

# Dynamic Surface Tension of a Charged Spherical Water Droplet

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Received April 19, 2023; revised May 11, 2023; accepted May 19, 2023

**Abstract**—In asymptotic calculations of the first order of smallness in the dimensionless amplitude of water droplet oscillations, the effect of the dynamic surface tension on the oscillation parameters was investigated using a model of an ideal incompressible liquid. It was shown that the effect of the dynamic surface tension strongly manifests itself at frequencies of external influences that are inversely proportional to the water relaxation time. At such frequencies, under the action of external influences, the electrical double layer is destroyed (the ordering of water molecules in the near-surface layer is disrupted). As a result, the free energy of the surface increases, and so does the liquid surface tension. The dynamic surface tension affects the acoustic radiation from the droplet by altering the coefficient of surface tension. The contribution to the electromagnetic radiation of the oscillating droplet is made by the disruption of the order of near-surface water molecules in the electrical double layer.

**Keywords:** dynamic surface tension, ordered dipole molecules, relaxation

**DOI:** 10.3103/S1068375524020054

## INTRODUCTION

The dynamic surface tension occurs in polar liquids and is characterized by a change in the coefficient of surface tension under short-term or periodical short-term force impacts on the droplet surface [1].

The physicochemical nature of the discussed process is related to the formation of an electrical double layer at the droplet surface. The essence of this phenomenon is that water molecules are dipoles, and the free surface of water in the near-surface layer orients the dipole molecules with negatively charged poles outward from the bulk of the droplet [2, 3], which has been repeatedly confirmed in experiments. As a result, the poles of electric dipoles with a positive charge are directed inside the droplet and attract negative ions from the bulk of the droplet. As a consequence, in the near-surface layer of the liquid with a thickness of about tens of nanometers, a diffuse space electric charge appears. Since this process occurs spontaneously, the surface free energy decreases. Thus, the formation of an electrical double layer gives rise to a certain order in the orientation of dipole molecules (dipoles are known [3] to be ordered by both dipolar and quadrupolar interactions of molecules) and to a decrease in the surface energy and in the coefficient of surface tension. It was experimentally observed that the maximum value of dynamic surface tension (with complete disorder of near-surface dipoles) is 25%

higher than its equilibrium value reached at maximum order [4]. There are numerous publications dedicated to dynamic surface tension (see, e.g., [5–8]).

External force impacts disrupt the ordered structure of the electrical double layer, increasing the surface energy and, consequently, the coefficient of surface tension.

A high-frequency capillary wave propagating along the liquid surface will disrupt the order of dipoles because the liquid particles under the surface wave are in a circular motion [9, p. 57]. This, in turn, leads to an increase in surface tension. The phenomenon of spontaneous restoration of order is called relaxation, and the characteristic time for the system to return to an equilibrium state is the characteristic relaxation time.

To account for the dynamic surface tension, let us use the well-known Maxwell's formula [5], where the coefficient of surface tension of a liquid,  $\sigma$ , takes into account the surface tension in the state of order of near-surface molecules and in the state of their complete disorder:

$$\sigma = \sigma_{\infty} - \frac{\sigma_*}{1 - i\omega_n \tau_r} = \sigma_0 - \frac{i\omega_n \sigma_* \tau_r}{1 - i\omega_n \tau_r};$$
$$\sigma_* = \sigma_{\infty} - \sigma_0. \quad (1)$$

Here,  $i$  is the imaginary unit,  $\tau_r$  is the characteristic relaxation time (the time it takes for the momentarily deformed near-surface layer of the liquid droplet to return to its equilibrium state),  $\sigma_0$  is the coefficient of surface tension of the equilibrium structure of the droplet (at zero frequency of external influence),  $\sigma_\infty$  is the maximum value of the coefficient of surface tension at very high frequencies (when the ordered structure of the electrical double layer is disrupted), and  $\sigma_*$  is the correction to  $\sigma_0$  for the deviation of the near-surface layer of the droplet from the equilibrium state due to the relaxation effect.

It is obvious that accounting for dynamic surface tension will complicate the spectrum of capillary oscillations of the liquid droplet.

In light of this, it seems reasonable to investigate the effect of dynamic surface tension on capillary-wave motions of the conducting surface of the droplet.

