

Kinematic Accuracy of Relative Manipulation by a Two-Handed Robot

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Abstract—Analytical formulas for the linear and angular errors of relative manipulation by a two-handed robot are derived, for use in determining its kinematic accuracy.

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Robots with two manipulators operating in a single working zone under the same control system are known as two-handed robots. Robots with two manipulators are capable of more complex operations than robots with a single manipulator. An important characteristic of such robots is the accuracy of relative motion of the output elements in assembly and other tasks. The manipulators are located on a single mobile platform or on a fixed base (Fig. 1). It is important to determine the angular and linear errors in relative motion of the robot grips.

To determine the linear error in relative manipulation, we find the radius vector of an arbitrary point of the output element for each manipulator [1]

$$\bar{r} = \bar{r}(q_i), \quad i = 1, \dots, n, \quad (1)$$

where n is the number of degrees of freedom for the manipulator; and q_i are the generalized manipulator coordinates. Differentiating Eq. (1) with respect to the time, we obtain

$$d\bar{r} = \sum_{i=1}^n \frac{\partial \bar{r}}{\partial q_i} dq_i. \quad (2)$$

Replacing the differentials by finite variables, we find the vector corresponding to the linear position error for a characteristic point in the grip of each manipulator. This point may be the origin of the coordinate system attached to the grip

$$\overline{\Delta r_k} = \sum_{i=1}^n \frac{\partial \bar{r}}{\partial q_i} \Delta q_i, \quad (3)$$

where Δq_i is the error of the programmed generalized coordinates; $k = 1, 2$.

For the characteristic point of the grip, the linear error may be written without differentials in the form [1]

$$\bar{v}_k = \sum_j \bar{\omega}_j \bar{r}_{jn} + \sum_i \bar{v}_i,$$

where j is the number of the rotary pair; i is the number of the translational pair; $\bar{\omega}_j$ is the vector corresponding to the relative angular velocity in pair j ; \bar{v}_i is the vector corresponding to the relative linear velocity in

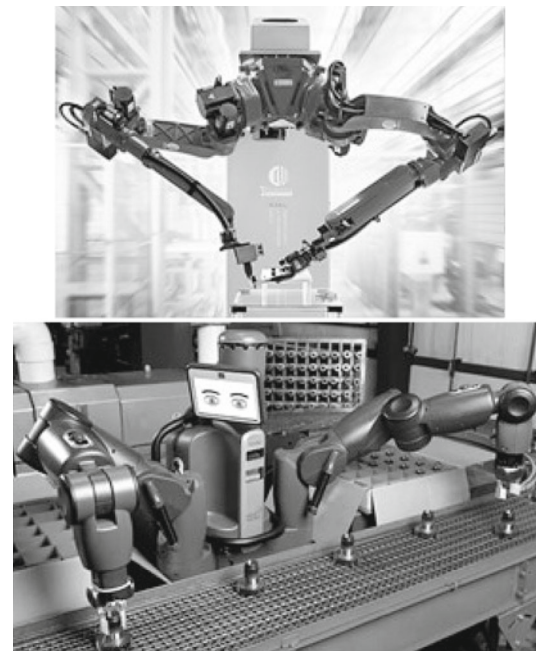


Fig. 1. Two-handed robots.

pair i ; and \bar{r}_{jn} is the radius vector connecting the center of pair j with the characteristic point of the grip.

In Eq. (1), we replace the differentials with finite displacements

$$\Delta \bar{r}_k = \sum_j \Delta \varphi_j \bar{e}_j \bar{r}_{jn} + \sum_i \Delta S_i \bar{e}_i,$$

where $\Delta \varphi_j$ is the error of the programmed coordinates in the rotary pairs; ΔS_i is the error of the coordinates in the translational pairs; and \bar{e}_j, \bar{e}_i are unit vectors corresponding to the kinematic pairs.

It follows from Eq. (2) that the linear error of each manipulator depends on the errors in the kinematic pairs and the position of the manipulator in the working zone.

To determine the relative velocities of points of the manipulator grips, we use the equation

$$\bar{v}_1 = \bar{v}_2 + \bar{v}_{12}.$$

Hence

$$\bar{v}_{12} = \bar{v}_2 - \bar{v}_1,$$

where \bar{v}_1 and \bar{v}_2 are the velocity vectors of points of the grips in the first and second manipulators, respectively; and \bar{v}_{12} is the relative velocity of the points.

