

Frequency Method of Measuring the Viscosity of Magnetorheological Fluids in a Rotary Viscosimeter

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Abstract—A frequency method of measuring the dynamic viscosity of rheological media is outlined for the example of a magnetorheological fluid. The method is based on the operational principle of a rotary viscosimeter, in which the torsion angle depends on the characteristics of the viscoelastic medium.

Keywords: frequency method, dynamic viscosity, torque, beat method, accuracy, procedural error, resolution, twist angle, torsion

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OUTLINE OF METHOD

Measurement methods have been developed for the small torques of shafts in various machines. These involve specific systems [1–3]: in particular, rotary viscosimeters [4–7]. In the latter, liquid viscosity is determined by measuring the torque in terms of the twist angle of a sprung torsion rod [6]. The phase shift between the reference rod and the displaced rod as a result of twisting is determined on the basis of the speed of the viscosimeter shaft, by measuring time intervals [4, 5, 7].

To measure the viscosity of the magnetorheological fluids used in vibrational shielding systems with magnetorheological converters, a new method has been proposed for measuring the torque of torsional shafts [8–13]. In that method, the torsional strain is determined by means of a broadband frequency modulation [6, 8, 14–16].

In the present work, we use two high-frequency sweep generators. The generators producing the reference signal and the displaced signal are denoted by G_r and G_d , respectively (Fig. 1).

In Fig. 2, we show the frequency contours of the reference signal f_r and displaced signal f_d [8, 16].

In determining the torques of the rotating torsional shafts by the given method, their twisting leads to a frequency shift between the reference signal f_r and displaced signal f_d (Fig. 3a) [6, 8, 14–16].

In the rotary viscosimeter, the dynamic viscosity of a magnetorheological fluid is determined by means of an elastic torsion rod rather than a torsional shaft [6]. The viscous drag of the magnetorheological fluid on the viscosimeter's rotor produces a torque that twists the elastic torsion rod [7]. We may use a spring as the elastic torsion rod. At the maximum torque, the spring's rigidity ensures mutual displacement of the modulation disks by no more than 3° [4–6].

An opposing torque at the elastic torsion rod (sensitive element) is created in determining the torque at the viscosimeter's rotor. That torque may be determined by the frequency method of measuring the torque of rotating torsional shafts [6, 8, 14–16].

The reference frequency-modulated (FM) signal from G_r is described as follows [17, 18]

$$\left. \begin{aligned} f_r(t) &= f_{r0} - \Delta f_m/2 + 2\Delta f_m t/T_m \text{ when } 0 < t < T_m/2; \\ f_r(t) &= f_{r0} + \Delta f_m/2 - 2(\Delta f_m/T_m)(t - T_m/2) \text{ when } T_m/2 < t < T_m; \\ f_r(t) &= f_r(t + kT_m) \text{ at any } t, \end{aligned} \right\} \quad (1)$$

where f_{r0} is the central frequency of the reference FM signal from G_r ; Δf_m is the frequency deviation; and T_m is the period of frequency modulation.

As the elastic torsion rod of the viscosimeter twists, the signal f_d displaced in time t_{tw} takes the form [17, 18]

$$\left. \begin{aligned} f_d(t) &= f_{d0} - \Delta f_m/2 + 2\Delta f_m (t - t_{tw})/T_m \text{ when } 0 < t < T_m/2; \\ f_d(t) &= f_{d0} + \Delta f_m/2 - 2(\Delta f_m/T_m)[(t - t_{tw}) - T_m/2] \text{ when } T_m/2 < t < T_m; \\ f_d(t) &= f_d[(t - t_{tw}) + kT_m] \text{ at any } t, \end{aligned} \right\} \quad (2)$$

