ISSN 1068-798X, Russian Engineering Research, 2020, Vol. 40, No. 11, pp. 893–900. © Allerton Press, Inc., 2020.
Russian Text © The Author(s), 2020, published in Vestnik Mashinostroeniya, 2020, No. 8, pp. 3–11.

Electroelastic Actuator for Nanomechanics

S. M. Afonin*

National Research University of Electronic Technology (MIET), Moscow, 124498 Russia

**e-mail: eduems@mail.ru*

Received September 17, 2020; revised October 10, 2020; accepted October 20, 2020

Abstract—Structural-parametric models of electroelastic actuators for nanomechanics are presented. The structure of the actuators is established, and their transfer functions are determined. Transfer functions are derived for a piezo actuator with a generalized piezo effect. The change in elastic pliability and rigidity of the actuator is established in the case of voltage control and also current control.

Keywords: electroelastic actuators, piezo actuator, structural-parametric model, transfer functions, structure, nanomechanics

DOI: 10.3103/S1068798X20110039

Electroelastic actuators with piezoelectric and electrostrictional effects are promising for use in nanomechanics and nanotechnology, nanobiology, the power industry, microelectronics, and astronomy. A piezo actuator is a piezomechanical device for setting mechanisms in motion or controlling them on the basis of the piezo effect and also for converting electrical signals into mechanical parameters: displacement and force. Nanometric precision of mechanical devices is ensured by precision mechatronic systems with electroelastic actuators. Cellular structures based on piezo actuators are used in nanomechanics. Piezo actuators for nanomechanical devices are used in photonics, adaptive optics, and nanotechnology in adjusting mirrors of annular laser gyroscopes, in information and energy transmission in laser systems, and for assembly and scanning purposes in electron, probe, and scanning-probe microscopes [1–12].

The structural-parametric model of an electroelastic actuator in nanomechanical devices provides a system of equations for Laplace transformation of the displacement of the tips of the device. Taking account of the actuator's electromechanical parameters, this system describes its structure and the conversion of the electrical energy to mechanical energy, as well as the corresponding Laplace transformations of the displacements and forces at the tips of the actuator.

In the present work, a structural-parametric model of an electroelastic actuator for nanomechanics is derived by means of mathematical physics, taking account of solutions of the wave equation for different boundary conditions. By means of Laplace transformation, the wave equation with partial derivatives of hyperbolic type is reduced to a linear differential equation. The transfer functions of nanoactuators are determined from the structural-parametric model of the actuator, in contrast to the expressions derived from equivalent electrical circuits of piezoconverters, piezoemitters, piezoreceivers, and piezovibrators with velocity and pressure as the output parameters [2, 8].

In the present work, the influence of the direct piezo effect on the voltage that acts at the actuator plates is taken into account; the change in elastic pliability due to the direct piezo effect is determined; and a structural-parametric model of an electroelastic actuator with feedback for use in nanomechanical applications is derived. We consider the following structural-parametric models and structures of the electroelastic actuator for nanomechanics: the case with an inverse piezo effect and constant elastic pliability and rigidity of the actuator; the case with inverse and direct piezo effects and variable elastic pliability and rigidity of the actuator; and the case with inverse and direct piezo effects and variable elastic pliability and rigidity of the actuator, when the direct piezo effect modifies the voltage on the actuator plates. On account of the reaction of the piezo actuator in the presence of the direct piezo effect, with different types of control (voltage or current control), the elastic pliability and rigidity of the actuator vary. Together with the piezomodulus, these are the main actuator parameters. When the tip of the piezo actuator moves at high speed, we take account of the influence of this speed through the direct piezo effect on the current through the piezo actuator and the voltage at its plates. For purposes of piezo actuator design and use in nanomechanics, we solve the wave equation with different boundary conditions and find the structural-parametric model of the actuator and its transfer functions in different operational frequency ranges.

The structural-parametric model and structure with feedback obtained here for a nanomechanical piezo actuator clearly reflect its ability to convert electrical energy to mechanical energy and the mutual dependence of its electromechanical parameters.

STRUCTURAL-PARAMETRIC MODEL AND TRANSFER FUNCTIONS OF ACTUATOR

We now consider the deformation of an electroelastic actuator corresponding to its stress state. If electric field strength E is created in the piezo actuator, strain S and mechanical stress T will appear. Correspondingly, if mechanical stress T is created in the piezo actuator, we will observe the appearance of electrical induction D and electrical charge at the plates of the piezo actuator.

The electroelastic equations of the nanomechanical actuators in the case of the inverse and direct piezo effect take the general form [6–8]

$$S_i = d_{mi}E_m + s_{ij}^E T_j;$$

$$D_m = d_{mi}T_i + \varepsilon_{mk}^T E_k,$$

where $i,\ j = 1, 2, \ldots, 6$ and $m,\ k = 1, 2, 3$; S_i is the relative displacement of the actuator cross section along the i axis; d_{mi} is the piezomodulus for a generalized piezo effect; $E_m(t) = U(t)/\delta$ is the electric field strength along the m axis; $U(t)$ is the voltage at the actuator plates; t is the time; δ is the actuator thickness; s_{ij}^E is the elastic pliability when $E = \text{const}$; T_j is the mechanical stress along the j axis; $D_m(t)$ is the electrical induction along the m axis; and ε_{mk}^T is the dielectric permittivity when $T = \text{const}$.

The equation for the forces on the face of the electroelastic actuator takes the form

$$TS_0 = F + M\frac{d^2\xi(x,t)}{dt^2},$$

where S_0 is the actuator area; F is the external force on the actuator; and M is the mass being moved.