### PROBLEM STATEMENT

Let us consider a stationary spherical droplet with radius  $R$  of an ideal, incompressible, perfectly conducting liquid with mass density  $\rho_1$ , carrying electric charge  $Q$ . The coefficient of surface tension of the interface between the media is denoted by  $\sigma$ . The external medium is an ideal incompressible dielectric medium with permittivity  $\epsilon_{\text{ex}}$  and mass density  $\rho_2$ .

The thermal motion of liquid molecules gives rise to a capillary wave motion with a very small amplitude ( $\leq 0.1$  nm [10]) on the droplet surface. However, because of the impacts of external forces (coagulation, fragmentation, collisions, friction against air, etc.), the capillary wave amplitude can significantly exceed this value [11].

Let us analyze the problem in a spherical coordinate system with the origin at the center of mass of the sphere using an axisymmetric formulation.

The motions of the liquid in the droplet and in the external medium are assumed to be potential, and the velocity field  $\vec{V}_j(r, \theta, t)$  can be expressed through the gradients of hydrodynamic potentials  $\psi_j(r, \theta, t)$  [9]:

$$\vec{V}_j(r, \theta, t) = \nabla \psi_j(r, \theta, t); \quad (2)$$

$$j = 1, 2.$$

Here, the subscripts  $j = 1$  and  $j = 2$  refer to the droplet and the medium, respectively.

The distortion of the equilibrium spherical shape  $\xi(\theta, t)$  that is caused by the capillary motion is assumed to be small along with the velocity fields of the liquid flow in the droplet and the medium,  $\vec{V}_j(r, \theta, t)$ , and the hydrodynamic potentials,  $\psi_j(r, \theta, t)$ .

The equation of the interface between the droplet and the medium at arbitrary time  $t$  is written as

$$r(\theta, t) = R + \xi(\theta, t),$$

where the amplitude of the surface wave motion of the droplet is much smaller than its equilibrium radius:

$$\max |\xi| / R \ll 1.$$

When modeling the droplet as a perfect conductor, it is assumed that the rate of redistribution of surface charge significantly exceeds the flow velocities of the liquid within it. Then the electric field in the vicinity of the droplet, which is created by its uniformly distributed charge, is considered quasi-stationary and described by the electric potential  $\Phi(r, \theta, t)$  related to the field intensity  $\vec{E}(r, \theta, t)$  by the relation

$$\vec{E}(r, \theta, t) = -\nabla \Phi(r, \theta, t). \quad (3)$$

The electric potential along the perturbed surface is denoted as  $\Phi_s$ .

### MATHEMATICAL FORMULATION OF THE PROBLEM

The mathematical formulation of the problem of capillary oscillations of a charged droplet consists of the equations of motion for the internal and external media of the droplet (Euler's equations):

$$\frac{d\vec{V}_j(r, \theta, t)}{dt} = -\frac{1}{\rho_j} \nabla P_j(r, \theta, t); \quad (4)$$

$$(j = 1, 2);$$

the continuity equations

$$\text{div } \vec{V}_j(r, \theta, t) = 0; \quad (j = 1, 2); \quad (5)$$

and Maxwell's equations

$$\text{rot } \vec{E} = 0; \quad \text{div } \vec{E} = 0. \quad (6)$$

The use of Eq. (2) allows one to transform equality (5) into Laplace's equations for the hydrodynamic potentials  $\psi_j(r, \theta, t)$

$$\Delta \psi_j(r, \theta, t) = 0; \quad (j = 1, 2); \quad (7)$$

Substitution of relation (3) into Eqs. (6) gives that the first equation in system (6) is satisfied identically, and the second is reduced to Laplace's equation for the electric field potential  $\Phi(r, \theta, t)$ :

$$\Delta \Phi(r, \theta, t) = 0. \quad (8)$$

From Eqs. (4), the expressions for the pressures in the droplet,  $P_1(r, \theta, t)$ , and in the external medium,  $P_2(r, \theta, t)$ :

$$P_j(r, \theta, t) = P_{0j} - \rho_j \frac{\partial \psi_j(r, \theta, t)}{\partial t}; \quad (j = 1, 2);$$

where  $P_{0j}$  are the constant pressures inside ( $j = 1$ ) and outside ( $j = 2$ ) the droplet.

