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Assessing the Fatigue Strength of Structural Materials by Assuming Equivalence of a Complex Stress State and Simple Extension. 1. Review of Experimental Data and Preliminary Analysis

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Abstract—The literature regarding the fatigue strength of certain steels is briefly reviewed. Terms, concepts, and numerical data are selected for subsequent use in equivalence criteria adapted to assessing the long-term fatigue strength of structural materials under combinations of alternating and steady loads.

Keywords: regular loading cycles, extreme stress state, static stress state, flexure, torsion, biaxial static tension, fatigue strength

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On the basis of known criteria [1, 2], it is possible to assess the likelihood that a structural material will pass to a limiting state just both under the action of static loads and under the action of some combination of static and alternating loads, or under alter-nating loads alone, with the creation of a regular loading cycle. Repetition of that cycle will lead ultimately to fatigue failure.

The loads on the material create a stress state which, when characterized by the primary normal stresses σ_1, σ_2, and σ_3, may be uniaxial or simple; or else multiaxial (in particular, biaxial) or complex.

The change in stress at a hazardous point of the material in the course of a regular loading cycle may be characterized by the mean stress σ_m and the amplitude σ_a of the cyclic stress [3, pp. 125, 126].

A simple stress state is a consequence of cyclic extension and compression or flexure of the material; a complex stress state in fatigue tests may be a consequence of diverse loading methods. The familiar criterial approach [1, 2] to calculating σ_m and σ_a is only applicable to the following loading methods:

• alternating torsion and/or flexure of tubular or nonhollow cylindrical samples, either with (asymmetric loading cycles) or without (symmetric loading cycles) static torsion and/or flexure [4–10];

• alternating loads on a tubular sample in the specific form of “internal pressure and an axial force varying in phase with the pressure” [4, p. 721], resulting in a zero-based loading cycle or a pulsating cycle of a loading [5, pp. 103, 274; 11, p. 60];

• extension or compression of a circular thin plate within a rigid hoop with catches, when the plate and hoop form a single unit; different hoop dimensions produce different ratios of the opposing primary normal stresses at the center of the plate [12, 13].

For example, in the following case, the fatigue strength under alternating loads cannot be assessed by the criterial approach in [1, 2]: when static loads are applied to a tubular sample with the creation of plane biaxial static tension, together with an alternating load in the form of flexure corresponding to stress of amplitude σ_a^t. In fact, when applied to 30ХГСА, ЭИ435, and ЭИ736 steels, this loading method reveals considerable sensitivity of the σ_a^t value “to biaxial static tension, especially with relatively low tensile stress” [14, 15].

Fatigue tests in cyclic extension and compression or flexure reveal a unique functional dependence of σ_a on σ_m, which may be represented as a σ_a–σ_m diagram [3, 5, 6, 16, p. 179]: the limiting amplitude σ_a of the normal stress is plotted against the mean stress σ_m in the cycle [3, p. 128]. Thus, for simple cyclic loading of any material, the function f_a determining the dependence of σ_a on σ_m is always known, and we may write a specific relation $\sigma_a = f_a(\sigma_m)$.

For a zero-based cycle, when the alternating load at the sample rises from zero to a maximum and then falls back to zero, we know that $\sigma_m = \sigma_a$ [3, c. 126]. In that case, it follows from the dependence $\sigma_a = f_a(\sigma_m)$ for a uniaxial stress state that, if $\sigma_m > 0$, then $\sigma_a = \sigma_0/2$, where σ_0 is the fatigue limit of the material in a zero-based tensile or flexural cycle. If $\sigma_m < 0$, by contrast, $\sigma_a = \sigma_{-\infty}/2$, where $\sigma_{-\infty}$ is the fatigue limit in a zero-based tensile or flexural cycle corresponding to minimum stress modulus of the cycle [9, pp. 125–130; 17].

Zero-based loading cycles that create a biaxial stress state at the hazardous point of the material provide additional information regarding its fatigue strength. For example, experimental data obtained by Rosh and Eichinger with tubular samples of soft (pipe) steel and cast steel in a million zero-based loading cycles were presented in [11, p. 59]. They may be represented by a continuous smooth curve or an approximating limiting contour, which resembles the contour employed in Mohr's well-known static strength theory for a plane stress state in the case of the maximum stress of zero-based cycles with $\sigma_1 \geq \sigma_2 = 0 \geq \sigma_3$ [11, p. 59]. In that case, the equation $\sigma_0 = \sigma_1 - \chi\sigma_3$ may be written, where $\chi = \sigma_0/\sigma_{-\infty} < 1$.