In calculations of the electroelastic actuator, we use the wave equation describing wave propagation in a long line with damping but no distortion [6–8, 12]. By Laplace transformation, the wave equation with partial derivatives of hyperbolic type is reduced to a linear differential equation with parameter p, where p is the Laplacian operator [9–12].

By applying the Laplace transformation to the wave equation, in the case of null initial conditions, we obtain a second-order linear differential equation in the form

$$\frac{d^2\Xi(x,p)}{dx^2} - \gamma^2\Xi(x,p) = 0.$$

Its solution is the function

$$\Xi(x,p) = Ce^{-x\gamma} + Be^{x\gamma}, \quad (1)$$

where $\Xi(x,p)$ is the Laplace transformation of the displacement of the actuator's cross section; $\gamma = p/c^E + \alpha$ is the propagation coefficient; c^E is the speed of sound in the actuator when $E = \text{const}$; and α is the damping factor, taking account of the damping due to energy scattering with the heat losses in wave propagation.

The structural-parametric model of a piezo actuator with voltage control is derived by solving the second-order linear differential equation, the equation of the inverse piezo effect, and the equation for the forces at the actuator's faces.

In solving the linear differential equation, the coefficients C and B are determined on the basis of the conditions

$$\Xi(0,p) = \Xi_1(p) \text{ when } x = 0;$$

$$\Xi(l,p) = \Xi_2(p) \text{ when } x = l.$$

We find that

$$C = \left(\Xi_1 e^{l\gamma} - \Xi_2\right)/[2\sinh(l\gamma)];$$

$$\text{and } B = \left(\Xi_2 - \Xi_1 e^{-l\gamma}\right)/[2\sinh(l\gamma)].$$

The solution of the linear differential equation takes the form

$$\Xi(x,p) = \{\Xi_1(p)\sinh[(l-x)\gamma] + \Xi_2(p)\sinh(x\gamma)\}/\sinh(l\gamma).$$

The equations for the forces at the actuator's faces are

$$T_j(0,p)S_0 = F_1(p) + M_1p^2\Xi_1(p) \text{ when } x = 0;$$

$$T_j(l,p)S_0 = -F_2(p) - M_2p^2\Xi_2(p) \text{ when } x = l.$$

We obtain a system of equations for the mechanical stress of the electroelastic actuator when $x = 0$ and $x = l$

$$\begin{cases} T_j(0,p) = \dfrac{1}{s_{ij}^E}\left.\dfrac{d\Xi(x,p)}{dx}\right|_{x=0} - \dfrac{d_{mi}}{s_{ij}^E}E_m(p); \\ T_j(l,p) = \dfrac{1}{s_{ij}^E}\left.\dfrac{d\Xi(x,p)}{dx}\right|_{x=l} - \dfrac{d_{mi}}{s_{ij}^E}E_m(p). \end{cases} \quad (2)$$

On the basis of Eq. (2), we may write the following structural-parametric model of the electroelastic actuator with a generalized piezo effect, in the case of voltage control and the parametric structure in Fig. 1

$$\begin{cases} \Xi_1(p) = \left[1/\left(M_1 p^2\right)\right]\left\{-F_1(p) + \left(1/\chi_{ij}^E\right)\left[d_{mi}E_m(p)\right.\right. \\ \left.\left. -\ [\gamma/\sinh(l\gamma)][\cosh(l\gamma)\Xi_1(p) - \Xi_2(p)]\right]\right\}; \\ \Xi_2(p) = \left[1/\left(M_2 p^2\right)\right]\left\{-F_2(p) + \left(1/\chi_{ij}^E\right)\left[d_{mi}E_m(p)\right.\right. \\ \left.\left. -[\gamma/\sinh(l\gamma)]\left[\cosh(l\gamma)\Xi_2(p) - \Xi_1(p)\ \right]\right]\right\}, \end{cases} \tag{3}$$

where $\chi_{ij}^E = s_{ij}^E / S_0$.

After appropriate transformations, we obtain

$$\begin{cases} \Xi_1(p) = \left[1/\left(M_1 p^2\right)\right]\left\{-F_1(p) + \left(1/\chi_{ij}^E\right)\left[d_{mi}E_m(p)\right.\right. \\ \left.\left. -\ \gamma\Xi_1(p)/\tanh(l\gamma) + \gamma\Xi_2(p)/\sinh(l\gamma)\right]\right\}; \\ \Xi_2(p) = \left[1/\left(M_2 p^2\right)\right]\left\{-F_2(p) + \left(1/\chi_{ij}^E\right)\left[d_{mi}E_m(p)\right.\right. \\ \left.\left. -\ \gamma\Xi_2(p)/\tanh(l\gamma) + \gamma\Xi_1(p)/\sinh(l\gamma)\right]\right\}. \end{cases} \tag{4}$$

We may write Eq. (3) in the form

$$\begin{cases} \Xi_1(p) = \left[1/\left(M_1 p^2\right)\right]\left\{-F_1(p) + C_{ij}^E l\left[d_{mi}E_m(p)\right.\right. \\ \left.\left. -\ [\gamma/\sinh(l\gamma)][\cosh(l\gamma)\Xi_1(p) - \Xi_2(p)]\right]\right\}; \\ \Xi_2(p) = \left[1/\left(M_2 p^2\right)\right]\left\{-F_2(p) + C_{ij}^E l\left[d_{mi}E_m(p)\right.\right. \\ \left.\left. -\ [\gamma/\sinh(l\gamma)][\cosh(l\gamma)\Xi_2(p) - \Xi_1(p)]\right]\right\}, \end{cases}$$

where $C_{ij}^E = S_0/\left(s_{ij}^E l\right) = 1/\left(\chi_{ij}^E l\right)$ is the rigidity of the actuator with a generalized piezo effect, in the case of voltage control.