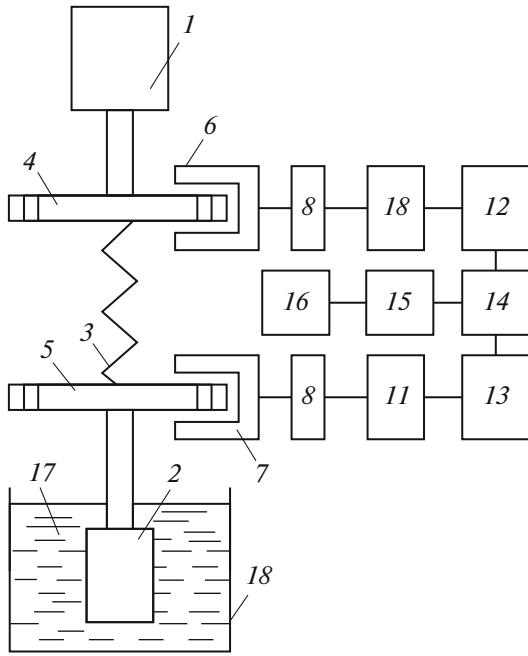


Fig. 1. Functional diagram of a rotary viscosimeter with frequency-based measurement of the torque: (1) electric motor; (2) measuring rotor; (3) elastic torsion rod; (4, 5) modulation disks; (6, 7) optical pairs; (8, 9) pulse converters; (10, 11) modulators for generators; (12) generator of reference signal (G_r); (13) generator of displaced signal (G_d); (14) mixer; (15) low-frequency filter; (16) beat-frequency counter; (17) test liquid; (18) measuring vessel.

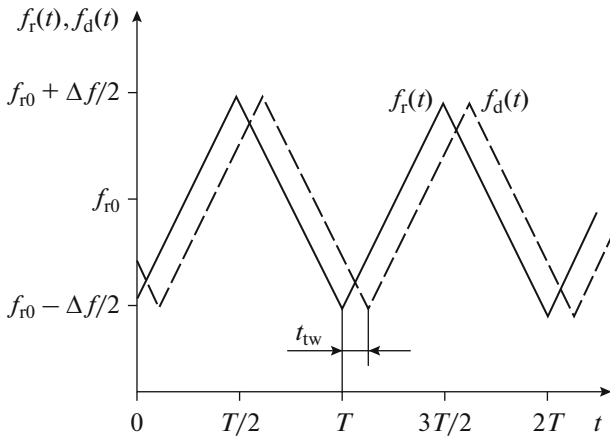


Fig. 2. Frequency variation of sweep generators producing reference and displaced signals in measuring the the viscosimeter's torque at zero intermediate frequency.

where f_{d0} is the central frequency of the displaced FM signal from G_d .

The signal f_d is delayed relative to f_r by the twisting time t_{tw} of the elastic torsion rod: $t_{tw} = \Delta\phi/\Omega_{ro}$. Thus, $f_d(t) = f_r(t - t_{tw})$.

In torque measurement with zero beats, the displaced signal f_d is summed with the reference signal at

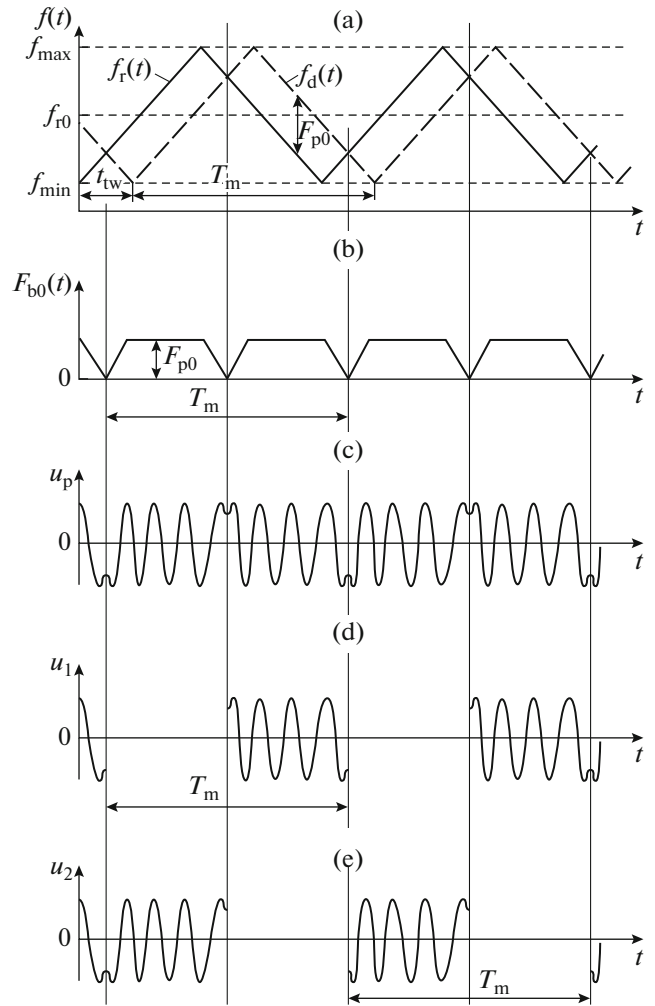


Fig. 3. Beat signal consisting of pulses with period T_m .

