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Deformation of Metals with Slight Elastoplastic Deformation

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Abstract—An analytical equation describing the quasi-static deformation curve of metals with slight elastoplastic deformation is proposed. This equation yields high precision in plasticity problems. The components of the equation are found from the experimental deformation curve.

Keywords: stress, strain, elastic modulus, strengthening modulus, elastoplastic deformation

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In many mechanics problems, it is necessary to establish the relation between the stresses and strains that arise in response to a load [1]. For engineering calculations, as a rule, a polygonal approximation is adopted. The deformation curve in extension is represented by a piecewise-linear dependence of the relative stress $\overline{\sigma} = \sigma/\sigma_y$ on the relative strain $\overline{\varepsilon} = \varepsilon/\varepsilon_y$. Here the yield point σ_y is assumed to be the stress corresponding to the limit of proportionality; and ε_y is the strain corresponding to the yield point.

In that case, the deformation curve may be described in each interval $\overline{\varepsilon}_k \le \varepsilon \le \overline{\varepsilon}_{k-1}$ by the equation

$$\overline{\sigma} = a_k + b_k\overline{\varepsilon},$$

where a_k and b_k are coefficients determined from the experimental deformation curve. Six intervals are used to describe the deformation curve of metals in one-time loading presented in [2]. In other words, 12 coefficients are calculated for the description of a single curve. This method is employed in ANSYS software.

The number of coefficients a and b may be reduced to two on the basis of a model in which the solid consists of a set of identical microvolumes characterized by different stress. Each microvolume undergoes linear strengthening on deformation. We assume that, with change in the strain, the stress in the normal cross section of the microvolume changes as follows

$$\sigma_i = E\varepsilon_i \text{ when } \varepsilon_i \le \varepsilon_y;$$
$$\sigma_i = E\varepsilon_i + E_s(\varepsilon_i - \varepsilon_y) \text{ when } \varepsilon_i > \varepsilon_y.$$

Here E is the elastic modulus; E_s is the strengthening modulus.

Suppose that there are n_o microvolumes in the normal cross section of the body. Then the stress in that cross section is

$$\sigma = \frac{1}{n_o}\sum_1^n E\varepsilon_i + \frac{1}{n_o}\sum_n^{n_o}\left[E\varepsilon_y + E_s(\varepsilon_i - \varepsilon_y)\right],$$

where n is the number of microvolumes that experience elastic deformation. The other $n - n_o$ microvolumes undergo elastoplastic deformation.

With increment dε in the strain of the body on extension, each microvolume will experience the same increment. The mean stress is then

$$\sigma + d\sigma = \frac{1}{n_o}\sum_1^n E(\varepsilon_i - d\varepsilon)$$
$$+ \frac{1}{n_o}\sum_n^{n_o}\left[E\varepsilon_y + E_s(\varepsilon_i - \varepsilon_y + d\varepsilon)\right].$$

Hence

$$d\sigma = E\frac{n}{n_o}d\varepsilon + E_s\left(1 - \frac{n}{n_o}\right)d\varepsilon.$$

The transition of individual microvolumes from the elastic state to the elastoplastic state on deformation is random. Suppose that this process has a Poisson distribution. Then the mean number of elastically deformed volumes in the given cross section when the strain changes by $\varepsilon - \varepsilon_y$ is

$$n = n_o e^{-\lambda(\varepsilon - \varepsilon_y)}, \tag{1}$$

where λ is a constant characterizing the rate at which the individual volume passes from state to the other.

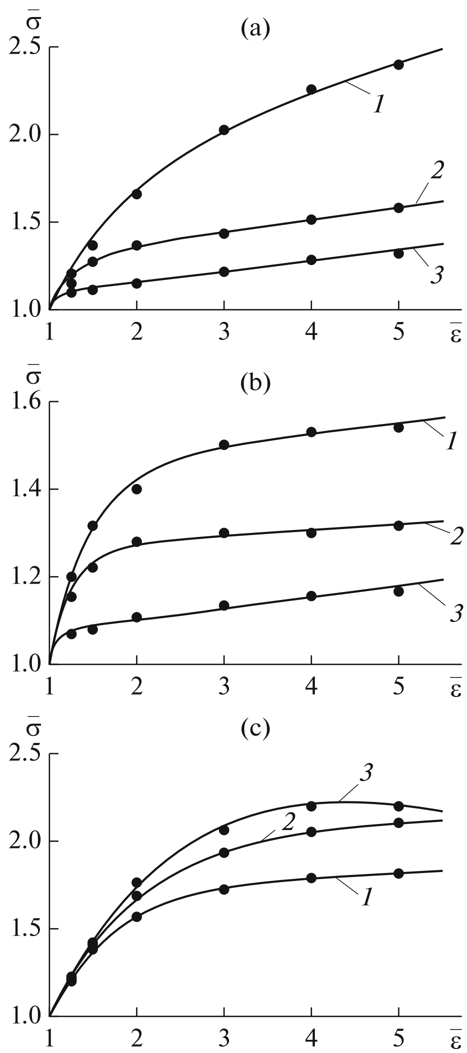

Fig. 1. Deformation curves of 18X2H4MA (*1*), X18H9T (*2*), and 40X structural steels (a); VT1 (*1*) titanium alloy and V95T (*2*) and V95 (*3*) aluminum alloys (b); and 18X2H4MA steel at elevated temperatures (c): 300°C (*1*), 400°C (*2*), and 500°C (*3*).

