

Minimization of Periodic Machining Errors

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Abstract—The factors responsible for periodic machining errors are analyzed, and corresponding mathematical models are proposed. Approaches to decreasing the periodic machining errors in machine tools are suggested.

Keywords: periodic machining errors, mechanical vibrations, vibration frequency, surface undulation, amplitude, height, pitch

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For machine tools, the periodic machining errors may be classified as follows: size variation within a batch of workpieces; deviation in the position of several surfaces for a single workpiece; and deviation in surface profile. Periodic errors in the surface profile may take the form of waves, characterized in terms of quantity, profile, and height. For example, a single wave on a surface of revolution may be regarded as eccentricity; two waves as an oval; and three or more waves as facets. They are all forms of noncircularity. Likewise, surface roughness with periodic characteristics may be regarded as a periodic machining error. A form of periodic machining error intermediate between a deviation in shape and in roughness is undulation.

The influence of periodic machining errors on the performance of joints in machine parts is negative, generally speaking. However, the influence of the parameters of periodic machining errors differs. For example, the wear of frictional pairs increases linearly with increase in height of the undulation and almost parabolically with increase in the pitch. The load capacity depends in a complex manner on the undulation: it declines sharply on moving away from certain optimal ratios of the undulation pitch and height. Certain profiles and tip radii significantly affect the microhardness distribution (Fig. 1).

The influence of periodic machining errors on the vibration and noise of machines is even more complex. For example, the vibrational acceleration of bearing balls increases linearly with increase in height of the undulation but depends quadratically on its pitch. In terms of the noise of operating couplings (contacting surfaces), periodic machining errors have significantly greater influence on higher spectral components than on lower components. Hence, it is not enough to characterize the periodic machining errors in the working surfaces of machine parts in terms solely of the pitch

and height of the fundamental harmonic, say. We also need to know the characteristics of all the spectral components.

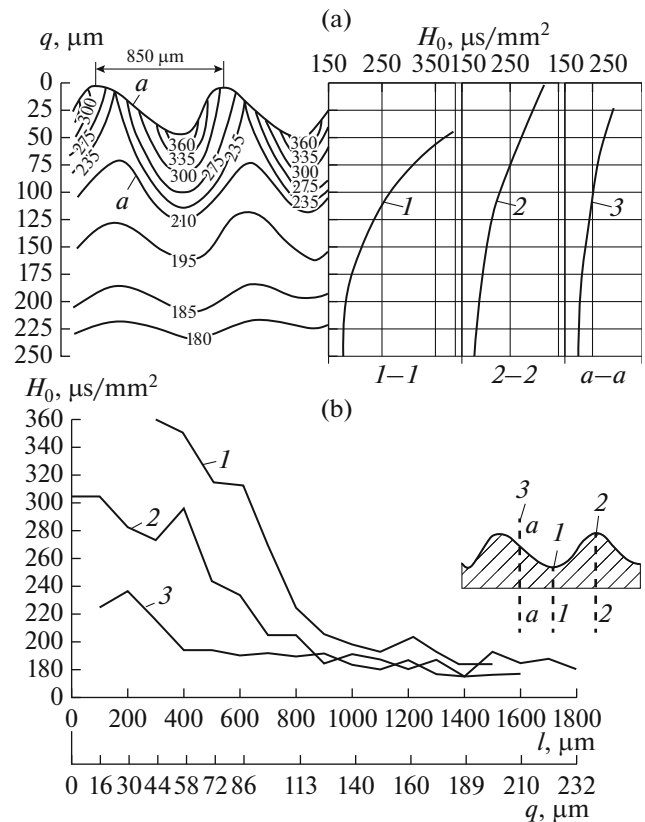


Fig. 1. Distribution of the cold hardening over the wave profile [6] (a) and depth of cold hardening q and degree of hardening on turning (b): $v = 175 \text{ m/min}$; $s = 0.3 \text{ mm/turn}$; $t = 1 \text{ mm}$; (1) at trough 1–1; (2) at crest 2–2; (3) over line a – a ; H_0 , microhardness; l , distance from tip to plane of transverse section.

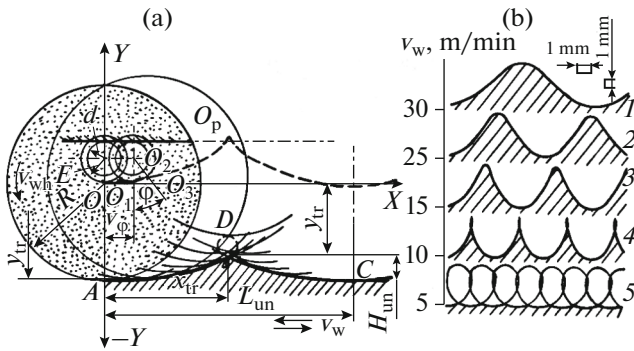


Fig. 2. Formation of undulation in plane grinding on account of wheel vibration (a); and influence of workpiece velocity v_w on undulation: $v_w = 30$ (1), 25 (2), 15 (3), 10 (4), and 5 (5) m/min (b) [22].