the mixer input. At the mixer output, a low-frequency filter isolates the beat frequency F_{b0} , which is equal to the absolute value $|F_{D0}|$ of the frequency difference between those signals (Figs. 3a and 3b). Taking account of the Eqs. (1) and (2), in ascending intervals $T_m/2$, we may write this signal in the form [17, 18]

$$\begin{aligned} |\Delta f(t)| &= |f_r(t) - f_d(t)| \\ &= |(f_{r0} - \Delta f_m/2 + 2\Delta f_m t/T_m) \\ &\quad - (f_{r0} - \Delta f_m/2 + 2\Delta f_m (t - t_{tw})/T_m)| \\ &= |f_{r0} - \Delta f_m/2 + 2\Delta f_m t/T_m \\ &\quad - f_{r0} + \Delta f_m/2 - 2\Delta f_m t/T_m + 2\Delta f_m t_{tw}/T_m| \\ &= |2\Delta f_m F_m t_{tw}| = |F_{b0}(t)|. \end{aligned}$$

From Eq. (1), we obtain the maximum frequency difference [17–19]

$$F_{b0max} = 2\Delta f_m t_{tw}/T_m = 2\Delta f_m F_m t_{tw}. \quad (3)$$

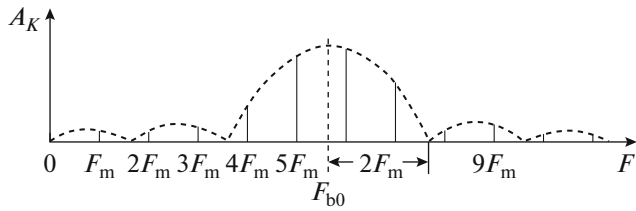


Fig. 4. Theoretical spectrum of the transformed beat signal, with discrete lines concentrated around the frequency F_{b0} .

Thus, by isolating the fundamental beat frequency at the mixer output corresponding to the reference and displaced FM signals, the true torque of the rotary viscosimeter may be measured.

RESOLUTION AND ACCURACY OF THE METHOD

The FM frequencies of the generators 12 and 13 (Fig. 1) are summed in mixer 14. The spectrum of the transformed beat signal (Fig. 3c) at the mixer output for many twist angles $\Delta\varphi$ of the viscosimeter's elastic torsion rod may be regarded as the superposition of the spectra of several individual signals (Figs. 3d and 3e) [19].

Consider the structure of a transformed beat signal (Fig. 3). We assume that the measurement time T_{me} is much greater than the frequency-modulation period T_m : $T_{me} \gg T_m$. Then the beat signal consists of individual pulses with period T_m .

These pulses have a discrete spectrum [19]. The interval between the individual spectral lines corresponds to the repetition frequency: $F_m = 1/T_m$. The amplitudes A_K of the spectral components of the beat signals are accommodated within the envelope of the continuous spectrum of a single pulse — in other words, within the function $\sin x/x$ (Fig. 4).

The spectrum of a single pulse is concentrated close to the frequency F_{b0} . The presence of inversion zones (phase shifts) within the beat signal (Fig. 3a) expands the spectral envelope. There is a phase shift (inversion zone) in the center of the interval T_m [19]. Accordingly, we may consider the pulse within the interval consisting of two pulses of length $T_m/2$. The spectral envelope of each pulse is doubled, so that the primary lobe occupies the frequency band from $F_{b0} - 2F_m$ to $F_{b0} + 2F_m$. Together with the basic spectral line F_{b0} , we now see secondary lines. Thus, the theoretical spectrum of the transformed beat signal consists of discrete lines concentrated around the frequency F_{b0} (Fig. 4) [19]. Decrease in the modulation frequency F_m increases the concentration of spectral lines of the transformed signal close to frequency F_{b0} .