Hence, tensile deformation beyond the elastic limit ($\varepsilon > \varepsilon_y$) may be described in the form

$$\sigma = \sigma_y + \int_{\varepsilon_y}^{\varepsilon} d\sigma$$

or

$$\bar{\sigma} = 1 + \frac{1}{\sigma_y}\int_{\varepsilon_y}^{\varepsilon} d\sigma. \quad (2)$$

If we integrate Eq. (2), taking account of Eq. (1), we obtain an analytical formula for the deformation

$$\sigma = 1 + D(\bar{\varepsilon} - 1) + \frac{1 - D}{B}\left[1 - e^{-B(\bar{\varepsilon}-1)}\right], \quad (3)$$

where $D = E_s/E$ is the relative strengthening modulus; $B = \lambda\sigma_y/E$ is a coefficient characterizing (accurately except for some constant factor) the rate of transition of an individual microvolume to an elastoplastic state on loading

We may determine D and B from the experimental curve by the least-squares method. In that case, they may expediently be calculated by means of the genifit function in Mathcad software. To that end, we proceed as follows.

1. On the basis of the experimental data, we write expressions for the vectors

$$\bar{\varepsilon} = \left(\bar{\varepsilon}_1\ \bar{\varepsilon}_2\ \bar{\varepsilon}_3\ \dots\ \bar{\varepsilon}_k\right)^{\mathrm{T}};$$
$$\bar{\sigma} = \left(\bar{\sigma}_1\ \bar{\sigma}_2\ \bar{\sigma}_3\ \dots\ \bar{\sigma}_k\right)^{\mathrm{T}},$$

where $\bar{\varepsilon}_k$ and $\bar{\sigma}_k$ are the experimental values for k points selected arbitrarily on the experimental curve; superscript T denotes transposition of the vectors.

2. We write the vector of initial values of B and D (for example, $B = 5$, $D = 0.1$)

$$g = (B\ \ D)^{\mathrm{T}}.$$

3. We write the auxiliary vector

$$Q(\bar{\varepsilon}, g) = \begin{vmatrix} 1 + g_1(\bar{\varepsilon} - 1) + \dfrac{1 - g_2}{g_1}\left[1 - e^{-g_1(\bar{\varepsilon}-1)}\right] \\ (\bar{\varepsilon} - 1)\dfrac{1 - g_2}{g_1}e^{-g_1(\bar{\varepsilon}-1)} - \dfrac{1 - g_2}{g_1^2}\left[1 - e^{-g_1(\bar{\varepsilon}-1)}\right] \\ (\bar{\varepsilon} - 1) - \dfrac{1}{g_1}\left[1 - e^{-g_1(\bar{\varepsilon}-1)}\right] \end{vmatrix},$$

where the first element of column vector $Q(\bar{\varepsilon}, g)$ corresponds to the function specified in Eq. (3); the second to the partial derivative of the specified function with respect to B; and the third to the partial derivative of the specified function with respect to D.

4. We obtain the solution by means of the operators

$$B = \mathrm{genifit}(\bar{\varepsilon}, \bar{\sigma}, g, Q)_1;$$
$$D = \mathrm{genifit}(\bar{\varepsilon}, \bar{\sigma}, g, Q)_2.$$

This model may be verified by comparison of the calculated and experimental data for some structural steels, aluminum alloys, and titanium alloys. As an example, the table presents the parameters of the deformation equation for some metals. In Fig. 1, we show deformation curves obtained from Eq. (3). The points on the curve correspond to experimental data [2].

Thus, we have obtained an analytical description of quasi-static deformation for a broad class of metals and alloys with slight elastoplastic deformation at normal and elevated temperatures. The equations obtained are simple to integrate, which is important in

Parameters of the deformation equation for structural steels, aluminum alloys, and titanium alloys

Material	Heat treatment	Test temperature, °C	Parameters	
			B	*D*
40X	Annealing	20	11.29	0.0630
18X2H4MA	Quenching at 950°C, tempering at 180°C	20	11.11	0.1640
		200	1.32	0.0210
		400	0.91	0.0083
		500	0.53	–0.2280
X18H9T	Quenching at 1050°C	20	3.20	0.0710
V95	Quenching, artificial aging	20	12.68	0.0260
V95T	Quenching, artificial aging	20	3.69	0.0130
VTT	Annealing at 700°C	20	2.19	0.0250

plasticity problems, and contain only two experimental parameters.

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