Switching to small linear displacements, we find from Eq. (3) that

$$\overline{\Delta r_{12}} = \overline{\Delta r_1} - \overline{\Delta r_2},$$

where $\overline{\Delta r_{12}}$ is the linear error in the relative position of the manipulators; and $\overline{\Delta r_1}$ and $\overline{\Delta r_2}$ are the linear errors of the first and second manipulators, respectively.

Switching to the moduli of the vectors $\overline{\Delta r_{12}}, \overline{\Delta r_1}, \overline{\Delta r_2}$, we write

$$|\overline{\Delta r_{12}}| = |\overline{\Delta r_1}| + |\overline{\Delta r_2}|.$$

As an example, we determine the kinematic error of a two-handed robot with six degrees of freedom, containing both rotary and translational pairs (Fig. 2).

In the motionless coordinate system $OXYZ$, the position of point C_3 in element 3 of the manipulator ABC_3 is determined by vector $\overline{OC_3} = \bar{r}_3$. The position of point C_4 in element 4 of manipulator EDC_4 is determined by vector $\overline{OC_4} = \bar{r}_4$.

The linear error of the two-handed robot is determined by the vector difference

$$\overline{\Delta r_{34}} = \bar{r}_4 - \bar{r}_3. \tag{4}$$

Note that each vector \bar{r}_3 and \bar{r}_4 includes an error that depends on many factors. The most significant is the error Δq_i in implementing the generalized coordinates of the manipulators.

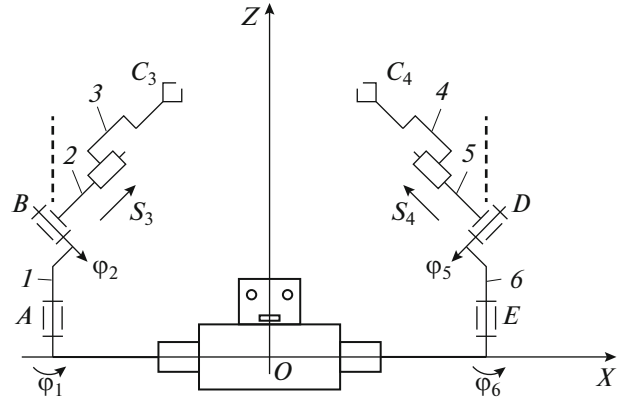


Fig. 2. Two-handed manipulator with six degrees of freedom.

For each manipulator

$$\overline{\Delta r_3} = \sum_{i=1}^3 \frac{\partial \bar{r}_3}{\partial q_i} \Delta q_i, \quad i = 1, 2, 3,$$

$$\overline{\Delta r_4} = \sum_{i=1}^6 \frac{\partial \bar{r}_4}{\partial q_i} \Delta q_i, \quad i = 4, 5, 6,$$

where $\overline{\Delta r_3}$ and $\overline{\Delta r_4}$ are the errors in vectors \bar{r}_3 and \bar{r}_4 , respectively; and q_i are the generalized coordinates of the manipulators.

From Eq. (4), we determine the modulus of the linear error

$$|\overline{\Delta r_{34}}| = |\overline{\Delta r_3}| + |\overline{\Delta r_4}|.$$

Note that determination of the linear error from Eq. (3) with differentiation leads to an unwieldy expression, especially if the manipulators have many degrees of freedom. Therefore, it is more expedient to use Eq. (4), in rewritten form

$$|\overline{\Delta r_{34}}| = \left| \sum_{i=1}^3 \Delta q_i \bar{e}_i \times \bar{r}_{i3} \right| + \left| \sum_{j=4}^6 \Delta q_j \bar{e}_j \times \bar{r}_{j4} \right|,$$

where $i = 1, 2, 3; j = 4, 5, 6$. The generalized coordinates are as follows

$$q_1 = \varphi_1; \quad q_2 = \varphi_2; \quad q_3 = S_3; \\ q_4 = S_4; \quad q_5 = \varphi_5; \quad q_6 = \varphi_6.$$

We may write the vectors $\overline{\Delta r_3}$ and $\overline{\Delta r_4}$ in the expanded form

$$\begin{cases} \overline{\Delta r_3} = \Delta \varphi_1 \bar{e}_1 \times (\overline{AB} + \overline{BC_3}) + \Delta \varphi_2 \bar{e}_2 \times \overline{BC_3}; \\ \overline{\Delta r_4} = \Delta \varphi_6 \bar{e}_6 \times (\overline{ED} + \overline{CD_4}) + \Delta \varphi_5 \bar{e}_5 \times \overline{DC_4}. \end{cases} \tag{5}$$

The vectors in Eq. (5) are determined in projections onto the Cartesian axes $OXYZ$, with specified manipulator positions.