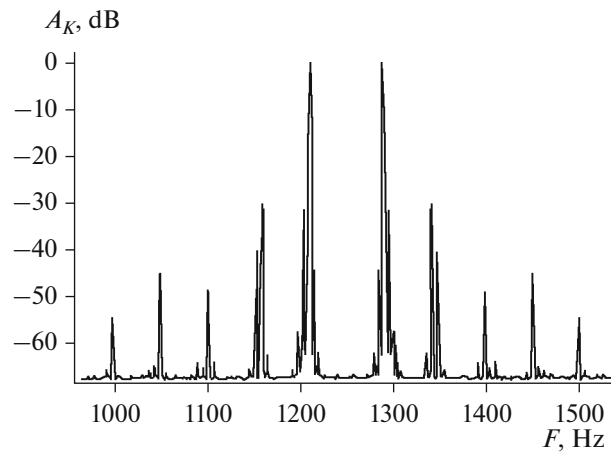


Fig. 5. Spectrum of the actual transformed beat signal with symmetric sawtooth frequency modulation; the central frequency is $F_{b0} = 1250$ Hz.

The spectrum of the actual transformed beat signal with symmetric sawtooth frequency modulation is shown in Fig. 5. The central frequency is $F_{b0} = 1250$ Hz. The beat frequency F_{b0} , which determines the twist angle $\Delta\varphi$ of the elastic shaft, is measured from the position of the spectral line of maximum amplitude. In the general case, that need not be the frequency F_{b0} (Figs. 4 and 5).

However, any change in $\Delta\varphi$ may only be detected from the change in amplitude of the spectral line [19]. This change corresponds to F_{b0} . Hence, with a discrete spectrum, the procedural error in measuring the twist angle $\Delta\varphi$ is

$$\delta\Delta\varphi_{\max} = \frac{\varphi F_{b0}}{2\Delta f_m} = \frac{\varphi F_m}{2\Delta f_m} = \frac{\pi F_m}{\Delta f_m} \text{ (rad)}.$$

For example, with frequency deviation $\Delta f_m = 20$ MHz and modulation frequency $F_m = 2.5$ Hz (the rotational frequency of the elastic torsion rod), the procedural error at $\Delta\varphi_{\max}$ is 4×10^{-7} rad (about 1"). This error may be neglected.

To decrease the error, we must increase the frequency deviation. As a rule, that is not difficult [17, 19].

With large modulation index $m = \Delta f_m/2F_m$, the frequency deviation is close to the width of the FM spectrum; for example, with sinusoidal modulation, $m = 0.01$. Thus, to decrease the procedural error, we must expand the spectrum of FM vibrations. The potential accuracy in torque measurement is then determined by the width of the frequency band for the signals from G_r and G_d [19].

That corresponds to minimum displacement time $\delta t_{\text{dis.min}}$ of the viscosimeter's modulation disks mounted on the elastic torsion rod

$$\delta t_{\text{dis.min}} = \frac{\Delta F_{b0}}{2\Delta f_m F_m} = \frac{F_m}{2\Delta f_m F_m} = \frac{1}{2\Delta f_m}. \quad (4)$$

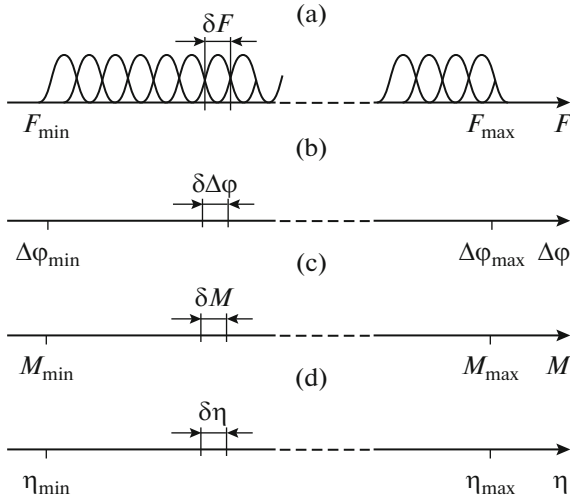


Fig. 6. Analysis of the torsional deformation (b) and torque (c) of the viscosimeter's rotating torsional shaft and the dynamic viscosity of the magnetorheological fluid (d) in terms of the signal (a) of the beat signal (parallel analysis).

According to Eq. (4), when $\Delta f_m = 20$ MHz, the minimum displacement time of the modulation disks is 25×10^{-9} s. In that case, the beat frequency is determined by the modulation frequency: for example, $F_{b0min} = 2.5$ Hz when $F_m = 2.5$ Hz.