On the basis of the structural-parametric model, we may determine the transfer functions of the actuator. Solution of Eq. (3) for the displacement of two faces in an actuator with a generalized piezo effect, in the case of voltage control, yields

$$\begin{cases} \Xi_1(p) = W_{11}(p)E_m(p) \\ +\ W_{12}(p)F_1(p) + W_{13}(p)F_2(p); \\ \Xi_2(p) = W_{21}(p)E_m(p) \\ +\ W_{22}(p)F_1(p) + W_{23}(p)F_2(p). \end{cases} \tag{5}$$

In Eq. (5), the generalized transfer functions take the form

$$W_{11}(p) = \Xi_1(p)/E_m(p)$$
$$= d_{mi}\left[M_2\chi_{ij}^E p^2 + \gamma\tanh(l\gamma/2)\right]/A_{ij};$$
$$W_{21}(p) = \Xi_2(p)/E_m(p)$$
$$= d_{mi}\left[M_1\chi_{ij}^E p^2 + \gamma\tanh(l\gamma/2)\right]/A_{ij};$$
$$W_{12}(p) = \Xi_1(p)/F_1(p)$$
$$= -\chi_{ij}^E\left[M_2\chi_{ij}^E p^2 + \gamma/\tanh(l\gamma)\right]/A_{ij};$$

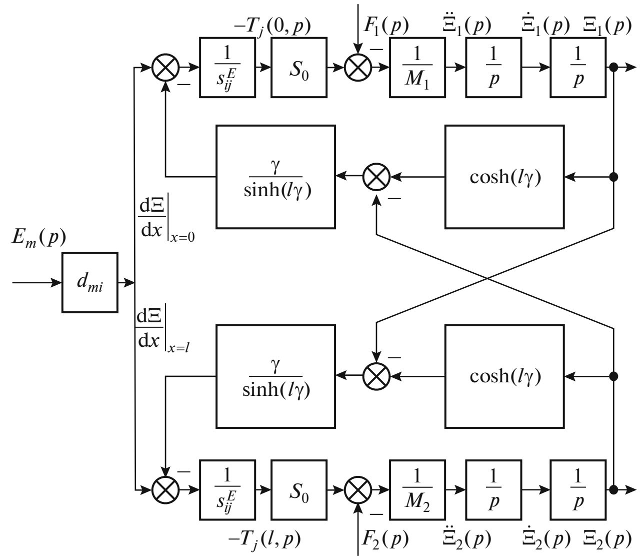

Fig. 1. Structure of electroelastic actuator with voltage control and zero resistance of the power source.

$$W_{13}(p) = \Xi_1(p)/F_2(p) = W_{22}(p)$$
$$= \Xi_2(p)/F_1(p) = \left[\chi_{ij}^E\gamma/\sinh(l\gamma)\right]/A_{ij};$$
$$W_{23}(p) = \Xi_2(p)/F_2(p)$$
$$= -\chi_{ij}^E\left[M_1\chi_{ij}^E p^2 + \gamma/\tanh(l\gamma)\right]/A_{ij},$$

where $\chi_{ij}^E = s_{ij}^E / S_0$ and

$$A_{ij} = M_1 M_2\left(\chi_{ij}^E\right)^2 p^4$$
$$+ \left\{(M_1 + M_2)\chi_{ij}^E/\left[c^E\tanh(l\gamma)\right]\right\}p^3$$
$$+ \left[(M_1 + M_2)\chi_{ij}^E\alpha/\tanh(l\gamma) + 1/\left(c^E\right)^2\right]p^2$$
$$+\ 2\alpha p/c^E + \alpha^2.$$

From Eq. (5), we obtain the matrix equation

$$\begin{pmatrix}\Xi_1(p)\\ \Xi_2(p)\end{pmatrix} = \begin{pmatrix}W_{11}(p) & W_{12}(p) & W_{13}(p)\\ W_{21}(p) & W_{22}(p) & W_{23}(p)\end{pmatrix}\begin{pmatrix}E_m(p)\\ F_1(p)\\ F_2(p)\end{pmatrix}.$$

We now consider the influence of the actuator's reaction associated with the counteremf in the direct generalized piezo effect with static deformation.

The maximum force $F_{\max}$ and mechanical stress $T_{j\max}$ developed by the piezo actuator with a generalized piezo effect in the case of a voltage source are as follows

$$F_{\max} = U\frac{1}{\delta}d_{mi}\frac{S_0}{s_{ij}^E};$$

$$\frac{F_{\max}}{S_0}s_{ij}^E = E_m d_{mi};$$

$$T_{j\max} s_{ij}^E = E_m d_{mi}.$$

Hence

$$T_{j\max} = E_m d_{mi} \big/ s_{ij}^E;$$

$$F_{\max} = E_m d_{mi} S_0 \big/ s_{ij}^E.$$

To simplify the equations, we use the electromechanical coupling coefficient [8, 10–12]

$$k_{mi} = d_{mi} \Big/ \sqrt{\varepsilon_{mk}^T s_{ij}^E}.$$

Then, for a piezo actuator of TsTS or PZT piezoceramic, we obtain the electromechanical coupling coefficients for the transverse, longitudinal, and shear piezo effects in the form

$$k_{31} = d_{31} \Big/ \sqrt{\varepsilon_{33}^T s_{11}^E};$$

$$k_{33} = d_{33} \Big/ \sqrt{\varepsilon_{33}^T s_{33}^E};$$

$$k_{15} = d_{15} \Big/ \sqrt{\varepsilon_{11}^T s_{55}^E}.$$

The maximum force $F_{\max}$ and maximum stress $T_{j\max}$ developed by an actuator with a generalized piezo effect, in the case of voltage control, may then be derived

$$\left.\begin{aligned} &F_{\max} = U \frac{1}{\delta} d_{mi} \frac{S_0}{s_{ij}^E} + F_{\max} \frac{1}{S_0} d_{mi} S_p \frac{1}{\varepsilon_{mk}^T S_p / \delta} \frac{1}{\delta} d_{mi} \frac{S_0}{s_{ij}^E};\\ &\frac{F_{\max}}{S_0} s_{ij}^E \left(1 - \frac{d_{mi}^2}{\varepsilon_{mk}^T s_{ij}^E}\right) = E_m d_{mi};\\ &k_{mi}^2 = \frac{d_{mi}^2}{\varepsilon_{mk}^T s_{ij}^E};\;\; T_{j\max}\left(1 - k_{mi}^2\right) s_{ij}^E = E_m d_{mi}. \end{aligned}\right\} \quad (6)$$