On the basis of these results, we may distinguish numerically with more clarity between two characteristic fatigue limits: $\sigma_{00} \approx (0.90–0.92)\sigma_0$ and $\tau_0 \approx 0.70\sigma_0$ for pipe steel ($\sigma_0 = 457$ MPa); and $\sigma_{00} \approx (0.95–0.98)\sigma_0$ and $\tau_0 \approx (0.68–0.78)\sigma_0$ for cast steel ($\sigma_0 = 270$ MPa). Here σ_{00} is the fatigue limit in a zero-based cycle with equal plane extension ($\sigma_1 = \sigma_2 = \sigma_{00}$ and $\sigma_3 = 0$); and τ_0 is the fatigue limit in a zero-based torsional cycle ($\sigma_1 = \tau_0$, $\sigma_2 = 0$, and $\sigma_3 = -\tau_0$). Other information may be obtained from [4, p. 721]: for example, $\sigma_{00} \approx 1.16\sigma_0$ and $\tau_0 \approx 0.62\sigma_0$ for low-carbon steel (0.20% C; 0.55% Mn), for which the strength in static tension is $\sigma_в = 438$ MPa, the yield point in static tension is $\sigma_y = 253$ MPa, and the fatigue limit in symmetric flexure or the fatigue strength under symmetrical cycling of a flexure is $\sigma_{-1} = 214$ MPa, while $\sigma_0 = 258$ MPa.

Regarding $\sigma_{-\infty}$, we know, for example, the following:

(1) For some plastic materials, $\sigma_{-\infty} = \sigma_0$ [10, p. 104; 18, p. 737], while for others (including forged iron) $\sigma_{-\infty} \approx 1.5\sigma_0$ [5, p. 98] or, more precisely, $\sigma_{-\infty} \approx \sigma_0(1 + k_*)/(1 - k_*)$, where $k_* \approx \tan 21°\sigma_{-1ex}/\sigma_y$ [5, p. 96]. The fatigue limit σ_{-1ex} for a symmetric extension–compression cycle is approximately related to σ_{-1}: $\sigma_{-1ex} = 0.85\sigma_{-1}$ [16, p. 182]; or $\sigma_{-1ex} = 0.7–0.9\sigma_{-1}$ [4, p. 605].

(2) On the basis of a million loading cycles, $\sigma_{-\infty} = 1.52\sigma_0$ for tubular samples of pipe steel and cast steel, according to the data of Rosh and Eichinger in [11, p. 59; 4, p. 637]. In addition, $\sigma_{-\infty} = 1.60\sigma_0$ for plane samples of low-carbon steel ($\sigma_u \approx 400$ MPa, $\sigma_y = 270$ MPa) and carbon steel ($\sigma_u \approx 700$ MPa, $\sigma_y = 392$ MPa).

(3) For steel rollers in sheet rolling, $\sigma_{-\infty} \approx 2.50–3.64\sigma_{-1}$ [9, p. 129].

(4) For gray iron, $\sigma_{-\infty} \approx 2.4–4.2\sigma_0$, with a mean value $\sigma_{-\infty} \approx 3.3\sigma_0$ [5, p. 98; 18, p. 738]; for iron containing globular graphite, $\sigma_{-\infty} \approx 4.1\sigma_0$ [9, p. 126].

For some steels, $\sigma_{-\infty}$ may be determined directly from the σ_a–σ_m diagram (presented, for example in [6, p. 32; 3, p. 149]; or by means of the modified Heywood formula with constant of proportionality $A_0 = \sigma_{-1}/\sigma_u$ (presented in [19, p. 191]), since $A_0 = 0.5$ was assumed arbitrarily for the equation $\sigma_{-1} = A_0\sigma_u$ in the original research [6, p. 28].

Experimental data regarding regular, synchronous, and in-phase loading, without limits on the number of loading cycles and without stress concentrations, were presented with sufficient accuracy in [4, 8–13]. Numerical analysis of those experimental data confirms that a criterion may be formulated for assessment of the equivalence of a complex alternating stress state and simple cyclic loading (extension and compression or flexure) on the basis of the function $\sigma_a = f_a(\sigma_m)$ and the criterial approach in [1]. In addition, the following assumptions must be made here:

(1) The regular cyclic loading may be represented as the sum of static and alternating loads.

(2) As a rule, the static loads on the material over the regular loading cycle correspond to the static stress state at the hazardous point of the material, which may be characterized by σ_m.

(3) The action of alternating loads leads to two significant stress states at the hazardous point of the material. Each of these may be regarded as an extreme stress state of the hazardous point of the material within the loading cycle. One extreme stress state corresponds to the maximum effect of the alternating loads; the other corresponds to the minimum effect of the alternating loads or even their opposite effect, taking account of the minus sign for compressive loads.

