributed loads. The equilibrium equations have the following form

$$\frac{dN_1}{dx} + \tau_1 = 0\,; \quad \frac{dN_n}{dx} - \tau_{n-1} + \tau_n = 0\,; \quad \frac{dN_m}{dx} - \tau_{m-1} = 0\,; \tag{1}$$

$$\frac{dQ_1}{dx} + \sigma_1 = 0\,; \quad \frac{dQ_n}{dx} - \sigma_{n-1} + \sigma_n = 0\,; \quad \frac{dQ_m}{dx} - \sigma_{m-1} = 0\,; \tag{2}$$

$$\frac{dM_1}{dx} + s_1\tau_1 - Q_1 = 0; \quad \frac{dM_m}{dx} + s_m\tau_{m-1} - Q_m = 0; \quad \frac{dM_n}{dx} + s_n\tau_{n-1} + s_n\tau_n - Q_n = 0, \tag{3}$$

where $n = 2,3,...,m-1$; N_n, Q_n, M_n are the longitudinal, lateral forces and linear bending moment in the beam n, respectively; τ_n, σ_n are the tangent and normal stresses, acting in the joint layer n; s_n is the half of the beam thickness n.

The tangent stresses in the joint layers are

$$\tau_n = \frac{1}{P_n}\left(u_{n+1} - u_n + s_n\frac{dw_n}{dx} + s_{n+1}\frac{dw_{n+1}}{dx}\right), \tag{4}$$

where P_n is the pliability of the connective layer for shift, $P_n = \delta_n G_n^{-1}$; G_n is the shift module of the corresponding joint layer; u_n, w_n are the longitudinal and lateral shifts of the nth beam.

The normal stresses in the joint layers are

$$\sigma_n = K_n\left(w_{n+1} - w_n\right), \tag{5}$$

where K_n is the tension–compression rigidity of the joint layer, $K_n = E_n^{(g)}\delta_n^{-1}$, $E_n^{(g)}$ is the modulus of elasticity of the joint layer; δ_n is the thickness of the corresponding joint layer.

The motion equations of beams take the following form

$$N_n = B_n \frac{du_n}{dx}; \quad M_n = -D_n \frac{d^2 w_n}{dx^2}; \quad (n = 1, 2, ..., m). \tag{6}$$

Here B_n and D_n are the tension–compression and bending beam rigidity, respectively; $B_n = 2s_n E_n$, $D_n = \frac{(2s_n)^3}{12} E_n$; E_n is the modulus of elasticity of the rod in the longitudinal direction.

The system of equations (1)–(6) can be reduced to a system of equations relative to the shifts, which has the following form

$$\boldsymbol{A}^{(4)} \frac{d^4 \boldsymbol{X}}{dx^4} + \boldsymbol{A}^{(2)} \frac{d^2 \boldsymbol{X}}{dx^2} + \boldsymbol{A}^{(1)} \frac{d\boldsymbol{X}}{dx} + \boldsymbol{A}^{(0)} \boldsymbol{X} = 0, \tag{7}$$

where $\boldsymbol{X} = (u_1, w_1, u_2, w_2, ..., u_m, w_m)^T$; $\boldsymbol{A}^{(4)}, \boldsymbol{A}^{(2)}, \boldsymbol{A}^{(1)}, \boldsymbol{A}^{(0)}$ are the symmetrical diagonal $\left(\boldsymbol{A}^{(4)}\right)$ and triple-diagonal matrices of $2m \times 2m$ dimension.

$$\boldsymbol{A}^{(k)} = \begin{pmatrix} \boldsymbol{W}_{k,1} & \boldsymbol{V}_{k,1} & & \dots & \\ \boldsymbol{V}_{k,1}^T & \boldsymbol{W}_{k,2} & \boldsymbol{V}_{k,2} & \dots & \\ & \boldsymbol{V}_{k,2}^T & \boldsymbol{W}_{k,3} & \dots & \\ \dots & \dots & \dots & \dots & \boldsymbol{V}_{k,m-1} \\ & & & \boldsymbol{V}_{k,m-1}^T & \boldsymbol{W}_{k,m} \end{pmatrix}; \quad \boldsymbol{W}_{4,n} = \begin{pmatrix} 0 & 0 \\ 0 & D_n \end{pmatrix}; \quad \boldsymbol{V}_{4,n} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix};$$