For manipulator ABC_3

$$\begin{aligned} \overline{[AB]} &= [l_1, 0, 0]^T, \quad l_1 = AB; \\ \overline{[BC]} &= [S_3 \cos \varphi_2 \cos \varphi_1, S_3 \cos \varphi_2 \sin \varphi_1, l_1 \\ &\quad + S_3 \sin \varphi_2]^T; \\ \overline{[e_1]} &= [0, 0, 1]^T; \quad \overline{[e_2]} = [\cos \varphi_1, \sin \varphi_1, 0]^T; \\ \overline{[e_3]} &= [\cos \varphi_2 \cos \varphi_1, \cos \varphi_2 \sin \varphi_1, \sin \varphi_2]^T. \end{aligned}$$

For manipulator EDC_4 , in view of symmetry, we have analogous expressions with switched subscripts.

The angular error in the relative position of two solids may be determined by means of the error matrix [2]. This is the transition matrix from a coordinate system fixed in one body to that fixed in the other body, which is rotated by three small Euler angles. The matrix includes the values of the three Euler angles. However, the matrix does not express the angular error by a single value.

A method of determining the angular error of a manipulator was proposed in [3]. This method is based on the d'Alembert–Euler theorem that there exists a single axis and angle of rotation permitting the rotation of a solid from one position to another. Therefore, the angular error of a manipulator is assumed to be the vector

$$\overline{\Delta \varphi} = \sum_j \overline{\Delta q_j}, \quad j = 1, \dots, k,$$

where k is the number of rotary pairs in the manipulator; and $\overline{\Delta q_j}$ are the rotational vectors in rotary pairs. Those vectors are added by the summation rule for small rotations.

Using the transition matrix between coordinate systems associated with the system elements, we write

$$\overline{\Delta \varphi} = \sum_j L_{ij} \Delta q_j \overline{e_j},$$

where $L_{ij} = \prod_{i=1}^n L_{i-1}$ is the third-order transition matrix from system j associated with element j to system i associated with the output element; Δq_j is the rotational error in the j th rotary pair; and $\overline{e_j}$ is the unit vector of the axis for pair j .

Analogously to the linear errors, the moduli of the angular errors of the first and second manipulators are added.

The error in the angular position of element 3 ABC_3 (Fig. 2) is

$$\overline{\Delta \Psi_3} = \overline{\Delta \Psi_1} + \overline{\Delta \Psi_2},$$

where the vectors $\overline{\Delta \Psi_1} = \Delta \Psi_1 \overline{e_1}$ and $\overline{\Delta \Psi_2} = \Delta \Psi_2 \overline{e_2}$ correspond to the angular errors in the position of elements 1 and 2.

The angular position errors of elements 2 and 3 are the same.

The angular position errors of element 4 EDC_4 is

$$\overline{\Delta \Psi_4} = \overline{\Delta \Psi_5} + \overline{\Delta \Psi_6},$$

where the vectors $\overline{\Delta \Psi_5} = \Delta \Psi_5 \overline{e_5}$ and $\overline{\Delta \Psi_6} = \Delta \Psi_6 \overline{e_6}$ correspond to the angular errors in the position of elements 5 and 6.

If the positions of points C_3 and C_4 are the same, we obtain the relative angular error

$$\overline{|\Delta \Psi_{34}|} = \overline{|\Delta \Psi_1|} + \overline{|\Delta \Psi_2|} + \overline{|\Delta \Psi_5|} + \overline{|\Delta \Psi_6|}.$$

CONCLUSIONS

We have obtained expressions for the linear and angular errors of a two-handed robot with six degrees of freedom

The linear error of relative manipulation by a two-handed robot depends on the position of the manipulator grips in the working zone and on the error of all the generalized coordinates of both manipulators.

The angular error of relative manipulation depends only on the total number of rotary pairs in the manipulators.

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REFERENCES

1. Vorob'ev, E.I. and Khatuntsev, D.I., Two-handed robots: specific algorithms of motion control, *Avtom., Sovrem. Tekhnol.*, 2016, no. 3, pp. 19–23.
2. Lur'e, A.I., *Analiticheskaya mekhanika* (Analytical Mechanics), Moscow: Fizmatlit, 1961.
3. Vorob'ev, E.I., Kozyrev, Yu.P., and Tserenko, V.I., *Pro-myshlennyye roboty agregatno-model'nogo tipa* (Industrial Modular Robots), Moscow: Mashinostroenie, 1988.
4. Tikhomirova, L., Amazon will completely replace human work with a robot in industrial plants, 2019. <https://versiya.info/tehnika-i-tehnologii/101175>.

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