Hence, we may write

$$\left.\begin{aligned} &T_{j\max} s_{ij}^D = E_m d_{mi};\\ &s_{ij}^D = \left(1 - k_{mi}^2\right) s_{ij}^E = k_s s_{ij}^E;\\ &k_s = 1 - k_{mi}^2 = s_{ij}^D \big/ s_{ij}^E,\;\; k_s > 0, \end{aligned}\right\}; \quad (7)$$

where k_s characterizes the change in elastic pliability.

From Eqs. (6) and (7)

$$F_{\max} = E_m d_{mi} S_0 \Big/ \left(s_{ij}^E k_s\right) = E_m d_{mi} S_0 \big/ s_{ij}^D;$$

$$T_{j\max} = E_m d_{mi} \big/ s_{ij}^D.$$

The actuator's elastic pliability s_{ij} satisfies the condition $s_{ij}^E > s_{ij} > s_{ij}^D$, while $s_{ij}^E / s_{ij}^D \le 1{,}2$. Correspondingly, $C_{ij}^E < C_{ij} < C_{ij}^D$, where $C_{ij} = S_0 / (s_{ij} l)$ is the rigidity of the piezo actuator; $C_{ij}^E = S_0 / \left(s_{ij}^E l\right)$ is its rigidity with voltage control; and $C_{ij}^D = S_0 / \left(s_{ij}^D l\right)$ is its rigidity with current control [10]. With open electrodes, the rigidity of the piezo actuator is greater than with closed electrodes. With increase in resistance of the power source and the matching circuits, the elastic pliability of the actuator declines, while its rigidity increases.

For a piezo actuator with a generalized piezo effect in the case of a power source with finite resistance, taking of the direct piezo effect, we may write the maximum force of the actuator in the form

$$F_{\max} = U \frac{1}{\delta} d_{mi} \frac{S_0}{s_{ij}^E} + F_{\max} \frac{1}{S_0} d_{mi} S_p \frac{1}{\varepsilon_{mk}^T S_p / \delta} k_u \frac{1}{\delta} d_{mi} \frac{S_0}{s_{ij}^E}.$$

Hence

$$\frac{F_{\max}}{S_0} s_{ij}^E \left(1 - \frac{d_{mi}^2 k_u}{\varepsilon_{mk}^T s_{ij}^E}\right) = E_m d_{mi};$$

$$T_{j\max}\left(1 - k_{mi}^2 k_u\right) s_{ij}^E = E_m d_{mi},\quad 0 \le k_u \le 1,$$

where the coefficient k_u characterizes the type of control (voltage or current control). In the case of current control, $k_u|_{R\to\infty} = 1$; in the case of voltage control, $k_u|_{R\to 0} = 0$.

The elastic pliability takes the form

$$s_{ij} = \left(1 - k_{mi}^2 k_u\right) s_{ij}^E = k_s s_{ij}^E;$$

$$k_s = 1 - k_{mi}^2 k_u,\quad k_s > 0;$$

$$\left.\left(1 - k_{mi}^2\right)\right|_{R\to\infty} \le k_s \le 1|_{R\to 0},\quad k_s|_{R\to\infty} = 1 - k_{mi}^2,$$

$$k_s|_{R\to 0} = 1,$$

where the coefficient k_s characterizes the change in elastic pliability.

For a piezo actuator with a generalized piezo effect in the case of a power source with finite resistance, when the structural-parametric model includes feedback with respect to the force (Fig. 2), we may write

$$U_{F\alpha}(p) = \frac{k_u (l/\delta) d_{mi}}{C_0} F_\alpha(p),\quad \alpha = 1, 2.$$

In the case of current control, when the resistance of the source is infinite, $k_u|_{R\to\infty} = 1$.

For an electroelastic actuator with current control, the mechanical stress when $x = 0$ and $x = l$ may be written in the form

$$\begin{cases} T_j(0, p) = \dfrac{1}{s_{ij}^D} \left.\dfrac{d\Xi(x, p)}{dx}\right|_{x=0} - \dfrac{g_{mi}}{s_{ij}^D} D_m(p);\\ T_j(l, p) = \dfrac{1}{s_{ij}^D} \left.\dfrac{d\Xi(x, p)}{dx}\right|_{x=l} - \dfrac{g_{mi}}{s_{ij}^D} D_m(p), \end{cases}$$