$$\boldsymbol{W}_{2,n} = \begin{pmatrix} B_n & 0 \\ 0 & -\frac{s_n^2}{P_n} - \frac{s_n^2}{P_{n-1}} \end{pmatrix}; \quad \boldsymbol{V}_{2,n} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{s_n s_{n+1}}{P_n} \end{pmatrix}; \quad \boldsymbol{W}_{1,n} = \begin{pmatrix} 0 & \frac{s_n}{P_n} - \frac{s_n}{P_{n-1}} \\ \frac{s_n}{P_n} - \frac{s_n}{P_{n-1}} & 0 \end{pmatrix}; \quad \boldsymbol{V}_{1,n} = \begin{pmatrix} 0 & \frac{s_{n+1}}{P_n} \\ -\frac{s_n}{P_n} & 0 \end{pmatrix};$$

$$\boldsymbol{W}_{0,n} = \begin{pmatrix} -\frac{1}{P_n} - \frac{1}{P_{n-1}} & 0 \\ 0 & K_n + K_{n-1} \end{pmatrix}; \quad \boldsymbol{V}_{0,n} = \begin{pmatrix} \frac{1}{P_n} & 0 \\ 0 & -K_n \end{pmatrix}.$$

As the layers numbered 0 and $m+1$ are absent then $P_0 = P_{m+1} = \infty$, $K_0 = K_{m+1} = 0$.

We are seeking a partial solution of system (7) in the form $\boldsymbol{X} = e^{\lambda x}\boldsymbol{h}$, where $\boldsymbol{h}$ is some vector. Substituting this expression in (7), we get

$$\left(\boldsymbol{A}^{(4)}\lambda^4 + \boldsymbol{A}^{(2)}\lambda^2 + \boldsymbol{A}^{(1)}\lambda + \boldsymbol{A}^{(0)}\right)\boldsymbol{h} = 0. \tag{8}$$

From here we get a characteristic equation

$$\det \boldsymbol{A}(\lambda) = 0, \tag{9}$$

where $\boldsymbol{A}(\lambda) = \boldsymbol{A}^{(4)}\lambda^4 + \boldsymbol{A}^{(2)}\lambda^2 + \boldsymbol{A}^{(1)}\lambda + \boldsymbol{A}^{(0)}$.

Equation (9) has a root $\lambda = 0$ of multiplicity equal to six. The total number of nonzero solutions of (7) are equal to $6(m-1)$. Hence, the general solution of Eq. (7) takes the following form

$$\boldsymbol{X} = \begin{pmatrix} u_1 \\ w_1 \\ \dots \\ u_m \\ w_m \end{pmatrix} = \sum_{k=1}^{6(m-1)} C_k \boldsymbol{h}_k e^{\lambda_k x} + \sum_{n=0}^{3} x^n \boldsymbol{H}_n. \tag{10}$$

Vectors $\boldsymbol{h}_k$ are the nontrivial solutions of a system of linear equations

$$\boldsymbol{A}(\lambda_k)\boldsymbol{h}_k = 0.$$

The vector $\boldsymbol{h}_k$ is determined with an accuracy up to an arbitrary factor C_k, as a defect of matrix $\boldsymbol{A}(\lambda_k)$ for all roots $\lambda_k \neq 0$ is equal to one. The vector $\boldsymbol{H}_n$ contains six integration constants $S_1, ..., S_6$ and they can be represented, for example, in the following form

$$\boldsymbol{H}_3 = S_1\boldsymbol{I}_1; \quad \boldsymbol{H}_2 = S_2\boldsymbol{I}_1 + 3S_1\boldsymbol{V}_2; \quad \boldsymbol{H}_1 = S_3\boldsymbol{I}_2 + S_4\boldsymbol{I}_1 - 2S_2\boldsymbol{V}_1; \quad \boldsymbol{H}_0 = S_5\boldsymbol{I}_2 + S_6\boldsymbol{I}_1 - 6S_1\boldsymbol{V}_0 - S_4\boldsymbol{V}_1,$$

where respectively

$$\boldsymbol{I}_1 = (0, 1, 0, 1, 0, ..., 1)^T; \quad \boldsymbol{I}_2 = (1, 0, 1, 0, 1, ..., 0)^T;$$

