ISSN 1068-798X, Russian Engineering Research, 2017, Vol. 37, No. 5, pp. 388–392. © Allerton Press, Inc., 2017.
Original Russian Text © A.N. Lisin, I.I. Nabokov, V.V. Mozalev, 2017, published in Vestnik Mashinostroeniya, 2017, No. 2, pp. 46–50.

Predicting the Fatigue Strength of Structural Elements

A. N. Lisin[a], I. I. Nabokov[a, *], and V. V. Mozalev[b]
[a] *Elektrostal Institute, Elektrostal, 144000 Russia*
[b] *OAO AK Rubin, Balashikha, 143900 Russia*
**e-mail: Nabokoff2008@mail.ru*

Abstract—The development of models relating the fatigue strength of materials to their static strength and plasticity is considered. Statistical strength theories were developed to describe the results of fatigue tests and to predict the strength of machine components with variable loads. Experiments show that the strength of materials depends significantly on structural defects, and the limiting stress is a statistical quantity.

Keywords: strength, fatigue strength, strength theories
DOI: 10.3103/S1068798X17050161

Tests show that the strength of materials depends largely on structural defects, while the limiting stress is a statistical quantity. Therefore, all the limiting surfaces in the primary stress aces may be represented by a set of mean stresses at which the material disintegrates instantaneously if the short-term strength is considered or over some period of time (the life) if the strength of a material with variable stress is considered.

Statistical theoretic of strength were developed to describe the results of fatigue tests and to predict the strength of machine components with variable loads [1]. Given the large spread in the results, a different approach to the calculation of parts has now been adopted: we cannot guarantee the strength with absolute confidence. We can only say that its probability of failure is relatively small in specific conditions [2].

One statistical theory rests on the hypothesis of a weak link [3]. In this approach, the body is assumed to consist of a large number of structural elements, each of which has its own local strength. The body as a whole disintegrates when at least one structural component fails. For massive bodies, this assumption must be regarded as an oversimplification: for the failure of the body as a whole, it is probably necessary for some group of components to fail. That principle is adopted in more complex theories.

In Afanas'ev's statistical theory, the fatigue strength is considered in simple and complex stress states [4]. A statistical model of fatigue failure describing the influence of stress concentrations and the absolute dimensions of the body is developed on the basis of different distribution functions. The model is verified by experiments with samples of aluminum, copper, brass, and austenitic and spring steels. However, all the formulas for the fatigue strength with complex stress states are based on the mechanical transfer of static-strength criteria, without verifying their validity; instead coefficients taking account of the asymmetry of the cycle and stress concentrations are introduced. In this approach, the metal is regarded as a conglomerate of grains with different stresses on account of the structural inhomogeneity and anisotropy. The probability of fatigue failure is determined as the probability that several grains with stresses exceeding the adhesive force exist simultaneously. Note that the appearance of a crack is attributed to the development of plastic stress exceeding the yield point and reaching the rupture strength in brittle failure and the cleavage strength in ductile failure [5].

Thus, according to Afanas'ev's model, the appearance of any fatigue crack is associated with local plastic deformation. The probability W of crack appearance is related to the macroscopic stress by a power law, which is determined from formulas for the strength of materials.

Volkov's statistical theory is evidently the most rigorous and promising [6]. It establishes a general statistical strength criterion: the tolerance on the probability of failure. In this approach, a material is regarded as a polycrystalline elastoplastic medium that is uniform macroscopically and nonuniform microscopically. On the basis of macroscopic continuity, an arbitrary bulk stress state may be considered. The theory is developed for a power law of the macroscopic deformation [6]. It combines new methods of calculating the strength of machine parts and structural components, taking account of the significant influence of structural inhomogeneity of the materials on failure, the scale effect, etc. These new methods employ statistics and probability theory. The development and application of this new strength theory was considered in [7].

On the basis of Volkov's criterion, the following proposition was stated in [8]: macroscopic failure of a solid appears on reaching a critical value of 0.5 for the relative damage of the primary area by microcracks, taking account of a stress tensor of the first kind (at the microlevel). The statistical conditions of the limiting state take account of the influence of the spherical tensor, the deviator, and the maximum normal and tangential stresses. On that basis, the results of strength tests for traditionally brittle materials such as cast iron, glass, gypsum, quenched steel, and carbon steel may be explained.