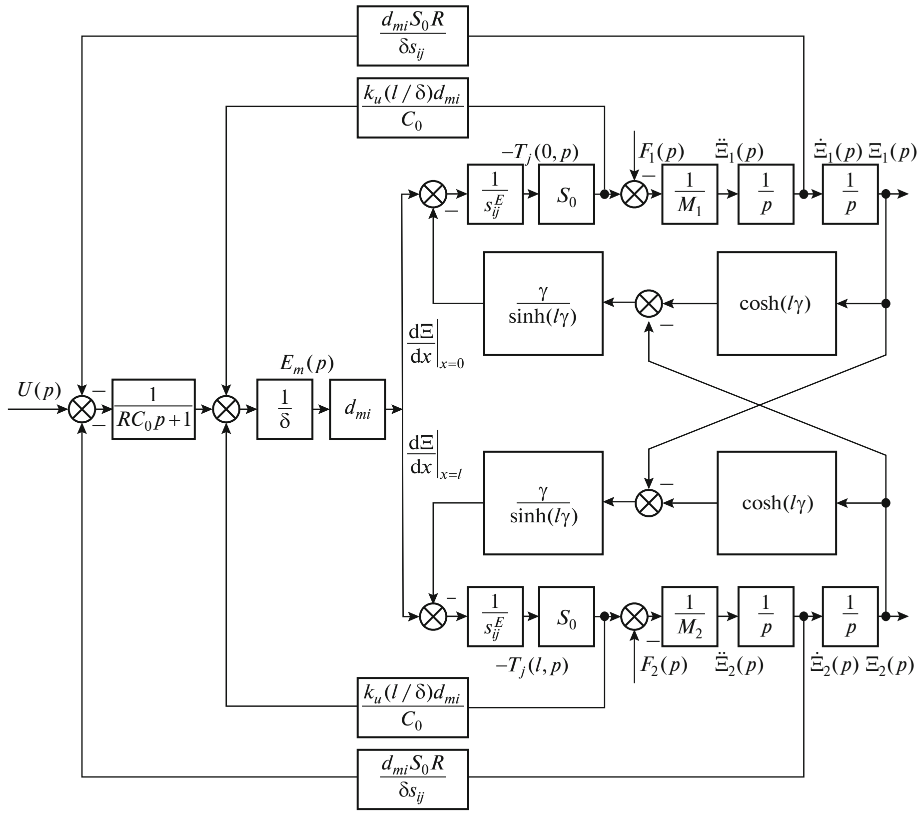

Fig. 2. Structure of electroelastic actuator with voltage control and finite resistance of the power source.

and thus the structural-parametric model of an electroelastic actuator in the case of a generalized piezo effect and current control takes the form

$$\begin{cases} \Xi_1(p) = \left[1\big/\left(M_1p^2\right)\right]\left\{-F_1(p) + \left(1\big/\chi_{ij}^D\right)\left[g_{mi}D_m(p)\right.\right. \\ \left.\left.-\left[\gamma/\sinh(l\gamma)\right]\left[\cosh(l\gamma)\Xi_1(p) - \Xi_2(p)\right]\right]\right\}; \\ \Xi_2(p) = \left[1\big/\left(M_2p^2\right)\right]\left\{-F_2(p) + \left(1\big/\chi_{ij}^D\right)\left[g_{mi}D_m(p)\right.\right. \\ \left.\left.-\left[\gamma/\sinh(l\gamma)\right]\left[\cosh(l\gamma)\Xi_2(p) - \Xi_1(p)\right]\right]\right\}, \end{cases} \tag{8}$$

where $\chi_{ij}^D = s_{ij}^D \big/ S_0$.

From Eq. (8), we obtain

$$\begin{cases} \Xi_1(p) = W_{11}(p)D_m(p) \\ + W_{12}(p)F_1(p) + W_{13}(p)F_2(p); \\ \Xi_2(p) = W_{21}(p)D_m(p) \\ + W_{22}(p)F_1(p) + W_{23}(p)F_2(p), \end{cases}$$

where the transfer functions are

$$W_{11}(p) = \Xi_1(p)\big/D_m(p) = g_{mi}\left[M_2\chi_{ij}^Dp^2 + \gamma\tanh(l\gamma/2)\right]\big/A_{ij};$$

$$W_{21}(p) = \Xi_2(p)\big/D_m(p) = g_{mi}\left[M_1\chi_{ij}^Dp^2 + \gamma\tanh(l\gamma/2)\right]\big/A_{ij};$$

$$W_{12}(p) = \Xi_1(p)\big/F_1(p) = -\chi_{ij}^D\left[M_2\chi_{ij}^Dp^2 + \gamma/\tanh(l\gamma)\right]\big/A_{ij};$$

$$W_{13}(p) = \Xi_1(p)\big/F_2(p) = W_{22}(p) = \Xi_2(p)\big/F_1(p) = \left[\chi_{ij}^D\gamma\big/\sinh(l\gamma)\right]\big/A_{ij};$$

$$W_{23}(p) = \Xi_2(p)\big/F_2(p) = -\chi_{ij}^D\left[M_1\chi_{ij}^Dp^2 + \gamma/\tanh(l\gamma)\right]\big/A_{ij}.$$

Here $\chi_{ij}^D = s_{ij}^D \big/ S_0$ and

$$A_{ij} = M_1M_2\left(\chi_{ij}^D\right)^2p^4 + \left\{(M_1+M_2)\chi_{ij}^D\big/\left[c^D\tanh(l\gamma)\right]\right\}p^3 + \left[(M_1+M_2)\chi_{ij}^D\alpha\big/\tanh(l\gamma) + 1\big/\left(c^D\right)^2\right]p^2 + 2\alpha p\big/c^D + \alpha^2.$$

