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New Equations of Motion of Vehicles

G. I. Mamiti
Gorsk State Agrarian University, Vladikavkaz
e-mail: avtofak.ggau@yandex.ru

Abstract—A new approach to calculating the forces on a wheeled vehicle is proposed, and the corresponding equation of motion is derived. On that basis, all the forces and torques may be referred to the center of the wheel–road contact spot.

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The motion of a wheeled vehicle (car, truck, tractor, motorcycle, etc.) depends on the interaction of the wheels and the supporting surface.

Consider the forces acting on a car that is accelerating up an incline (Fig. 1) [1]. In Fig. 1, we may express the inclination α in terms of the ratio

$$H_\alpha / B_\alpha = \tan\alpha = i_\alpha.$$

Gravitational force $G = mg$ and inertial force $P_j = mj$, which opposes the acceleration j, are applied to the car's center of mass C, with coordinates a, b, h. (Here m is the mass of the car and g is the acceleration due to gravity.) The component $P_\alpha = G\sin\alpha$ of the gravitational force, which is parallel to the supporting plane, opposes the vehicle's ascent; component $G\cos\alpha$, which is normal to the supporting surface, presses the car to the road. The drag force $P_w = kFv^2$ acts at the height h_w of the center of wind action. Here $k = c_w\rho/2$ is the flow coefficient; c_w is the frontal (aerodynamic) drag; ρ is the density of the air; F is the car's frontal area; v is its speed in still air. The drag force P_x of the trailer may be applied to the towing hook at height h_x.

The resultant normal (Z_1 and Z_2) and tangential (X_1 and X_2) reactions are applied from the road to the wheels. Subscripts 1 and 2 denote the front driven wheel and rear driving wheel of the vehicle; X_1 and X_2 are the resultants of all the tangential forces at wheel–road contact, disregarding the reaction associated with the forces P_w, P_j, P_α, and P_x.

Consider the motion of the driving wheel in acceleration (Fig. 2). The wheel is subjected to the vertical (G_2) and longitudinal (X_1) forces due to the vehicle mass and the resistance of the driven wheel; the tractional torque M; the reactions Z_2 and X_2 of the road; and the drag torque M_{f2}. (The influence of P_w, P_j, P_α, and P_x will be taken into account later.) Since wheel rotation is nonuniform, an inertial torque opposes the angular acceleration ε_{wh}

$$M_\varepsilon = J_{wh}\varepsilon_{wh},$$

where J_{wh} is the moment of inertia of the wheel and the associated rotating components of the transmission.

The torque M_e developed by the vehicle's engine is transmitted to the axles of the driving wheels, creating the tractional torque

$$M = (M_e - J_{fl}\varepsilon_{fl})i\eta,$$

where i and η are the gear ratio and efficiency of the transmission; J_{fl} and ε_{fl} are the moment of inertia and angular acceleration of the engine's flywheel.

The tractional torque M sent to the wheel produces a drag force of the road at the road–tire contact; this is the tractional force P. Obviously, the tractional force cannot exceed the maximum adhesive force of the tire at the road

$$P_\varphi = \varphi Z_2.$$

Here φ is the adhesion coefficient of the tire at the road; Z_2 is the normal reaction of the road at the driving wheel.

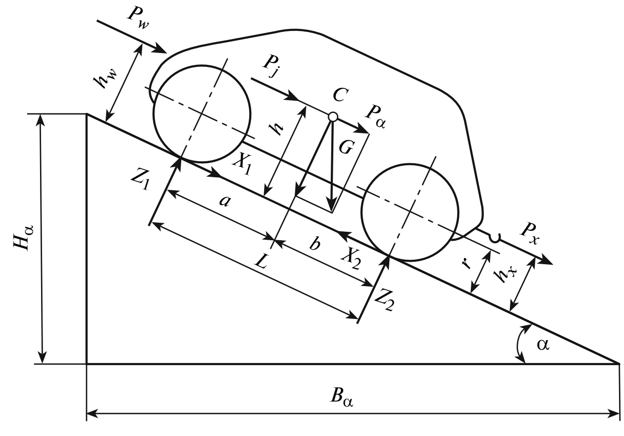

Fig. 1. Forces on a vehicle as it accelerates up an incline.

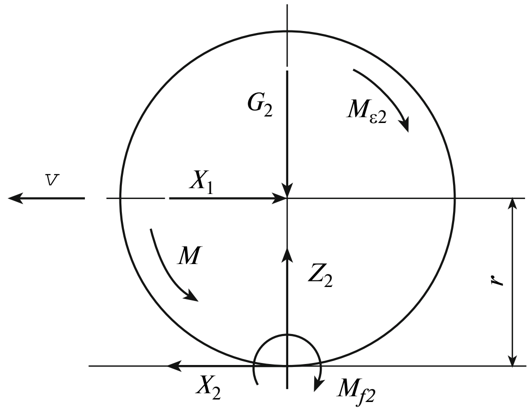

Fig. 2. Forces and torques on the driving wheel in acceleration.

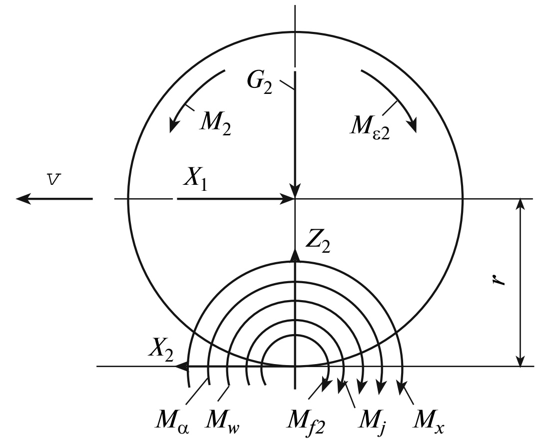

Fig. 3. Force diagram for the car.