$$R_{i,j} = -s_i - s_j + 2\sum_{k=i}^{j} s_k, \ (j > i); \quad \varphi_{i,j} = \begin{cases} R_{i,j}B_j, & i < j; \\ -R_{j-1,i}B_{j-1}, & i \geq j; \end{cases}$$

$$\xi_i = \sum_{k=2}^{m} \varphi_{i,k}, \quad \psi_i = \sum_{k=1}^{i}\sum_{j=i+1}^{m} R_{k,j}B_kB_j; \quad \theta_i = \sum_{k=1}^{i} P_k\psi_k; \quad \beta = \sum_{k=1}^{m} B_k;$$

$$\boldsymbol{V}_2 = \frac{1}{\beta}(\xi_1 \quad 0, \quad \xi_2 \quad 0, \quad ..., \quad \xi_m, \quad 0)^T;$$

$$\boldsymbol{V}_1 = (0, \quad 0, \quad R_{1,2}, \quad 0 \quad R_{1,3}, \quad 0 \quad ..., \quad R_{1,m}, \quad 0)^T;$$

$$\boldsymbol{V}_0 = \frac{1}{\beta}(0, \quad 0, \quad \theta_1, \quad 0 \quad \theta_2, \quad 0 \quad ..., \quad \theta_{m-1}, \quad 0)^T.$$

The constants C_k, $k = 1, 2, .., 6(m-1)$ and $S_1, S_2, ..., S_6$ are obtained from $6m$ boundary conditions. Three boundary conditions, namely, longitudinal shift or longitudinal force; lateral shift; rotation angle, bending moment or a lateral force, are set on each of the two ends of the separate beam.

THROUGH-THICKNESS STRESS DISTRIBUTION IN THE ADHESIVE JOINT

We use the given model for analyzing the stress state of an adhesive joint of a multilayer composite rod with metal ending (Fig. 2).

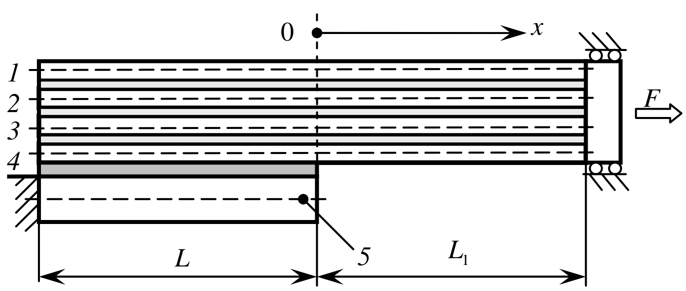

Fig. 2. The scheme of a compound.

The origin will be placed at the boundary of bonding. The multilayer composite will be considered in accordance with the model described above, and the ending will be considered as a homogeneous beam. The metal has a relatively high modulus of the interlayer shift and the uniform beam model describes well the stress state of the metal ending. Let the composite rod consists of N monolayers, in this case, the problem is solved for $N+1$ layers at bonding site. The length of the bonding site is L, the length of the composite rod protruding beyond the limit of compound is L_1. Let us denote shifts and forces in the rods outside the field of bonding $\left(x\in[0;L_1]\right)$ as u_n, w_n, N_n, Q_n, M_n, ($n=1,..,N$) and inside the field of bonding $\left(x\in[-L;0]\right)$ ad $\bar{u}_n, \bar{w}_n$ $\bar{N}_n$, $\bar{Q}_n$, $\bar{M}_n$ ($n=1,..,N+1$). The boundary conditions corresponding to Fig. 2 are as follows

$$\bar{N}_n(-L)=\bar{M}_n(-L)=\bar{Q}_n(-L)=0\,;\;\;(n=1,2,...,N)\,;$$

$$\bar{u}_{N+1}(-L)=\bar{w}_{N+1}(-L)=\left.\frac{d\bar{w}_{N+1}}{dx}\right|_{x=-L}=0\,;$$

$$\bar{N}_{N+1}(0)=\bar{M}_{N+1}(0)=\bar{Q}_{N+1}(0)=0\,;\bar{u}_n(0)=u_n(0)\,;\bar{w}_n(0)=w_n(0)\,;$$

$$\bar{N}_n(0)=N_n(0)\,;\bar{M}_n(0)=M_n(0)\,;\bar{Q}_n(0)=Q_n(0)\,;$$

$$N_n(L_1)=F_n\,;w_n(L_1)=\left.\frac{dw_n}{dx}\right|_{x=L_1}=0\,.$$

Obviously, if the length L_1 is many times greater than the thickness of the multilayer rod, the differences in the boundary conditions for the isolated rods at the end $x=L_1$ have no effect on the stress state of the compound, since the local stresses are rapidly damped with removal from the end of the rod.