Afanas'ev developed a statistical theory of the strength of concrete and related materials that fail with little deformation in a complex stress state [4] somewhat before the publication of [8].

All the theoretical and experimental data in the literature indicate that reliable information regarding the strength of blocks and thin samples made of structural materials may only be obtained by developing a physical theory of failure and employing statistical analysis.

Despite the extensive research on the topic, we have yet to find satisfactory means of predicting the fatigue strength of materials, for the following reasons: the problem is complex; many types of variable stress state and test conditions are encountered; and very different methods of surface and bulk treatment are used for existing metals and alloys. Over time, these factors become even more complex, since important machine parts and structural components generally operating in repeating or alternating loads and their life is completely determined by the cyclic strength of the material.

The strength of solids is studied by specialists in solid-state physics, materials science, and the mechanics of deformable continuous media. Despite their common goals, these fields use different methods; however, they exchange their findings [9]. There is a continuing search for new laws applicable to the complex systems of interest here. Fatigue loading is characterized by exchange of energy with the environment (energy absorption and liberation) and by external stimuli that vary over time.

In research on the physical nature of fatigue, the formation and development of fatigue cracks is analyzed; fatigue structures and their variation, hyperfatigue, fatigue at cryogenic and elevated temperatures, and fatigue at high loading frequencies are studied in detail; and the role of crystalline defects and strengthening and failure of metals has been shown. Amorphous phases are found to exist at the crack tip, and excited atom–vacancy states are also encountered.

At present, in predicting the fatigue strength, it is not enough to confine our attention to a continuum model of the body, on the basis of continuum mechanics [10]. Regardless of which strength criteria are chosen (deformational, energetic, or other criteria), crack behavior and kinetics must be taken into account, and the period in which they grow must be assessed, since research shows that the nucleation and development of fatigue cracks occurs over much of the life of the body. This must be taken into account in the model of fatigue failure [11].

We know that the surface layer of a part is subject to the most load in all stress states. The surface layer is where energy and matter are exchanged with the environment. In cyclic loading, damage accumulates predominantly in the surface layers. The considerable influence of the surface state of the metal on its cyclic strength is associated with the earlier development of fatigue cracks in the surface layers than in the rest of the metal. That may be attributed to the accumulation of the critical dislocation density in that layer.

After surface treatment, a layer with different structure and stress state is formed to some depth; this is known as the modified layer [12]. The distribution of the physicomechanical and chemical properties over this layer is complex and is determined by the surface-treatment technology and conditions. The fatigue strength is mainly controlled by treatments at the surface of the part. Therefore, in predicting fatigue characteristics such as the fatigue limit and fatigue life, we must take account of the influence of the modified surface layer and primarily the relation between the characteristics (surface roughness, residual stress) and the distribution of the residual stress over the depth of the surface layer.

The following factors indicate the need to develop scientific principles for predicting the fatigue strength.

(1) The fatigue failure and wear determine when the components of highly loaded systems become inoperable. The elimination of those processes demands close study of their development in dynamic mechanical and thermal treatment [13].

(2) In manufacturing, it is always important to increase the reliability, durability, and working life of components, with simultaneous decrease in their mass and maintenance or reduction of their cost. That calls for new or improved technologies.

(3) An effective approach in improving technological processes is to produce designs with a known relation between the life and the factors determining the machining conditions and method. However, the production of components has an indirect influence on the life and durability: in machining, the structure and physicomechanical properties of the metal and, in particular, the modified surface layer are formed. The properties of the modified layer differ from those of the remaining metal and essentially determine the life and durability of the part.

