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Determining Worm-Mill Profiles

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Abstract—The proposed mathematical model permits shaping of worm mills for the machining of parts with helical surfaces. The model includes a module generating matrices for the spatial transformation of coordinate systems. An algorithm is presented for formulating a model of the worm-mill surface. Shaping is completed in one stage, without determining the profile of the conjugate helical rack.

Keywords: worm mills, shaping, profiling, helical surfaces, mathematical model, running-in, numerical methods

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Today, parts with helical (screw) surfaces are mainly machined by means of disk tools. If several helical surfaces are present on the part, it is more efficient to use worm mills. However, the design of worm mills for that purpose is a complex task, which has yet to be fully understood.

Tooth-cutting worm mills are often produced on the basis of the properties of a common normal [1]. This method consists of two stages. The first is to determine the profile of the conjugate helical rack. This method is inapplicable if the position of the normal cannot be determined—for example, if the initial profile is specified by the coordinates of individual points or a spatial curve. In addition, it cannot be used for the machining of more complex surfaces, such as tapered helical surfaces.

These problems may be circumvented by employing numerical methods in which the profile of the worm mill is established in a single stage.

The proposed system includes four basic modules; (1) formulation of a model of the surface to be machined; (2) analysis of this model; (3) formulation of a model of the tool surface; (4) analysis of that model.

To solve specific production problems, we need only use one or two modules. For predesign analysis of a new tool for machining a complex part, it is best to use all four modules.

The first module formalizes the numerical representation of the points in the initial surface of the part, on the basis of a coordinate system tied to the tool's generating surface [2]. The initial data are as follows:

(1) the coordinates x_i , y_i and the number i of points on the surface of the part;

(2) the number f of coordinate transformations and the number (order) n of each transformation;

(3) the angular displacements xy, yz, zx and linear displacements Ax, Ay, Az characterizing the transformation of the coordinate systems.

The basis of the module is the initial matrix

$$MO = \begin{pmatrix} \cos(xy)\cos(zx) & \sin(-xy) & \sin(zx) & Ax \\ \sin(xy) & \cos(yz)\cos(xy) & \sin(-yz) & Ay \\ \sin(-zx) & \sin(yz) & \cos(zx)\cos(yz) & Az \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (1)$$

The program implementing the first module permits automatic formulation of the matrices $M1$, $M2$, $M3$, $M4$ from the initial matrix MO in Eq. (1). The matrices $M1$, $M2$, $M3$, $M4$ correspond to successive transformation of the coordinate systems from the profile of the part to the desired tool surface in accordance with the specified values of f and n and the numerical value characterizing each transformation.

In Fig. 1, we show the shaping of the worm mill for machining a part with a helical channel of arbitrary profile. This process includes four coordinate transformations ($f = 4$) in the following sequence.