Introducing the control parameter $\Psi = E, D$ for the actuator, we now write the transfer functions in the general form

$$W_{11}(p) = \Xi_1(p)\big/E_1(p) = d_{mi}\left[M_2\chi_{ij}^\Psi p^2 + \gamma\tanh(l\gamma/2)\right]\big/A_{ij};$$

$$W_{21}(p) = \Xi_2(p)/E_m(p)$$
$$= d_{mi}\left[M_1\chi_{ij}^{\Psi}p^2 + \gamma\tanh(l\gamma/2)\right]/A_{ij};$$
$$W_{12}(p) = \Xi_1(p)/F_1(p)$$
$$= -\chi_{ij}^{\Psi}\left[M_2\chi_{ij}^{\Psi}p^2 + \gamma/\tanh(l\gamma)\right]/A_{ij};$$
$$W_{13}(p) = \Xi_1(p)/F_2(p) = W_{22}(p)$$
$$= \Xi_2(p)/F_1(p) = \left[\chi_{ij}^{\Psi}\gamma/\sinh(l\gamma)\right]/A_{ij};$$
$$W_{23}(p) = \Xi_2(p)/F_2(p)$$
$$= -\chi_{ij}^{\Psi}\left[M_1\chi_{ij}^{\Psi}p^2 + \gamma/\tanh(l\gamma)\right]/A_{ij},$$

where $\chi_{ij}^{\Psi} = s_{ij}^{\Psi}/S_0$; and

$$A_{ij} = M_1M_2\left(\chi_{ij}^{\Psi}\right)^2 p^4$$
$$+ \left\{(M_1 + M_2)\chi_{ij}^{\Psi}/\left[c^{\Psi}\tanh(l\gamma)\right]\right\}p^3$$
$$+ \left[(M_1 + M_2)\chi_{ij}^{\Psi}\alpha/\tanh(l\gamma) + 1/\left(c^{\Psi}\right)^2\right]p^2$$
$$+ 2\alpha p/c^{\Psi} + \alpha^2.$$

Hence, the structural-parametric model for an electroelastic actuator with a generalized piezo effect (Fig. 1) takes the form

$$\begin{cases} \Xi_1(p) = \left[1/\left(M_1p^2\right)\right]\left\{-F_1(p) + \left(1/\chi_{ij}^{\Psi}\right)\left[d_{mi}\Psi_m(p)\right.\right. \\ \left.\left. -[\gamma/\sinh(l\gamma)]\ [\cosh(l\gamma)\Xi_1(p) - \Xi_2(p)]\right]\right\}; \\ \Xi_2(p) = \left[1/\left(M_2p^2\right)\right]\left\{-F_2(p) + \left(1/\chi_{ij}^{\Psi}\right)\left[d_{mi}\Psi_m(p)\right.\right. \\ \left.\left. -[\gamma/\sinh(l\gamma)]\ [\cosh(l\gamma)\Xi_2(p) - \Xi_1(p)]\right]\right\}, \end{cases} \quad (9)$$

where $\chi_{ij}^{\Psi} = s_{ij}^{\Psi}/S_0$.

To take account of the velocity of the actuator with a generalized piezo effect associated with the counteremf due to the direct piezo effect, we introduce feedback in the structural-parametric model

$$U_{\dot{\Xi}\alpha}(p) = \frac{d_{mi}S_0R}{\delta s_{ij}}\dot{\Xi}_\alpha(p), \quad \alpha = 1, 2.$$

From Eq. (9), we obtain the transfer functions for an electroelastic actuator fixed at one end in the frequency range $0 < \omega < 0.01c^{\Psi}/l$ when $M_2/m \gg 1$, in the case of an inertial load and voltage or current control

$$W_{21}(p) = \frac{\Xi_2(p)}{E_m(p)} = \frac{d_{mi}l}{\left(T_{ij}^{\Psi}\right)^2 p^2 + 2T_{ij}^{\Psi}\xi_{ij}^{\Psi}p + 1};$$

$$W_{23}(p) = \frac{\Xi_2(p)}{F_2(p)} = -\frac{1/C_{ij}^{\Psi}}{\left(T_{ij}^{\Psi}\right)^2 p^2 + 2T_{ij}^{\Psi}\xi i_{ij}^{\Psi}p + 1};$$

$$W_{21}(p) = \frac{\Xi_2(p)}{U(p)} = \frac{d_{mi}(l/\delta)}{\left(T_{ij}^{\Psi}\right)^2 p^2 + 2T_{ij}^{\Psi}\xi_{ij}^{\Psi}p + 1};$$

$$T_{ij}^{\Psi} = \sqrt{M_2 s_{ij}^{\Psi}l/S_0} = \sqrt{M_2/C_{ij}^{\Psi}};\ \ C_{ij}^{\Psi} = S_0/\left(s_{ij}^{\Psi}l\right);$$
$$\xi_{ij}^{\Psi} = \alpha\delta\sqrt{m/M_2}/3.$$

Here $T_{ij}^{\Psi} = T_{ij}^{E}, T_{ij}^{D}$, $\xi_{ij}^{\Psi} = \xi_{ij}^{E}, \xi_{ij}^{D}$, $C_{ij}^{\Psi} = C_{ij}^{E}, C_{ij}^{D}$, and $s_{ij}^{\Psi} = s_{ij}^{E}, s_{ij}^{D}$ are, respectively, the time constant, damping factors, rigidity, and elastic pliability of the actuator with control parameter $\Psi = E, D$, where E and D correspond to voltage and current control, respectively.

In the case of a power source with finite resistance and a generalized piezo effect, taking account of the elastic pliability and rigidity of the electroelastic actuator, the feedback with respect to the force may be written in the form

$$U_F(p) = \frac{k_u(l/\delta)d_{mi}}{C_0}F_2(p).$$

For current control, with infinite source resistance, $k_u|_{R\to\infty} = 1$; for voltage control, with zero source resistance, $k_u|_{R\to 0} = 0$.

Correspondingly, for an electroelastic actuator with a generalized piezo effect, in the case of a power source with finite resistance, we introduce the following feedback in the structural-parametric model

$$U_{\dot{\Xi}}(p) = \frac{d_{mi}S_0R}{\delta s_{ij}}\dot{\Xi}_2(p).$$

We now consider structures with distributed and point parameters for an electroelastic actuator fixed at one end with inertial elastic loading and voltage control, with finite source resistance. From Eqs. (4) and (5), as $M_1 \to \infty$, we obtain a structure with distributed parameters (Fig. 3).