The frequency deviation with the required procedural error is found from Eq. (3) and the relation

$$\frac{1}{2\Delta f_m} = \frac{\delta\Delta\varphi_{\max} T_m}{\varphi} = \frac{\delta\Delta\varphi_{\max}}{\varphi F_m} = \delta t_{\text{dis.min}}; \quad (5)$$

$$\Delta f_m = \frac{\varphi_{\max} F_m}{2\delta\Delta\varphi_{\max}} = \frac{\pi F_m}{\delta\Delta\varphi_{\max}}.$$

The values of $\delta t_{\text{dis.max}}$ and $\Delta\varphi_{\max}$ depend on the mechanical properties of the elastic torsion rod—that is, on the shear modulus G and density ρ of the material, which determine its strength [6, 8, 14–16].

To assess the resolution and accuracy of the frequency method of viscosity measurement for magnetorheological fluids, we need to consider the beat-frequency spectrum for each time corresponding to a specific discrete value of the twist angle $\Delta\varphi$. To that end, we pass from serial to parallel spectral analysis [19].

In parallel spectral analysis, all the successive values of $\Delta\varphi$ appear simultaneously at the input of the parallel analyzer, at some specific time. There is a set of discrete filters limiting the ranges of frequency ($F_{b0min} - F_{b0max}$), twist angle ($\Delta\varphi_{\min} - \Delta\varphi_{\max}$), torque ($M_{\text{to.min}} - M_{\text{to.max}}$), and dynamic viscosity of the magnetorheological fluid ($\eta_{\min} - \eta_{\max}$), as illustrated in Fig. 6 [19].

If the bandwidth of each filter is ΔF_f , the number required is $n_f = (F_{b0max} - F_{b0min})/\Delta F_f$.

ASSESSING THE RESOLUTION AND ACCURACY

Two twist angles may be distinguished if the difference between their beat frequencies F_{b01} and F_{b02} is greater than the transmission band of the filter: $F_{b02} - F_{b01} \geq \Delta F_f$, where

$$F_{b01} = 2\Delta f_m F_m \left(\frac{T_m \Delta\varphi_1}{\varphi} \right) = 2\Delta f_m \left(\frac{\Delta\varphi_1}{\varphi} \right);$$

$$F_{b02} = 2\Delta f_m F_m \left(\frac{T_m \Delta\varphi_2}{\varphi} \right) = 2\Delta f_m \left(\frac{\Delta\varphi_2}{\varphi} \right).$$

Thus, the resolution condition is

$$\frac{2\pi\Delta F_f}{2\Delta f_m} = \frac{\pi\Delta F_f}{\Delta f_m};$$

$$\Delta\varphi_2 - \Delta\varphi_1 \geq \frac{2\pi\Delta F_f}{2\Delta f_m} = \frac{\pi\Delta F_f}{\Delta f_m}.$$

Then the resolution is [17, 19]

$$\delta\Delta\varphi = \frac{2\pi\Delta F_f}{2\Delta f_m} = \frac{\pi\Delta F_f}{\Delta f_m}.$$

The filter's bandwidth ΔF_f must be consistent with the time of the corresponding transformed beat signal, which consists of individual radio pulses with carrier frequency F_{b0} (Figs. 3b and 3c). For example, with symmetric sawtooth modulation, the pulse length may be assumed to be $T_m/2$ [19]. Therefore, the transmission band of the matched filter is $\Delta F_{f.ma} \approx 2/T_m$. Then the potential resolution for symmetric sawtooth modulation of the signals from G_r and G_d (Fig. 2) is [17, 19]

$$\delta\Delta\varphi_{\text{pot}} = \frac{2\pi\Delta F_f}{\Delta f_m} = \frac{2\pi F_m}{\Delta f_m}.$$

In that case, the resolution matches the discreteness in measuring the twist angles $\Delta\varphi$ of the elastic torsion rod and is determined by the frequency bandwidth of the reference and displaced signals from G_r and G_d —that is, the frequencies $f_i(t)$ and $f_d(t)$. This indicates that the resolution with respect to $\Delta\varphi$ is ultimately determined by the spectral width of the transformed beat signal [19].

If the mean spectral frequency F_{b0} of the fundamental beat frequencies with variation in the twist angles falls in the frequency band ΔF_f of the filter, then we may assume that F_{b0} is equal to the filter's resonant frequency.