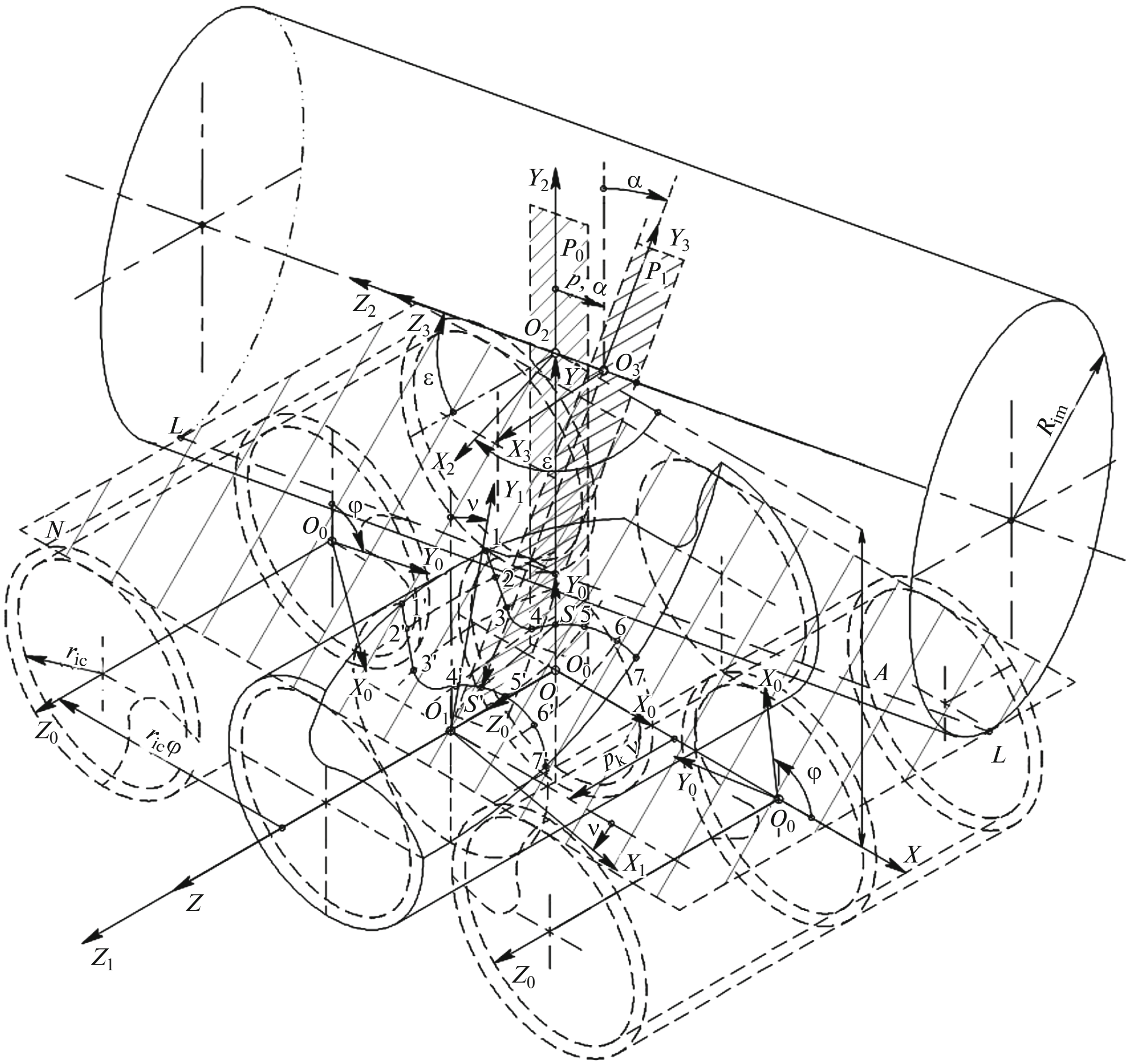

Fig. 1. Formulating a three-dimensional numerical model of the worm mills.

(1) Displacement along the X axis (n = 1) of coordinate system $O_0X_0Y_0Z_0$ by a distance $Ax = -r_{ic}\varphi$, where r_{ic} is the radius of the initial cylindrical part to be machined, and consistent rotation (rolling) around the Z axis by an angle $xy = \varphi$, which is a variable ensuring the rolling motion of the initial cylinder with respect to the plane N tangential (in terms of the line L–L) to the initial cylinder of a worm mill of radius R_{im}.

On the basis of the initial data, the program implementing the module formulates transformation matrix $M1$ from matrix MO in Eq. (1) in accordance with the established algorithm

$$M1 = \begin{pmatrix} \cos(\varphi) & -\sin(\varphi) & 0 & -r_{ic}\varphi \\ \sin(\varphi) & \cos(\varphi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (2)$$

(2) Rotation around axis Z (n = 3) of coordinate system $O_1X_1Y_1Z_1$ by an angle $xy = -\nu$, which is a variable ensuring the helical motion that creates the machined surface of the part and consistent displacement along this axis by a distance $Ax = -p_c\nu$. Here $p_c = r_c/\tan\omega$; r_c is the external radius of the part; and ω is the inclination of the helical channel produced.

According to the algorithm, the corresponding transformation matrix $M2$ takes the form

$$M2 = \begin{pmatrix} \cos(\nu) & \sin(\nu) & 0 & 0 \\ -\sin(\nu) & \cos(\nu) & 0 & 0 \\ 0 & 0 & 1 & -p_c\nu \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

(3) Displacement along the Y axis (n = 2) of the coordinate system $O_2X_2Y_2Z_2$ by a distance $Ay = -A =$

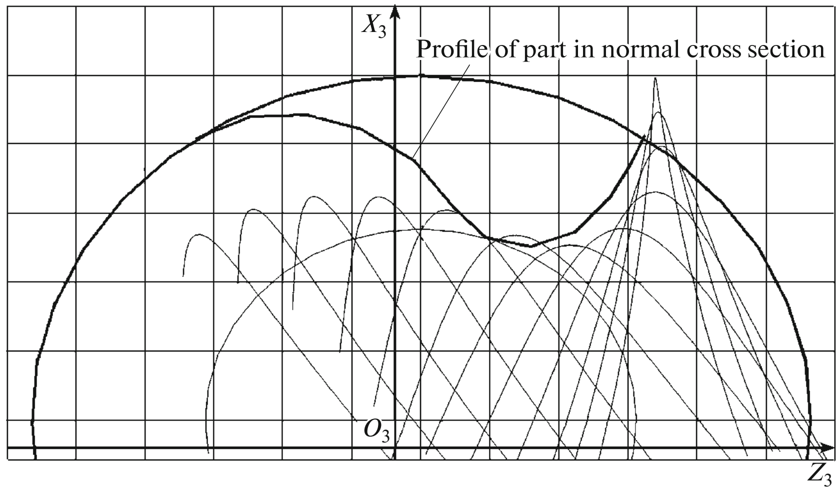

Fig. 2. Graphical formulation of the numerical model of the initial surface with rolling parameter $\varphi_j = \varphi_{\max}$.