Accordingly, we need to solve two problems:

(a) after investigating the influence of the physicomechanical properties of the metal by means of existing models—for example, that in Eq. (1)—and the corresponding influence of the modified surface layer

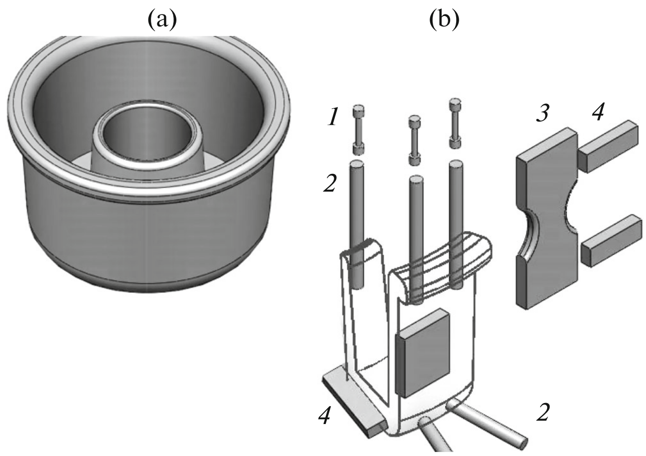

Fig. 1. Cutting of samples (b) from a stamped wheel workpiece (a): (*1*) for static tests (Gagarin sample); (*2*) for fatigue flexure tests with rotation; (*3*) for observations of the growth of surface fatigue cracks; (*4*) beam samples for determination of crack resistance.

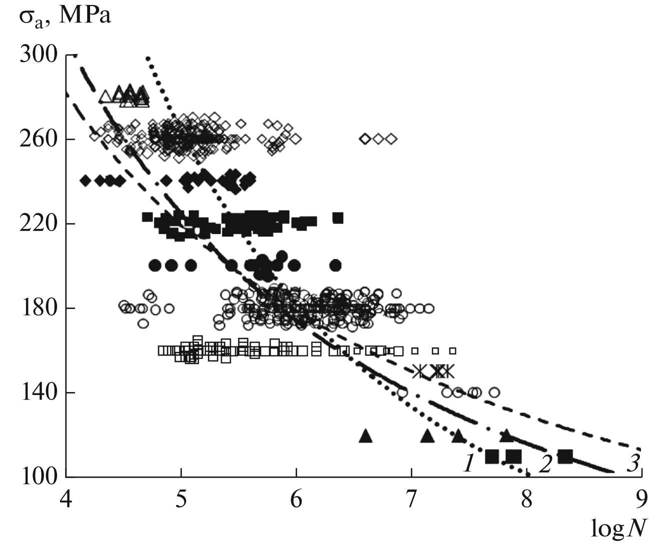

Fig. 2. Results of fatigue tests of samples cut from a stamped AK6 alloy workpiece.

on the fatigue strength, we must develop a method of predicting the characteristics of the surface layer;

(b) we must identify and implement technologies capable of ensuring the required parameters of the surface layer.

(4) Reliable methods of predicting the fatigue strength of metallic parts are required; these methods must take account of the influence of the modified surface layer on the fatigue limit and fatigue life. The fatigue characteristics must be determined at any stage of the process and in any machining conditions so that the optimal result ensuring the specified or maximum fatigue strength may be selected. It is difficult to obtain the required set of data experimentally.

Since the operational properties of important machine parts are determined when the manufacturing technology is designed, we rule out the use of forgings, rods, and other standard prefabricated structures, since their properties will be inferior to those in individual billet production [14]. Thus, the properties of the part must be taken into account in developing the manufacturing technology. This is also the case for vehicle wheels.

For example, new technologies are used to create the required properties of drums (disks) in the wheels of Formula 1 cars [15, 16]. Wheels produced from prefabricated forgings or stamped blanks cannot match those produced by the new technologies, as shown in practice in [15–17].

Accordingly, rather than new materials, we should focus on materials technologies—that is, manufacturing technologies using modified materials. In other words, attention is directed not to the material (an alloy, say), but to the overall manufacturing technology. Within this framework, we should be aware that, when a part is made from a new material produced by nanotechnologies, it will not necessarily have all of the properties of the initial material. To ensure that it does, we need to improve the manufacturing technology for that specific material. In that case, we obtain an experimental technology that provides the basis for the mass-production technology.

A technological process is a combination of strictly defined methods and operating conditions that need to be optimized [18]. The creation and introduction of new production methods for workpieces is one of the most important stages in improving technological processes [19]. Workpieces that are close in properties and shape to the final product simplify the machining process, reduce the consumption of materials, and lower the cost of the final part, while the part's performance is improved [20].