After replacing the hyperbolic cotangent with two terms of the power series and introducing the coefficient k_d of the direct electroelastic effect and the coefficient k_r of the inverse electroelastic effect in Fig. 3 $\left(k_d = k_r = \dfrac{d_{mi}S_0}{\delta s_{ij}}\right)$, we obtain a structure with point parameters as $M_1 \to \infty$ (Fig. 4).

Transformation of Fig. 4 yields the structure with point parameters for the actuator in Fig. 5.

Taking account of the structure with point parameters, we find the transfer function in the form

$$W(p) = \frac{\Xi_2(p)}{U(p)} = \frac{k_r}{RC_0M_2p^3 + (M_2 + RC_0k_v)p^2 + (k_v + RC_0C_{ij} + RC_0C_e + Rk_rk_d)p + C_{ij} + C_e},$$

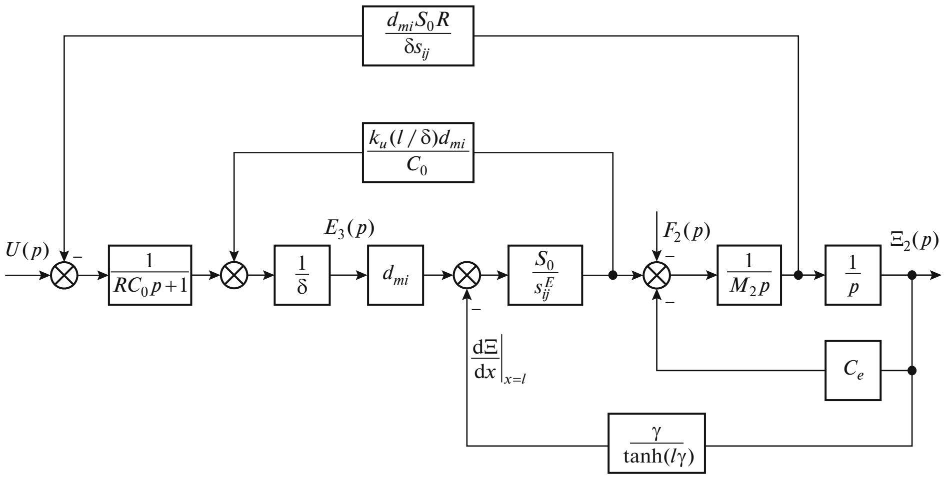

Fig. 3. Structure of an electroelastic actuator fixed at one end, with distributed parameters, in the case of an inertial elastic load and voltage control with finite resistance of the power source.

where $\Xi_2(p)$ and $U(p)$ are Laplace transforms of the tip motion and the voltage at the actuator plates; C_{ij} is the rigidity of the electroelastic actuator ($C_{ij}^E < C_{ij} < C_{ij}^D$); and k_ν is its damping factor.

When $Rk_r k_d \ll k_\nu$ or $Rk_r^2 \ll k_\nu$ we may write

$$W(p) = \frac{\Xi_2(p)}{U(p)} = \frac{k_r}{(RC_0 p + 1)\left(M_2 p^2 + k_\nu p + C_{ij} + C_e\right)}.$$

When $R = 0$, we write the transfer function in the form

$$W(p) = \frac{\Xi_2(p)}{U(p)} = \frac{k_r}{M_2 p^2 + k_\nu p + C_{ij}^E + C_e}$$

$$= \frac{k_r \big/ \left(C_{ij}^E + C_e\right)}{\left(\left(M_2 \big/ \left(C_{ij}^E + C_e\right)\right) p^2 + \left(k_\nu \big/ \left(C_{ij}^E + C_e\right)\right) p + 1\right)}.$$

Hence

$$W(p) = \frac{\Xi_2(p)}{U(p)} = \frac{d_{mi}(l/\delta)}{\left(1 + C_e/C_{ij}^E\right)\left(T_t^2 p^2 + 2T_t \xi_t p + 1\right)},$$

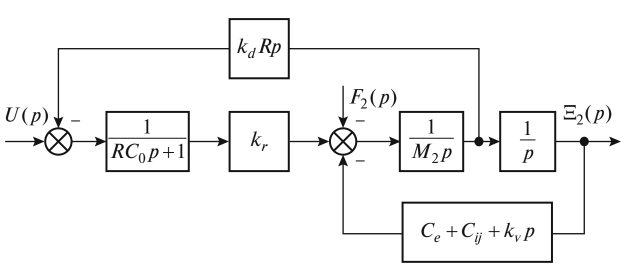

Fig. 4. Structure of an electroelastic actuator fixed at one end, with point parameters, in the case of an inertial elastic load and voltage control with finite resistance of the power source.

where

$$T_t = \sqrt{M_2 \big/ \left(C_{ij}^E + C_e\right)};$$

$$\xi_t = k_\nu \Big/ \left(2\left(C_{ij}^E + C_e\right)\sqrt{M_2\left(C_{ij}^E + C_e\right)}\right);$$

$$C_{ij} = S_0 \big/ \left(s_{ij}^E l\right) = 1 \big/ \left(\chi_{ij}^E l\right).$$

For the transfer function of an electroelastic actuator with a transvers piezo effect and voltage control, when $R = 0$

$$W(p) = \frac{\Xi_2(p)}{U(p)} = \frac{d_{31} h/\delta}{\left(1 + C_e/C_{11}^E\right)\left(T_t^2 p^2 + 2T_t \xi_t p + 1\right)}. \quad (10)$$

Where

$$T_t = \sqrt{M_2 \big/ \left(C_{11}^E + C_e\right)};$$

$$\xi_t = \alpha h^2 C_{11}^E \Big/ \left(3c^E \sqrt{M\left(C_{11}^E + C_e\right)}\right);$$

$$C_{11}^E = S_0 \big/ \left(s_{11}^E h\right) = 1 \big/ \left(\chi_{11}^E h\right).$$

In Eq. (10), δ and h are the actuator's thickness and height; and T_t and ξ_t are its time constant and damping coefficient.