On the eve of the sixth technological age, as Kondrat'ev argues (some say it has already begun [1]), we need to review the progress already made in terms of machining precision—especially periodic machining errors—and formulate goals for the future. That is the purpose of this article.

The theory of surface precision and quality was developed in the second scientific age (1941–1970) [2]. Most researchers agreed that vibration of individual components in the elastic mechanical system is responsible for undulation, regarded as the basic form of periodic machining error [3–31]. Various factors may produce the vibration. In grinding, undulation may appear on account of clogging of the grinding wheel; reverse rotation of the cylindrical workpiece; or imbalance in the wheel [4].

Vibration also arises on account of jolts associated with poor belt adjustment, gear wobble, unsatisfactory balancing of the wheel, and inadequate wheel trimming. Such vibration is accompanied by undulation [5].

Surface undulation is due to vibrational displacement of points within the machine tool and depends on its vibrational stability, wheel imbalance, nonuniform wheel feed, incorrect trimming, and clogging [7].

High-frequency tangential vibration of the tool (for example, in turning)—that is, self-oscillation—produces undulation [6]. In self-oscillation, periodic machining errors in the form of surface roughness are converted to undulation with increasing height. With different values of the feed and the cutting depth and different cutter alloys, we note change in the extrema of the cutting speeds, which influence the undulation height [6]. A very important finding (Fig. 1) is that the appearance of undulation is accompanied by change in the physicomachanical properties—for example, the microhardness—not only of the surface layer where the undulation is seen but also in a subsurface layer. These changes may penetrate to a depth that is orders of magnitude greater than the height of the

undulation. That is the primary danger of undulation in terms of the surface performance in couplings.

Cutter vibrations were measured in [8] and used in [28]. Such vibrations tend to decrease. In centerless grinding, the formation of undulation is promoted by large grinding depth and transverse workpiece supply; high workpiece speeds; incorrect positioning of the workpiece's center of rotation relative to the centers of the wheel; large inclination of the blade's supporting surface; wheel imbalance; large wheel hardness; small grain size of the wheel; and nonuniform wear of the wheel [7].

Periodic machining errors with a relatively small number of waves are due to a different factor: elastic deformation of the workpiece by the forces applied during its attachment (item 1 in Table 1) [29].

The simplest formula for the undulation height was presented in [9] (item 2 in Table 1); it has since been used by many authors. Conditions ensuring undulation height less than the vibrational spread $2A$ were described in [10] (item 3 in Table 1). The same phenomenon was discovered independently in 1969 and described in [22] (item 13 in Table 1); it is sometimes known as wave clipping.

The formation of periodic machining errors as a result of the discreteness of the cutting process was analyzed at length in [30] (item 5 in Table 1). Practically all the basic cutting methods were investigated, and recommendations were made for the reduction of periodic machining errors.

The derivation of the undulation profile with specified center trajectory of the grinding wheel was presented in general form in [23].

The envelope of wheel positions with vibration under the action of the centrifugal force in grinding (Fig. 2) corresponds to the discriminant line described by the equation $\partial F(x, y, c)/\partial C = 0$, where x and y are the coordinates of the wheel's center trajectory.

For plane grinding, suppose that we solve the equation of a family of circles $(X_{un} - x)^2 + (Y_{un} - y)^2 - R^2 = 0$ and determine its derivative with respect to the parameter t , taking into account that x and y are functions of t : $(X_{un} - x)\dot{x} + (Y_{un} - y)\dot{y} = 0$. On that basis, we may find the coordinates of the undulation profile in general form

$$X_{un} = x \pm \frac{R\dot{y}}{\sqrt{(\dot{x})^2 + (\dot{y})^2}}; \tag{1}$$

$$Y_{un} = y \mp \frac{R\dot{x}}{\sqrt{(\dot{x})^2 + (\dot{y})^2}}. \tag{2}$$

On the basis of Eqs. (1) and (2), if we specify the variation in the relative trajectory of the tool and workpiece, we may find the equation of the undulation profile for the workpiece surface in any type of machining.