Tests of new products confirm this conclusion—for example, in relation to the fatigue strength. To determine the uniformity of the distribution of mechanical properties for AK6 alloy, we conduct tests of samples cut from the selected zones of stamped wheel workpieces (Fig. 1). Aviation and automobile wheels are made from this alloy [21]. Statistical characteristics of the fatigue strength are plotted in Fig. 2 on the basis of the test data [22]. Curves *1–3* are obtained from test results for the maximum number of samples. Each experimental point represents the test result for a single sample. The notation for the points is as follows: (□) results corresponding to curve *1*; (○) results corresponding to curve *2*; (■) results corresponding to curve *3*.

We now consider the results of fatigue tests in flexure, with rotation of the samples. (The diameter of the samples in the working cross section is 8 mm.) The

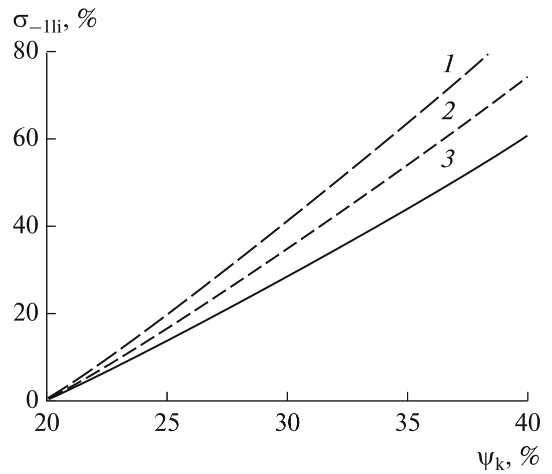

Fig. 3. Predicted increase in the fatigue limit σ_{-1li} as a function of the relative residual constriction ψ_k when S_k = 500 (*1*), 400 (*2*), and 300 MPa (*3*).

visible scattering of the experimental data must be taken into account in specifying the life of the part, as in assessing its quality in terms of the fatigue strength.

We analyze the relation between the mechanical properties of AK6 alloy by means of the equation

$$e_a = \frac{1}{4(N/2)^{m_1}} \ln \frac{1}{1-\psi_k} + \frac{S_k}{E(2N)^{m_2}}, \quad (1)$$

where $e_a = \sigma_a/E$ is the amplitude of the rated strain; S_k is the alloy strength; E is its elastic modulus; N is the sample life to failure; ψ_k is the relative residual constriction (plasticity); and m_1 and m_2 are empirical parameters.

The model in Eq. (1) may be rewritten to describe the fatigue curve

$$\sigma_a = D(N/2)^{-m_1} + F(N/2)^{-m_2}, \quad (2)$$

with the coefficients

$$D = \frac{E}{4} \ln \frac{1}{1-\psi_k} \text{ and } F = \frac{S_k}{4^{m_2}}.$$

The first term in Eq. (2) takes account of the alloy's plastic properties; the second takes account of its strength. We may also say that the first describes the behavior of the alloy on passing from few-cycle fatigue to multicyclic fatigue, while the second describes its behavior in multicyclic fatigue. Note that the model in Eq. (2) experimentally confirms the independence of the S_k and ψ_k characteristics; otherwise, their correlation would need to be evaluated.

On the basis of these data, it is possible to predict the influence of the manufacturing technology on the fatigue strength of the new material. This calls for an experiment that is not easily repeated: numerous tests of samples, with a base of 10^8–$10^{8.5}$ cycles (Fig. 2). According to fatigue curve *2*, the fatigue limit increases markedly (by 20–40%) with 25–30% increase in plasticity (Fig. 3). The fatigue life is increased by an order of magnitude in that case.

Thus, the development of new models relating the fatigue strength of materials with their statistical strength and plasticity is necessary and expedient.