As a model problem, we consider the adhesive joint of a four-layer rod ($N=4$, as shown in Fig. 2), made of carbon fiber reinforced plastic (CFRP) with aluminum ending. Lengths of sections are $L=25$ mm and $L_1=100$ mm. The parameters of all joint members are shown in Table.

Table

n	s_n, mm	E_n, GPa	δ_n, mm	G_n, GPa	$E_n^{(g)}$, GPa
1	0.125	210	0.1	1.5	4.35
2	0.125	210	0.1	1.5	4.35
3	0.125	210	0.1	1.5	4.35
4	0.125	210	0.25	0.9	2.65
5	1.5	72	–	–	–

Suppose that the equal longitudinal stretching unit forces $F_n=1$ N/m, $n=1,2,3,4$ are applied to all layers of the composite rod. The maximum stresses in the joint layers appear at the left end of the joint $(x=-L)$ and their graphs in the neighborhood of this point ($x\in[-L;-0.8L]$) are shown in Fig. 3.

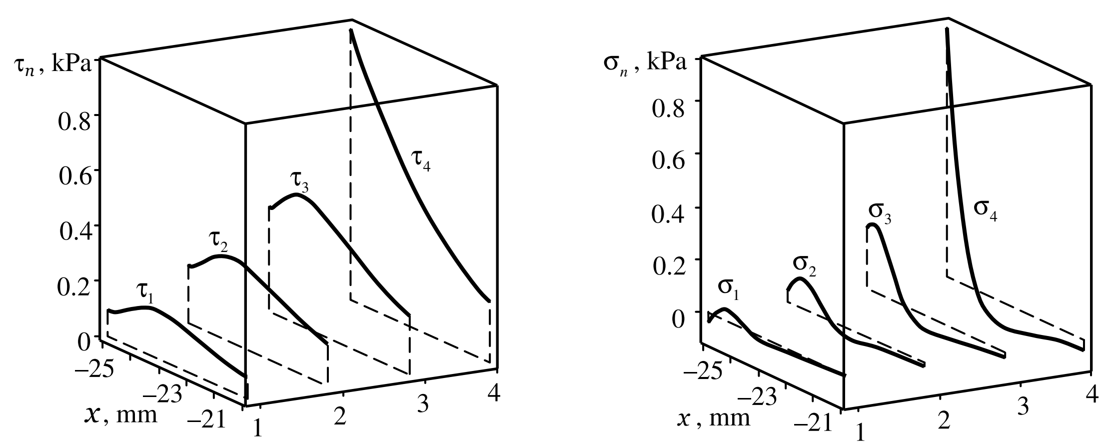

Fig. 3. Stress state at the joint layers close to the left end of the compound: tangent stresses (a) and normal stresses (b) in the transverse direction.

As is seen from graphs (Fig. 3), stresses in the bonding layer of the multilayer rod (τ_3 and σ_3) that are nearest to the adhesive layer, although are lower in magnitude than the stresses in the adhesive, however, they are relatively high. Therefore, in the case of the low interlayer strength of the multilayer composite, compound destruction can occur in the form of bundles along the composite tie layer.

The technique proposed can be verified by comparing the computational results with the results of analyzing the stress state of the compound by the classical model [0, 0]. In this case, the composite rod assumed to be homogeneous, and its modulus of elasticity is calculated by the rule of mixtures using the data from table. Calculations showed that the stresses in the adhesive layer (τ_4 and σ_4), calculated by the technique proposed are different from those calculated by the classical Goland–Reissner model only by a few percents. Distribution of shear stresses in the thickness of connected parts in a homogeneous rod has a quadratic dependence [0, 0] (generalization of the Zhuravskii formula) that is close to the linear one, and differs noticeably from the results shown in Fig. 3a. However, the through-thickness distribution of normal stresses in the transverse direction in the compound cannot be determined in general, if we consider the composite rod as a homogeneous beam. This is due to the fact that according to the bending theory the longitudinal fibers of beams do not press on each other.