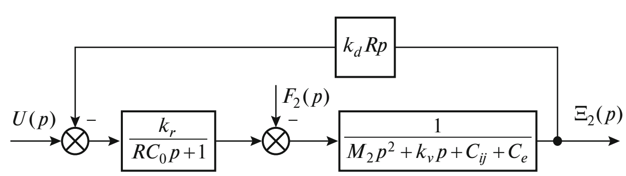

Fig. 5. Transformed structure of an electroelastic actuator fixed at one end, with point parameters, in the case of an inertial elastic load and voltage control with finite resistance of the power source.

From Eq. (10), by Laplace transformation, we determine the transient characteristic of an actuator with transvers piezo effect and voltage control

$$\xi(t) = \xi_m \left(1 - \frac{e^{-\frac{\xi_t t}{T_t}}}{\sqrt{1-\xi_t^2}} \sin(\omega_t t + \varphi_t) \right).$$

Here

$$\xi_m = \frac{d_{31}(l/\delta)U_m}{1 + C_e/C_{11}^E}; \quad \omega_t = \sqrt{1-\xi_t^2}\Big/T_t;$$
$$\varphi_t = \arctan\left(\sqrt{1-\xi_t^2}\Big/\xi_t\right),$$

where ξ_m is the steady displacement; and U_m is the voltage amplitude.

For a piezo actuator of TsTS ceramic that is rigidly fixed at one end, with a transvers piezo effect and voltage control, in the case of an inertial elastic load, when $M_1 \to \infty$ and $m \ll M_2$, we find that $\xi_m = 160$ nm and $T_t = 0.4 \times 10^{-3}$ s for a step voltage of amplitude $U_m = 50$ V and the following parameters: $d_{31} = 2 \times 10^{-10}$ m/V; $h/\delta = 20$; $M_2 = 4$ kg; $C_{11}^E = 2 \times 10^7$ N/m; and $C_e = 0.5 \times 10^7$ N/m.

CONCLUSIONS

We have determined the structural-parametric model and structure of an electroelastic actuator for nanomechanics with a generalized piezo effect, taking account of the counteremf due to the direct piezo effect. Structural-parametric models have been obtained for actuators with transverse, longitudinal, shear, and generalized piezo effects and with voltage and current control. The differences in the structural-parametric models for actuators with voltage and current control are established. On introducing feedback in the models, the conversion of electrical energy to mechanical energy in the actuator is clearly evident.

The maximum forces and stresses developed by nanomechanical piezo actuators with transverse, longitudinal, shear, and generalized piezo effects are determined, and the elastic pliability and rigidity of the actuator are determined for the different types of control (voltage or current control).

Likewise, transfer functions are determined for actuators with transverse, longitudinal, shear, and generalized piezo effects and voltage and current control.

REFERENCES

1. Schultz, J., Ueda, J., and Asada, H., *Cellular Actuators*, Oxford: Butterworth-Heinemann, 2017.
2. Cady, W.G., *Piezoelectricity: An Introduction to the Theory and Applications of Electromechanical Phenomena in Crystals*, New York: McGraw-Hill, 1946.
3. Afonin, S.M., Structural-parametric model and transfer functions of electroelastic actuator for nano- and microdisplacement, in *Piezoelectrics and Nanomaterials: Fundamentals, Developments and Applications*, Parinov, I.A., Ed., New York: Nova Science, 2015, ch. 9, pp. 225–242.
4. Nikol'skii, A.A., *Tochnye dvukhkanal'nye sledyashchie elektroprivody s p'ezokompensatorami* (Precise Two-Channel Tracking Electric Actuators with Piezocompensators), Moscow: Energoatomizdat, 1988.
5. Panich, A.E., *P'ezokeramicheskie aktyuatory* (Piezoceramic Actuators), Rostov-on-Don: Yuzhn. Fed. Univ., 2008.
6. Afonin, S.M., Structural parametric model of a piezoelectric nanodisplacement transducer, *Dokl. Phys.*, 2008, vol. 53, no. 3, pp. 137–143.
7. Afonin, S.M., Structural-parametric model of a piezonanomotor, *Vestn. Mashinostr.*, 2001, no. 5, pp. 29–33.
8. *Physical Acoustics: Principles and Methods*, Vol. 1, *Part A: Methods and Devices*, Mason, W., Ed., New York: Academic, 1964.
9. Polyanin, A.D., *Spravochnik po lineinym uravneniyam matematicheskoi fiziki* (Handbook on Linear Equations in Mathematical Physics), Moscow: Fizmatlit, 2001.
10. Afonin, S.M., Compression and elastic-pliability diagrams of nano-scale piezomotors, *Vestn. Mashinostr.*, 2003, no. 9, pp. 16–18.
11. Afonin, S.M., Multilayer piezoelectric nano- and micromotors with a transverse piezo effect, *Russ. Eng. Res.*, 2014, vol. 34, no. 11, pp. 671–679.
12. Afonin, S.M., A structural-parametric model of electroelastic actuator for nano- and microdisplacement of mechatronic system, in *Advances in Nanotechnology*, Bartul, Z. and Trenor, J., Eds., New York: Nova Science, 2017, vol. 19, ch. 8, pp. 259–284.

Translated by B. Gilbert