On that basis, we will now formulate a criterion characterizing the equivalence of a complex alternating stress state and simple cyclic loading (extension and compression or flexure). We will also compare the criterion with the Gough experimental data, first published in 1949 and partially accessible in [4, 13]. In particular, information is given there regarding the mechanical properties of chromonickel steel (0.24% C, 0.20% Si, 0.57% Mn, 3.06% Ni, 1.29% Cr, 0.54% Mo, and 0.25% V). For such Cr–Ni steel, after normalization at 900°C, quenching in oil at 850°C, and tempering at 640°C, the properties are as follows: $\sigma_u = 1025$ MPa, $\sigma_y = 970$ MPa, torsional strength $\tau_u = 890$ MPa, torsional yield point $\tau_y = 735$ MPa, and fatigue limit $\sigma_{-1} = 595$ MPa in symmetric plane flexure and τ_{-1} = 363 MPa in symmetric torsion. In addition, $\tau_0 \approx 705$ MPa and $\sigma_0 \approx 1087$ MPa.

To verify that the chosen criterion agrees with experimental data and apply it in practice, we need the function $\sigma_a = f_a(\sigma_m)$, which may expediently be determined numerically by means of a polynomial taking account both of the known mechanical properties of the specific steel and the known characteristic relations and generalized information regarding the dependence of σ_a on σ_m, as follows:

(1) When $\sigma_m = 0$, $\sigma_a = \sigma_{-1}$. When $\sigma_m = \sigma_u$ or $\sigma_m = \sigma_u^t$, $\sigma_a = 0$. Here σ_u^t is the strength of the material in tests of a cylindrical sample in static flexure, for example ($\sigma_u^t > \sigma_u$) [10].

(2) As an approximation, σ_u^t may be determined as the mean of two ratios, according to the data in [4, p. 605; 19, p. 193]. Thus, $\sigma_u^t \approx (\sigma_u/s + \sigma_{-1}\tau_u/\tau_{-1})/2$, where s = 0.7–0.9.

(3) The decrease in the amplitude σ_a "with increase in the static component of the stress may be less in flexure than in axial loading, since the sample cross section does not decrease in testing, even if the yield point increases" [5, pp. 94, 95].

(4) When $\sigma_m = \sigma_u$ or $\sigma_m = \sigma_u^t$, the polynomial has a tangent whose inclination β to the σ_m axis must be no less than the –45° inclination for the static-loading line bounding the maximum stress of the cycle (equal to the sum of absolute values σ_a and σ_m) in the case of σ_u or σ_u^t [19, p. 192].

(5) When $\sigma_m = \sigma_0/2$, the amplitude $\sigma_a = \sigma_0/2$. Analogously, when $\sigma_m = -\sigma_{-\infty}/2$, we know that $\sigma_a = \sigma_{-\infty}/2$.

(6) It is expedient to express $\sigma_a = f_a(\sigma_m)$ as a polynomial up to the value $\sigma_m \geq -\sigma_{-\infty}/2$, if data are available regarding the inflection point of the σ_a–σ_m curve when $\sigma_m = -\sigma_{-\infty}/2$, beyond which , as a rule, in cyclic compression, there is a transition "from fracture to shear failure ... with the appearance of considerable plastic compressive strain" [9, pp. 125–127].

(7) In the range $-\sigma_{-\infty}/2 < \sigma_m \leq 0$, the function $\sigma_a = f_a(\sigma_m)$ may be concave in the direction of the σ_m axis [9, p. 126].

Finally, for Cr–Ni steel, a graphically smooth function $\sigma_a = f_a(\sigma_m)$ may be obtained on the basis of the following data:

• primary data: $\sigma_0 \approx 1087$ MPa, σ_{-1} = 595 MPa [13], $\sigma_{-\infty} \approx 1415$ MPa, $\sigma_u^t = (1025/0.75 + 595 \times 890/370)/2 \approx 1400$ MPa;

• supplementary data: σ_a^t = 565 MPa when σ_m = 272 MPa [13];

• auxiliary data (adopted in order to obtain an acceptable polynomial curve): σ_a^t = 640 MPa when σ_m = –300 MPa and σ_a^t = 680 MPa when σ_m = –600 MPa.

As a result, the coefficients of the sixth-order polynomial corresponding to the function $\sigma_a = f_a(\sigma_m)$ in the form

$$\sigma_a = m_6\sigma_m^{*6} + m_5\sigma_m^{*5} + m_4\sigma_m^{*4} + m_3\sigma_m^{*3} + m_2\sigma_m^{*2} + m_1\sigma_m^{*} + m_0$$

take the following values: m_6 = 0.2391, m_5 = –0.3707, m_4 = –0.2402, m_3 = 0.1978, m_2 = 0.0914, m_1 = –0.1433, and m_0 = 0.595. The polynomial is then plotted on the basis of the value $\sigma_m^* = \sigma_m/1000$ for the range $-\sigma_{-\infty}/2 < \sigma_m \leq \sigma_u^t$ when $\tan\beta \approx -37°$.