Suppose that F_{b0} may correspond with the same probability to any value within the band ΔF_f . We know that, with uniform distribution density of a random quantity within a given interval, its standard deviation

is equal to that interval divided by $2\sqrt{3}$ [19]. Therefore, from Eq. (5)

$$\delta\Delta\varphi_{\text{pot}} \approx \frac{1}{2\sqrt{3}} \frac{\pi\Delta F_f}{\Delta f_m}.$$

To simplify the spectral analysis, we may decrease the number of filters, while increasing their transmission band ΔF_f . However, that decreases the resolution and accuracy [19].

To ensure that the twist angles of the elastic torsion rod and the frequency F_{b0} correspond in making use of the potential resolution, we must employ a very linear modulation law. The required linearity is assessed in terms of the relative deviation of the rate of frequency variation: $\gamma = \Delta F_{b0}/F_{b0}$.

If

$$\Delta F_{b0} = (\Delta F_{b0\text{max}} - F_{b0\text{min}})/2n_f,$$

where n_f is the number of filters for the corresponding twist angles, then $\gamma = 0.25\%$ when $(F_{b0\text{max}} - F_{b0\text{min}})/F_{b0} = 0.5$ and $n_f = 100$. With the electronic components currently available, that is perfectly acceptable [19].

DETERMINING THE DYNAMIC VISCOSITY ON THE BASIS OF THE TWIST ANGLE

In measuring the dynamic viscosity of a magnetorheological fluid in a rotary viscosimeter, the torque applied to the internal cylinder is determined from the formula

$$M_k = FR, \quad (6)$$

where M_k is the torque on the rotor; F is the force applied to the cylinder; and R is its radius [4–8, 20].

The force on the internal cylinder is determined from Newton's law [4, 5, 20]

$$F = \eta \frac{(\Omega_{ro}S)}{l}, \quad (7)$$

where $\Omega_{ro} = 2\pi F_{ro}$ is the angular velocity of the cylinder; S is its working area; l is the gap between the sleeve and the internal cylinder; and η is the viscosity of the magnetorheological fluid.

Substituting Eq. (7) into Eq. (6), we obtain [4, 5]

$$M_k = \eta \frac{(\Omega_{ro}SR)}{l}. \quad (8)$$

If $\Omega_{ro} = 2\pi F_{ro}$, and the cylinder dimensions are constant, then the torque M_k is equal to the retarding torque M_{re} due to the viscous drag of the magnetorheological fluid, which is proportional to its viscosity η [4, 5]

$$M_{re} = k_{di}\Omega_{ro}\eta,$$

where the coefficient $k_{di} = SR/l$ depends on the dimensions of the sleeve and rotor; S is the working

area of the cylinder; R is its radius; and l is the gap between the sleeve and the rotor.

Hence, knowing the torque on the internal cylinder, we may determine the dynamic viscosity of the magnetorheological fluid [4–8, 20].

In determining the dynamic viscosity on the basis of the frequency, the torque on the viscosimeter's turning torsional shaft is calculated from the formula [6, 8, 14–16]

$$M_{to} = \frac{\Delta\varphi}{H} GJ_r = \frac{2\pi F_{b0}}{2\Delta f_m} \frac{GJ_r}{H} = \frac{F_{b0}}{2\Delta f_m} 2\pi \frac{GJ_r}{H}, \quad (9)$$

where $(F_{b0}/2\Delta f_m)2\pi = \Delta\varphi$ is the twist angle (rad) of the viscosimeter's torsional shaft [6, 8, 14–16].

Equating the torques in Eqs. (8) and (9), we find that

$$M_{to} = \eta \frac{(\Omega_{ro}Sr)}{l} = \frac{F_{b0}}{2\Delta f_m} 2\pi \frac{GJ_r}{H}. \quad (10)$$

From Eq. (10), we may determine the dynamic viscosity η of the magnetorheological fluid in the rotary viscosimeter [6, 8]

$$\eta = \left(\frac{F_{b0}}{2\Delta f_m} 2\pi \frac{GJ_r}{H} \right) \left(\frac{l}{2\pi F_{ro}SR} \right) = \frac{F_{b0}}{2\Delta f_m} \frac{GJ_r}{H} \frac{l}{F_{ro}SR}.$$

Thus, on the basis of the frequency method of measuring the torque at the torsion shaft, the change in dynamic viscosity of the magnetorheological fluid under the action of external factors may be assessed so as to permit preliminary regulation of the working fluid in the magnetically controlled damper, taking account of the magnetic field.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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