Table 1

No.	Operation; cause of error	Formulas for periodic machining errors	Date, source
1	Reaming; attachment forces	Height of undulation $H_{un} = \frac{0.242P_t(K+1)^3}{Eb(K-1)^3}$	1961 [29]
2	Wheel grinding; wheel imbalance	Height of undulation $H_{un} = A \left\{ 1 + \cos \left[(p-1) \frac{180^\circ}{n} \right] \right\}, 1 \leq p \leq m, \frac{n_{wh}}{n_w} = n + \frac{i}{m}$	1962 [9]
3	Wheel grinding; wheel imbalance Wave-clipping condition	Height of undulation $W_Z = A \left[1 - \cos \left\{ 0.79 \frac{v_w}{W_s} \sqrt{\frac{1}{A} \left(\frac{1}{d_s} + \frac{1}{D_s} \right)} \right\} \right],$ $0 \leq 0.79 \frac{v_w}{W_s} \sqrt{\frac{1}{A} \left(\frac{1}{d_s} + \frac{1}{D_s} \right)} \leq \pi$	1962 [10]
4	Wheel grinding; oval wheel	Height of undulation $H = (D_{max} - D_{min})/2, L = \pi dn/2N$	1963 [11]
5	Blade cutting; discreteness of cutting, turning by broad cutters	Undulation, height of undulation $H = \sqrt{(R-t)^2 + a_t^2 R/2t} - (R-t),$ $H \approx (R-t)/2 \tan^2 \delta, L = s_0/\tan \lambda$ Facet height $H = \pm L_{cm} \cos(\xi/2 - \sigma) \mp \sqrt{r^2 L_{cm}^2 \sin^2(\xi/2 - \theta) - (R-t)}$	1964 [30]
6	Wheel grinding; vibration	Height of undulation $H_{max} = \frac{l^2}{4} \left(\frac{1}{d} + \frac{1}{D} \right) = \frac{70v_n^2}{f^2} \left(\frac{1}{d} + \frac{1}{D} \right),$ $H = 2a_0 \cos^2 \frac{\pi l_t}{l} + l_t^2 \left(\frac{1}{d} + \frac{1}{D} \right)$	1965 [12]
7	Plunged grinding; forced vibration	Height of undulation $H_{max} = A \left\{ 1 + \cos \left[\frac{n\pi(i-1)}{f} \right] \sin \frac{a}{2} \right\}$	1967 [14]
8	Plane milling' mill wobble	Height of undulation $H = D/2 - \sqrt{(D/2)^2 - (S_0/2)^2}$	1968 [31]
9	Wheel grinding; vibration	Height of undulation $H_{max} = s_{li}/f (1/d + 1/D)$	1968 [17]
10	Wheel grinding; wheel imbalance	$H_n/2 = a_1 \sqrt{1 + 2a_{n-1} k k_{gr} \cos \gamma + a_{n-1}^2 k^2 k_{gr}^2}$	1968 [18]
11	Plane grinding; vibration of machining system.	Amplitude of undulation $H = 2K_{un} \sqrt{A^2 + B^2 - 2AB \cos \varphi},$ $A = \frac{h(d - \omega^2)}{(a - \omega^2)(d - \omega^2) - ac}, a = \frac{j_2 + k}{m_2}, c = \frac{j_2 + k}{m_1}, d = \frac{j_1 + j_2 + k}{m_1}, h = \frac{P_0}{k},$ $B = \frac{ch}{(a - \omega^2)(d - \omega^2) - ac}$	1968 [19]
12	Turning; workpiece and spindle imbalance	Facet height $H = K \frac{(f_n/f_e)^3}{\sqrt{[1 - (f_n/f_e)^2]^2 + 4D^2 (f_n/f_e)^2}}$	1969 [21]
13	Internal grinding; imbalance of electric motor, wave-clipping condition	Coordinates of undulating surface $X_{un} = t$ $+ \frac{rA \times 2\pi n_{mo}/v_n \times 1000 \cos(2\pi n_{mo}/v_w \times 1000)t}{\sqrt{1 + [A \times 2\pi n_{mo}/V_n \times 1000 \cos(2\pi n_{mo}/v_w \times 1000)t]^2}},$ $Y_{un} = A \sin \left(\frac{2\pi n_{mo}}{v_w \times 1000} \right) t - \frac{r}{\sqrt{1 + [A \times 2\pi n_{mo}/v_w \times 1000 \cos(2\pi n_{mo}/v_w \times 1000)t]^2}}$	1970 [22]

Table 1 (Contd.)