REFERENCES

1. Kozlov, P.N., The criterion of equivalence of a complex stressed state to a simple tension for structural materials, *Vestn. Mashinostr.*, 2019, no. 6, pp. 41–46.
2. Kozlov, P.N., Three options for recording of the criterion of the equivalence of a complex stress state to simple tension for construction materials, *Vestn. Mashinostr.*, 2020, no. 2, pp. 13–18.
3. Serensen, S.V., Kogaev, V.P., and Shneiderovich, R.M., *Nesushchaya sposobnost' i raschety detalei mashin na prochnost'* (Bearing Capacity and Strength Calculations of Machine Parts), Moscow: Mashinostroenie, 1975.
4. Ponomarev, S.D., Biderman, V.L., Likharev, K.K., et al., *Raschety na prochnost' v Mashinostroenie. Tom 3. Inertsionnye nagruzki. Kolebaniya i udarnye nagruzki. Vynoslivost'. Ustoichivost'* (Calculations for Strength in Machine Engineering, Vol. 3: Inertial Loads. Vibrations and Shock Loads. Endurance. Strength), Ponomarev, S.D., Ed., Moscow: Mashgiz, 1959.
5. Forrest, P.G., *Fatigue of Metals*, Amsterdam: Elsevier, 1962.
6. Heywood, R.B., *Designing Against Fatigue*, London, Chapman and Hall, 1962.
7. Kogaev, V.P., *Raschety na prochnost' pri napryazheniyakh, peremennykh vo vremeni* (Strength Calculations at Stresses, Variable in Time), Moscow: Mashinostroenie, 1977.
8. Serensen, S.V. Fatigue resistance under complex stress state and symmetric cycle, in *Izbrannye trudy. Tom 2. Ustalost' materialov i elementov konstruktsii* (Selected Research Works, Vol. 2: Fatigue of Materials and Construction Elements), Kiev: Naukova Dumka, 1985.
9. Oleinik, N.V., *Nesushchaya sposobnost' elementov konstruktsii pri tsiklicheskom nagruzhenii* (Bearing Capacity of Constructional Elements under Cyclic Loading), Kiev: Naukova Dumka, 1985.
10. Oding, I.A., *Dopuskayemye napryazheniya v mashinostroyenii i tsiklicheskaya prochnost' metallov* (Allowable Stresses in Machine Engineering and Cyclic Strength of Metals), Moscow: Mashgiz, 1962.

11. Troshchenko, V.T., *Prochnost' metallov pri peremennykh nagruzkakh* (Strength of Metals under Variable Loads), Kiev: Naukova Dumka, 1978.
12. Kudryavtsev, I.V., *Vnutrennie napryazheniya kak rezerv prochnosti v mashinostroenii* (Internal Stresses as a Resource of Strength in Machine Engineering), Moscow: Mashgiz, 1951.
13. Uzhik, G.V., Strength of metals and the effect of stress concentration during bending with torsion under of asymmetric cycles of variable loads, *Vestn. Mashinostr.*, 1954, no. 4, pp. 11–14.
14. Lebedev, A.A., Shkanov, I.N., and Kozhevnikov, Yu.L., Criteria relating to the fatigue life of steels subjected to alternating loads under conditions of uniaxial and biaxial static strain, *Strength Mater.*, 1972, vol. 4, no. 5, pp. 1433–1437.
15. Lebedev, A.A., Koval'chuk, B.I., Giginyak, F.F., et al., *Mekhanicheskie svoistva konstruktsionnykh materialov pri slozhnom napryazhennom sostoyanii: Spravochnik* (Mechanical Properties of Constructional Materials in Complex Stress State: Handbook), Kiev: Naukova Dumka, 1983.
16. Troshchenko, V.T. and Sosnovskii, L.A., *Soprotivlenie ustalosti metallov i splavov: spravochnik* (Fatigue Resistance of Metals and Alloys: A Handbook), Kiev: Naukova Dumka, 1987, part 1.
17. Orlov, M.R., Morozova, L.V., Naprienko, S.A., et al., Fatigue fracture of constructional steel 20Kh3MVF under cyclic compression, *Elektrometallurgiya*, 2017, no. 3, pp. 32–40.
18. Belyaev, N.M., *Soprotivlenie materialov* (Strength of Materials), Moscow: Nauka, 1965.
19. Pisarenko, G.S. and Lebedev, A.A., *Deformirovanie i prochnost' materialov pri slozhnom napryazhennom sostoyanii* (Deformation and Strength of Materials under Complex Loading), Kiev: Naukova Dumka, 1976.

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