CONCLUSIONS

Thus, the study performed shows that the joint destruction can occur in the form of composite material exfoliation along the connective layers, nearest to the adhesive layer.

The technique proposed is not sensible to the packet thickness over joint length ratio, in contrast to the classic beam theory.

This technique can be used for computation of the construction part joints of the layered composite materials with the stepped joint thickness change, the load-bearing element joints and for computation of the multiple-shift joints with arbitrary number of adhesive layers. Thermal stresses can also be taken into account in the model.

REFERENCES

1. Karpov, Ya.S., *Soedineniya detalei i agregatov iz kompozitsionnyh materialov* (Joining of Parts and Assemblies Made of Composite Materials), Kharkov: KhAI, 2006.
2. Sirotkin O., Grishin, V.I., and Litvinov, V.B., *Proektirovanie, raschet i tehnologiya soedinenii aviatsionni tekhniki* (Design, Calculation and Joint Technology of Aviation Equipment), Moscow: Mashinostroenie, 2006.
3. Artyukhin, Yu.P., *Napryazhenija v kleevyh soedineniyah* (Stresses in Adhesive Joints), in *Issledovanyja po teorii plastin i obolochek* (Studies on Plate and Shell Theory), 1973, no. 10, pp. 3–27.
4. Lucas F.M. da Silva, P.J.C. das Neves, Adams, R.D., and Spelt, J.K., Analytical Models of Adhesively Bonded Joints—Part I: Literature Survey, *Int. J. of Adhesion & Adhesives*, 2009, vol. 29, pp. 319–330.
5. Zhao, Bo and Lu, Z.-H., A Two-Dimensional Approach of Single-Lap Adhesive Bonded Joints, *Mechanics of Advanced Materials and Structures*, 2009, vol. 16, pp. 130–159.
6. Zhao, Bo., Lu, Z.-H., and Lu, Y.-N., Closed-Form Solutions for Elastic Stress–Strain Analysis in Unbalanced Adhesive Single-Lap Joints Considering Adherend Deformations and Bond Thickness, *Int. J. of Adhesion & Adhesives*, 2011, vol. 31, pp. 434–445.
7. Vasil'ev, V.V. and Bokov, Yu.V., *Issledovaniya napryazhennogo sostoyaniya kleevogo soedinenija kompotsionnogo materiala s metallicheskim listom* (Investigations of Stress State Adhesive Joint of the Composite Material with a Metal Sheet), in *Proektirovanie, raschet i ispytaniya konstruktsii iz kompotsionnykh materialov* (Design, Calculation, and Testing of Structures from Composite Materials), 1979, no. 7, pp. 4–10.
8. Sawa, T., Temma, K., Nishigaya, T., and Ishikawa, H., A Two-Dimensional Stress Analysis of Adhesive Butt Joints of Dissimilar Adherends Subjected to Tensile Loads, *J. Adhesion Sci. Technol.*, 1995, vol. 9, no. 2, pp. 215–236.
9. Banea, M.D. and da Silva, L.F.M., Adhesively Bonded Joints in Composite Materials: an Overview, Proc. of the Institution of Mechanical Engineers, Part L: *Journal of Materials Design and Applications,* 2009, vol. 223, no. 1, pp. 1–18.

10. Galliot, C., Rousseau J., and Verchery, G., Drop Weight Tensile Impact Testing of Adhesively Bonded Carbon/Epoxy Laminate Joints, *Int. J. of Adhesion & Adhesives*, 2012, vol. 35, pp. 68–75.

11. Kut'inov, V.F., Calculation of Adhesive Joints, in *Proektirovanie, raschet i ispytaniya konstruktsii iz kompotsionnykh materialov* (Design, Calculation, and Testing of Structures from Composite Materials), 1979, no. 7, pp. 14–30.

12. Rzhanitsyn, A.R., *Sostavnye sterzhni i plastiny* (Composite Rods and Plates), Moscow: Stroiizdat, 1986.