No.	Operation; cause of error	Formulas for periodic machining errors	Date, source
14	Grinding; noncircular wheel, dwelling process	Coordinates of undulating surface: $x = \rho \left[\cos \varphi + 2 \sin \frac{a}{2} \cos \left(\varphi + \frac{a}{2} \right) \right] + v_w t$, $y = \rho \left[\sin \varphi - 2 \sin \frac{a}{2} \sin \left(\varphi + \frac{a}{2} \right) \right]$, $H_{un} = \rho H r_{wh}, H_{un_1} e^{(1-i)k}$	1970 [23]
15	Turning; spindle imbalance	Noncircularity $\Delta R = \sqrt{x_0^2 - 2x_0 y_0 \sin \varphi + y_0^2} / 2$	1970 [24]
16	Grinding; vibration of spindle chuck on account of wheel eccentricity E	Coordinates of machined surface $X_{un} = E \sin \frac{2\pi t}{L} \pm \frac{v_w R}{60 v_{wh} L} t + \frac{ER \sin(2\pi t/L)}{E \cos(2\pi t/L) \pm v_w}$, $Y_{un} = E \left(\cos 2\pi t + \frac{1}{L_{un}} \right) - R, X_{un} = t + \frac{rA \times 2\pi \cos 2\pi t}{L_{un} L_{un}}$, $Y_{un} = A \sin \frac{2\pi t}{L_{un}} - R, L_{un} = \frac{v_{wh} \times 1000}{nk}$	1970 [23]
17	Internal grinding; wheel imbalance, incorrect wheel mounting	$R_{un} = \frac{j_{iny}}{j_{inx}}, H_{un_1} = 2 \frac{P}{j_{inx} r_k} \left(1 - \frac{1}{C} \right), H_{un_2} = 4 \left(\frac{M_1}{j_{ix}} \right) \frac{k\omega^2}{g}; k = \frac{\Delta\omega}{\omega}$, $H_{un_3} = 2(A_{gr} - A_n) = \frac{4BL}{l_c} \Sigma P_i$	1970 [27]
18	Wheel grinding with dwelling; forced vibration	Profile radius $R_{un} = \sqrt{\rho^2 + R^2 + \frac{2\rho^2 Rq}{\sqrt{\rho^2 q^2 + \mu^2}}}$, $\varphi_{un} = \arctan \frac{b \sin(\varphi q) - \arccos(\varphi q)}{b \cos(\varphi q) + \arcsin(\varphi q)}$	1970 [23]

In Fig. 2b (for plane grinding), we see that, with variation in the workpiece speed from 30 to 5 m/min, loops appear in its profile (compare curves 4 and 5).

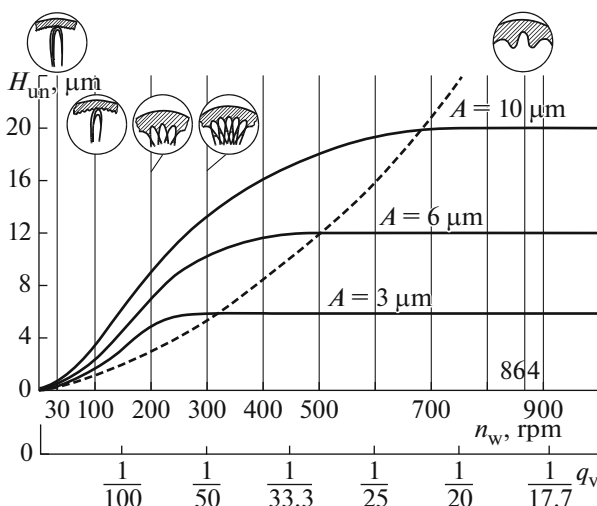


Fig. 3. Influence of the amplitude A of relative vibrations of the wheel and workpiece, the velocity ratio $q_v = v_w/v_{wh}$, and the workpiece speed n_w on the undulation height in internal ball grinding [23].

The wheel passes its contact point with the workpiece twice, and some of its tip is removed. This is known as clipping; the corresponding conditions were first derived for internal spherical grinding of ball-bearing races in [22].

It follows from Eqs. (1) and (2) that the formation of undulation may be controlled by selecting the optimal relative trajectory of the tool and workpiece. The key factors here are the static and dynamic rigidity of the system, the eigenfrequencies, and the velocity of the system components.

A new method of measuring the relative vibration of the system components was presented in [13]. In experiments regarding plane grinding on a 3B71M machine tool, such measurements showed that the grinding headstock, the grinding wheel and spindle, and the table with the plate and workpiece exhibit an extremely unfavorable combination of vibrational eigenfrequencies [23].

In internal ball grinding on an L3-5M machine tool (Fig. 3), two ranges of workpiece velocity (indicated by a dashed line) are observed. In the first, the height of the undulation is equal to the vibrational spread; in the second, it is less.

A new method of internal grinding has been proposed, such that the rocking motion of the headstock ceases in the course of dwelling. In this method, the dynamic rigidity K_d of the machine tool is increased. The height of the undulation is decreased from 0.87 to 0.46 μm [23]. New grinding cycles are proposed to decrease the periodic machining errors [23]. Table 1 presents mathematical models of the periodic machining errors.

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