In addition, it is necessary that the sought potentials  $\psi_j(r, \theta, t)$  and  $\Phi(r, \theta, t)$  should satisfy conditions

of boundedness (natural boundary conditions) at the origin and at infinite distances from the droplet:

$$r \rightarrow 0: \psi_1(r, \theta, t) \rightarrow 0; \quad (9)$$

$$r \rightarrow \infty: \psi_2(r, \theta, t) \rightarrow 0; \quad \Phi(r, \theta, t) \rightarrow 0. \quad (10)$$

At the interface between the media,  $r = R + \xi(\theta, t)$ , the hydrodynamic and electrostatic boundary conditions should be satisfied:

the equality of the normal components of the motion velocities of the internal and external media:

$$\begin{aligned} \vec{n}_1 \times \nabla \psi_1(r, \theta, t) &= \vec{n}_2 \times \nabla \psi_2(r, \theta, t) \\ &\equiv \vec{n} \times \nabla \psi(r, \theta, t); \quad \vec{n} = \vec{n}_1 = -\vec{n}_2; \end{aligned}$$

the kinematic condition:

$$\frac{\partial \xi(\theta, t)}{\partial t} \approx \vec{n} \times \nabla \psi(r, \theta, t);$$

the dynamic condition for the normal component of the stress tensor:

$$P_1(r, \theta, t) - P_2(r, \theta, t) + P_q(r, \theta, t) - P_\sigma(r, \theta, t) = 0;$$

$$P_q(r, \theta, t) = \frac{\varepsilon_{\text{ex}}}{8\pi} (\nabla \Phi(r, \theta, t))^2;$$

$$P_\sigma(r, \theta, t) = \sigma \operatorname{div} \vec{n}(r, \theta, t)$$

and the condition of the constancy of the electric potential on the droplet surface:

$$\Phi(r, \theta, t) = \Phi_s.$$

Let us supplement the problem formulated above with the additional integral conditions for the conservation of the droplet volume and the absence of motion of its center of mass:

$$\int_V r^2 dr \sin \theta d\theta d\varphi = \frac{4}{3} \pi R^3; \quad \int_V \vec{r} r^2 dr \sin \theta d\theta d\varphi = 0;$$

$$V = [0 \leq r \leq R + \xi(\theta, t), 0 \leq \theta \leq \pi, 0 \leq \varphi \leq 2\pi];$$

and the constancy of the total charge of the droplet:

$$-\frac{\varepsilon_{\text{ex}}}{4\pi} \oint_S \vec{n}(r, \theta, t) \times \nabla \Phi(r, \theta, t) dS = Q;$$

$$S = [r = R + \xi(\theta, t), 0 \leq \theta \leq \pi, 0 \leq \varphi \leq 2\pi].$$

In the above expressions,  $\vec{n}_1$  is the external normal vector to the droplet,  $\vec{n}_2$  is the external normal vector to the external medium,  $P_q(r, \theta, t)$  is the pressure of electric forces, and  $P_\sigma(r, \theta, t)$  is the pressure of surface tension forces.

Let us seek the solution to formulated problem (7)–(11) using classical methods of perturbation theory [12] in the linear approximation in the small parameter  $\varepsilon \equiv |\xi|/R \ll 1$ , which is the ratio of the amplitude of capillary oscillations to the droplet radius. By the direct perturbation method, the perturbation of the equilibrium shape  $\xi(\theta, t)$ , the potentials  $\psi_j(r, \theta, t)$  and

$\Phi(r, \theta, t)$ , and the pressures  $P_j(r, \theta, t)$ ,  $P_q(r, \theta, t)$ , and  $P_\sigma(r, \theta, t)$  are represented as asymptotic expansions:

$$\xi(\theta, t) = \xi^{(1)}(\theta, t) + O(\varepsilon^2);$$

$$\psi_j(r, \theta, t) = \psi_j^{(1)}(r, \theta, t) + O(\varepsilon^2); \quad j = 1, 2;$$

$$\Phi(r, \theta, t) = \Phi^{(0)}(r, \theta) + \Phi^{(1)}(r, \theta, t) + O(\varepsilon^2);$$

$$P_j(r, \theta, t) = P_j^{(0)}(r, \theta, t) + P_j^{(1)}(r, \theta, t) + O(\varepsilon^2);$$

$$P_q(r, \theta, t) = P_q^{(0)}(r, \theta) + P_q^{(1)}(r, \theta, t) + O(\varepsilon^2);$$

$$P_\sigma(r, \theta, t) = P_\sigma^{(0)}(r, \theta) + P_\sigma^{(1)}(r, \theta, t) + O(\varepsilon^2),$$

where the superscript denotes the order of smallness in  $\varepsilon$  for the corresponding components.