REFERENCES

1. Agamirov, L.V., Development of statistical evaluation of the fatigue characteristics of materials and indicators of reliability of structural elements of aviation equipment, *Extended Abstract of Doctoral (Tech.) Dissertation*, Moscow: Moscow Aviat. Inst., 1994.
2. Stepnov, M.N., Chernyshev, S.L., Kovalev, I.E., and Zinin, A.V., *Kharakteristika soprotivleniya ustalosti. Raschetnye metody otsenki* (Characteristics of Fatigue Resistance: Estimation Methods), Moscow: Tekhnol. Mashinostr., 2010.
3. Ivanova, V.S. and Terent'ev, V.F., *Priroda ustalosti metallov* (Nature of Metal Fatigue), Moscow: Metallurgiya, 1975.
4. Afanas'ev, N.N., Statistical theory of fatigue resistance of metals, *Zh. Tekh. Fiz.*, 1940, vol. 10, no. 19, pp. 1553–1568.
5. Afanas'ev, N.N., *Statisticheskaya teoriya ustalostnoi prochnosti metallov* (Statistical Theory of Fatigue Strength of Metals), Kiev: Akad. Nauk UkrSSR, 1953.
6. Volkov, S.D., *Statisticheskaya teoriya prochnosti* (Statistical Theory of Strength), Moscow: Mashgiz, 1960.
7. Volkov, S.D., The theory of macrocraks: Part 1. The simplest models, *Probl. Prochn.*, 1981, no. 2, pp. 44–48.
8. Volkov, S.D., Dubrovina, G.I., and Sokovnin, Yu.P., Stability of material resistance in fracture mechanics, *Probl. Prochn.*, 1978, no. 6, pp. 65–69.
9. Konovalov, L.V., Engineering taking for account fatigue as required condition for creation of efficient mechanical systems, *Vestn. Mashinostr.*, 1993, no. 3, pp. 3–11.
10. Ivanova, V.S. and Terent'ev, V.F., *Priroda ustalosti metallov* (Nature of Metal Fatigue), Moscow: Metallurgiya, 1975.
11. Selikhov, A.F. and Chizhov, V.M., *Veroyatnostnye metody v raschetakh prochnosti samoleta* (Probabilistic Calculations of an Aircraft Strength), Moscow: Mashinostroenie, 1987.
12. Serensen, S.V., Fatigue of materials and construction elements, in *Izbrannye trudy* (Selected Research Works), Kiev: Naukova Dumka, 1985.
13. Stepnov, M.N., *Statisticheskie metody obrabotki rezul'tatov mekhanicheskikh ispytanii: spravochnik* (Statistical Analysis of the Results of Mechanical Tests: Handbook), Moscow: Mashinostroenie, 1985.
14. Surkov, A.I., Probabilistic evaluation of strength at variable loads along the median endurance limits of different-size samples, *Probl. Prochn.*, 1982, no. 12, pp. 42–50.
15. Basyuk, S.T., Evteev, F.I., and Kovalev, S.I., Expert system for the engineering and manufacture of aircraft

wheels, *Vopr. Aviats. Nauki Tekh., Ser. Tekhnol. Legkikh Splavov*, 1987, no. 11, pp. 58–61.

16. Basyuk, S.T., *Ob"emnaya shtampovka zagotovok iz legkikh splavov na gidravlicheskikh pressakh* (Large Pressing of Light-Alloy Billets on Hydraulic Presses), Moscow: Sport i Kul'tura–2000, 2011.
17. Basyuk, S.T., *Intensifikatsiya deformatsii sdviga pri izgotovlenii polufabrikatov* (Intensive Shift Deformation in Production of Semi-Finished Products), Moscow: Sport i Kul'tura–2000, 2010.
18. Makhutov, N.A., *Deformatsionnye kriterii razrusheniya i raschet elementov konstruktsii na prochnost'* (Deformation Criteria of Destruction and Calculation of Construction Elements for Strength), Moscow: Mashinostroenie, 1981.
19. Lisin, A.N., Evaluation of the characteristics of fatigue resistance and the survivability of vehicle wheels, *Extended Abstract of Doctoral (Tech.) Dissertation*, Moscow: Russ. State Technol. Univ., 2012.
20. Mozalev, V.V. and Lisin, A.N., The application of statistical strength theories in the function analysis of distribution curve of fatigue, *Aviakosm. Tekh. Tekhnol.*, 2014, no. 4, pp. 41–45.
21. Bogdanoff, J.L. and Kozin, F., *Probabilistic Models of Cumulative Damage*, New York: Wiley, 1985.
22. Khazanov, I.I., Agafonov, Yu.A., and Mozalev, V.V., Optimization of routine tests during operation of aviation wheels by technical state, *Tr. Gos. Nauchno-Issled. Inst. Grazhd. Aviats.*, 1980, no. 183, pp. 17–22.

Translated by Bernard Gilbert