The adhesion coefficient φ is equal to the ratio of the force producing uniform slip of the wheel and the road's normal reaction. The longitudinal adhesion coefficient φ_x and transverse adhesion coefficient φ_y depend on the direction of the slip. The adhesion coefficient takes account of the friction and mechanical coupling in tire–road interaction.

Thus, whatever torque is developed by the motor or brake, the maximum possible torque in terms of tire–road adhesion is

$$M_{\max} = \varphi Zr,$$

where r is the wheel radius; Z is the road's normal reaction.

The tangential reaction of the road on the driving wheel in acceleration (Fig. 2) takes the form

$$\begin{aligned} X_2 &= (M - M_\varepsilon - M_{f2})/r - X_1 \\ &= (M_e - J_{fl}\varepsilon_{fl})i\eta/r - J_{wh2}\varepsilon_{wh2}/r - M_{f2}/r - X_1, \end{aligned} \tag{1}$$

where $X_1 = J_{wh1}\varepsilon_{wh}/r + M_{f1}/r$ is the resultant tangential reaction of the road on the driven wheel; $J_{fl}\varepsilon_{fl}$ is the inertial torque opposing flywheel acceleration; i is the gear ratio of the transmission; J_{wh1} and J_{wh2} are the moments of inertia of the front and rear wheels, respectively; $M_{\varepsilon1} = J_{wh1}\varepsilon_{wh}$ and $M\varepsilon_2 = J_{wh2}\varepsilon_{wh}$ are the inertial torques opposing wheel acceleration; M_{f1} and M_{f2} are the drag torques at the front and rear wheels of the vehicle.

Analysis of Eq. (1) shows that only the engine torque reduced to the axle of the driving wheels, the drag torques of the wheels, and the inertial torques opposing wheel and flywheel acceleration are taken into account in determining the reactions X_1 and X_2.

We now determine the sum of the reactions

$$X_2 + X_1 = (M_e - J_{fl}\varepsilon_{fl})i\eta/r - J_{wh}\varepsilon_{wh}/r - M_f/r,$$

where J_{wh} is the moment of inertia of all the vehicle's wheels; M_f is the drag torque of all the vehicle's wheels.

All the forces and torques on the housing of the machine act only at contact of the driving wheels and the supporting surface. We now reduce them to the center of the wheel–road contact spot. These forces create torques relative to the axes of rotation of the driving wheels (Fig. 3)

$$M_w = P_w(h_w - r);\ M_j = P_j(h - r);$$
$$M_\alpha = P_\alpha(h - r);\ M_x = P_x(h_x - r).$$

That, in turn, creates opposing tangential reactions from the supporting surface

$$P_{\tau w} = M_w/r = P_w(h_w - r)/r;$$
$$P_{\tau j} = M_j/r = P_j(h - r)/r;$$
$$P_{\tau\alpha} = M_\alpha/r = P_\alpha(h - r)/r;$$
$$P_{\tau x} = M_x/r = P_x(h_x - r)/r.$$

Then the vehicle's equation of motion takes the form

$$\begin{aligned} &X_2 + X_1 - P_j(h - r)/r - P_w(h_w - r)/r \\ &- P_\alpha(h - r)/r - P_x(h_x - r)/r = 0. \end{aligned} \tag{2}$$

Hence, we may write

$$\begin{aligned} &(M_e - J_{fl}\varepsilon_{fl})i\eta/r - J_{wh}\varepsilon_{wh}/r - M_f/r - mj(h - r)/r \\ &- G\sin\alpha(h - r)/r - kFv^2(h_w - r)/r \\ &- P_x(h_x - r)/r = 0. \end{aligned}$$

Since $\varepsilon_{wh} = j/r$ and $\varepsilon_{fl} = \varepsilon_{wh}i = ji/r$, we obtain the equation of motion of the wheeled vehicle in the form

$$\begin{aligned} &M_e i\eta/r - J_{fl}ji^2\eta/r^2 - J_{wh}j/r^2 - fG\cos\alpha \\ &- mj(h - r)/r - G\sin\alpha(h - r)/r - kFv^2(h_w - r)/r \\ &- P_x(h_x - r)/r = 0. \end{aligned}$$

Hence, we may write

$$P - mj\left(\frac{h-r}{r} + \frac{J_{\mathrm{fl}}\eta i^2 + J_{\mathrm{wh}}}{mr^2}\right) - G\left(f\cos\alpha + \frac{h-r}{r}\sin\alpha\right) - kFV^2\frac{h_w - r}{r} - P_x\frac{h_x - r}{r} = 0. \quad (3)$$

In Eq. (3), we now define the rotating-mass coefficient

$$\delta = \frac{h-r}{r} + \frac{J_{\mathrm{fl}}\eta i^2 + J_{\mathrm{wh}}}{mr^2}$$

and the road's drag

$$\psi = f\cos\alpha + \frac{h-r}{r}\sin\alpha.$$

Hence, we obtain the equation of motion

$$P - mj\delta - \psi G - kFV^2\frac{h_w - r}{r} - P_x\frac{h_x - r}{r} = 0,$$

where $P = M_e i\eta/r$ is the tractional force with uniform vehicle motion.

Note that Eq. (3) is fundamentally different from the equation of motion presented in the Russian and foreign literature (in [1–3], for example): it takes account of forces applied to the vehicle's housing at the driving wheels by means of the corresponding torques relative to the wheels' axis of rotation. In previous derivations of the equation of motion, these torques were incorrectly projected onto the plane of the road.

Thus, in response to doubts regarding the conventional force diagram for a wheeled vehicle and the corresponding equation of motion [4], a new equation of motion has been derived [5].

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