$-(r_{ic} + R_{im})$ and rotation by an angle $zx = -\varepsilon$ corresponding to the skewing of the axes of the worm mill and the part. On the basis of the initial data, the first module forms transformation matrix $M3$ from the initial matrix MO in Eq. (1)

$$M3 = \begin{pmatrix} \cos(\varepsilon) & 0 & \sin(\varepsilon) & 0 \\ 0 & 1 & 0 & -(r_{ic} + R_{im}) \\ -\sin(\varepsilon) & 0 & \cos(\varepsilon) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (4)$$

(4) Rotation around axis Z_2 ($n = 3$) of coordinate system $O_3X_3Y_3Z_3$ by an angle $xy = \alpha$, which is a variable related to the coordinates x_{2i}, y_{2i} of the point considered, and consistent displacement along this axis by distance $Az = p_m\alpha$, where p_m is the helical parameter of the mill. As an example, note that, for point $S'(x_{2s'}, y_{2s'})$, $\tan\alpha = x_{2s'}/y_{2s'}$.

According to the algorithm, the corresponding transformation matrix $M4$ takes the form

$$M4 = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & p_m\alpha \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (5)$$

The resulting numerical model MR is a column matrix of the form

$$MR = \begin{pmatrix} x_{3i} \\ y_{3i} \\ z_{3i} \\ 1 \end{pmatrix} = M\Sigma \begin{pmatrix} x_i \\ y_i \\ 0 \\ 1 \end{pmatrix}, \quad (6)$$

where $M\Sigma = M4 \cdot M3 \cdot M2 \cdot M1$.

Software has been written for the operations based on the transformation matrices in Eqs. (1)–(5). As a result of those operations, we obtain the numerical model in Eq. (6), where the coordinate y_{3i} is the distance from the axis Z_3 to the given point i after the transformations just itemized. The coordinate z_{3i} determines the distance to the coordinate plane $O_3X_3Y_3$.

To determine the tool profile, we may use the following shaping algorithm.

(1) Specification of the rolling motion by angle φ_j in the range from $+\varphi_{\max}$ to $-\varphi_{\max}$, in increments established on the basis of the required precision (mean value 0.01π).

(2) Specification of the number i of the point with coordinates x_i, y_i for each value of φ_j. In Fig. 1, $i = 1, ..., 5$.

(3) Specification of the change in the angle ν in the range from $+\nu_{\max}$ to $-\nu_{\max}$, for each angle φ_j and value of i. The value of ν depends on the profile of the part, the inclination of the helical channel, and the radius of the initial worm circumference, which is $(0.1–0.2)\pi$. In this case, points $i = 1, ..., S, ..., 7$ move to the positions $i = 1', ..., S', ..., 7'$. The increment in ν is established on the basis of the required precision (mean value 0.01π).

(4) As a result of displacement of the point S to position S', say, plane P_0 passing through that point is moved to position P_1. Correspondingly, the plane is turned by angle α and moved consistently from center O_2 over a distance $p_m\alpha$ to point O_3.

As a result of these actions, when $\varphi_j = \varphi_{\max}$, we note the appearance of curves representing the intersection of the lines 1–1', 2–2', ..., S–S', ..., 7–7' (Fig. 2) with plane P_1, which moves helically around axis Z_3 with helical parameter p_m.

In Fig. 3, we show all the curves for the complete range of angular variation $\varphi_{\min} \le \varphi_j \le \varphi_{\max}$. Software

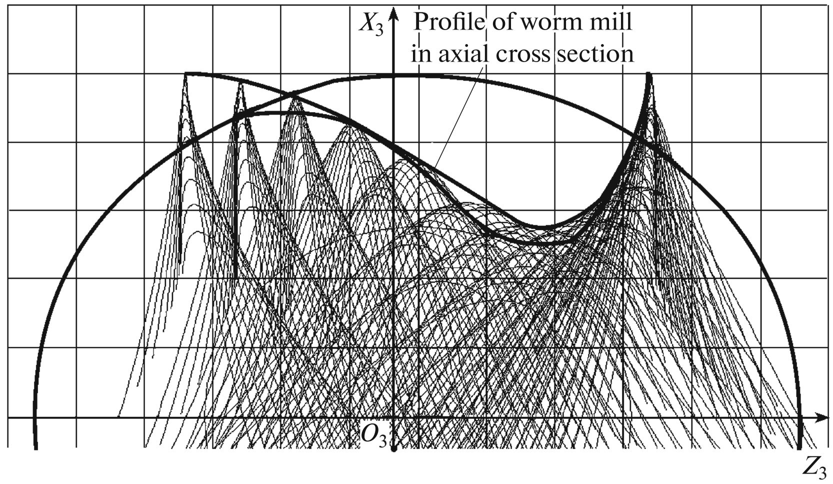

Fig. 3. Graphical formulation of the numerical model of the shaped worm mill.

constructing the envelope of these curves permits determination of the desired worm-mill profile.

The proposed approach may be used for the shaping and analysis of worm mills with a protuberance [3]. It sets no constraints on the shape of the generatrix and directrix of the helical surface produced nor on the method used to specify the generatrix and directrix.

The proposed system may be used not only to plot curves and determine the desired profile but also to formulate mathematical expressions for that purpose on the basis of the initial matrix *MO* in Eq. (1) and the transformation matrices. By that means the profile of the worm mill may be determined in a single stage without determining the profile of the conjugate helical rack.

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