## ANALYSIS

In the general case, let us decompose the complex frequency of oscillations of the  $n$ th mode of the droplet into real and imaginary parts,

$$\omega_n \equiv \operatorname{Re} \omega_n + i \operatorname{Im} \omega_n \equiv \omega_n' + i\eta_n,$$

and solve the problem using standard methods (see, e.g., [13]). The frequency is assumed to be complex because, in a droplet of a polar liquid, such as water, capillary surface waves disrupt the order of molecules in the near-surface layer and alter the free energy of the water surface, leading to damping. In the end, using Eq. (1), one can derive the dispersion equation for capillary oscillations of the droplet, taking into account dynamic surface tension:

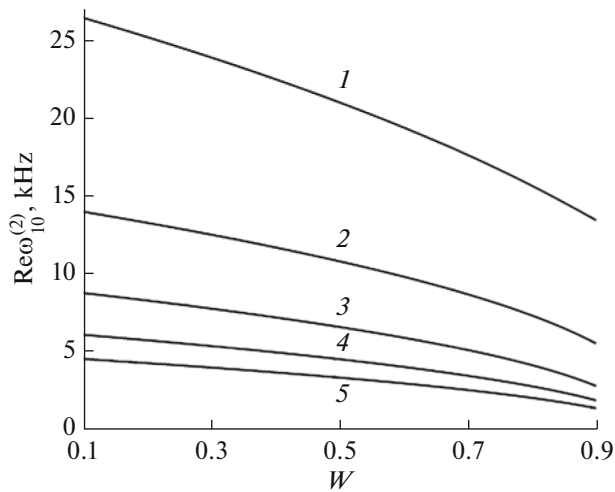
$$\omega_n^3 + i \frac{\omega_n^2}{\tau_r} - \omega_n \left( \omega_{n0}^2 + \frac{\sigma_*}{\sigma_0} \omega_{n0}'^2 \right) - i \frac{\omega_{n0}^2}{\tau_r} = 0; \quad n \geq 2; \quad (12)$$

$$\omega_{n0}'^2 = \frac{\sigma_0}{R^3} \frac{n(n-1)(n+1)(n+2)}{(\rho_1(n+1) + \rho_2 n)};$$

$$\omega_{n0}^2 = \frac{\sigma_0}{R^3} \frac{n(n-1)(n+1)(n+2)}{(\rho_1(n+1) + \rho_2 n)} \left[ 1 - \frac{W}{(n+2)} \right];$$

$$W = \frac{Q^2}{4\pi\sigma_0 R^3},$$

where the subscript 0 refers to the equilibrium state of the droplet surface;  $n$  is the mode number;  $\omega_n$  is the complex frequency of oscillations of the  $n$ th mode of the charged droplet with the disrupted ordered structure of the electrical double layer,  $\operatorname{Re} \omega_n \equiv \omega_n'$  is the natural frequency of oscillations of the charged droplet with the disrupted ordered structure of the electrical double layer;  $\eta$  is the damping ratio;  $W$  is the Rayleigh parameter [14], characterizing the stability of the droplet with respect to its own charge;  $\omega_n$  is the frequency of oscillations of the  $n$ th mode,  $\omega_{n0}$  and  $\omega_{n0}'$  are the frequencies of oscillations of the  $n$ th modes of the



**Fig. 1.** Dependences of the real component  $\text{Re } \omega_{10}^{(2)}$  of the complex frequency of oscillations of a charged droplet on the Rayleigh parameter  $W$  at  $n = 10$ ;  $\epsilon_{\text{ex}} = 1$ ;  $\rho_1 = 1 \text{ g/cm}^3$ ;  $\rho_2 = 1.3 \times 10^{-3} \text{ g/cm}^3$ ;  $\sigma_0 = 73 \text{ dyn/cm}$ ;  $\sigma_\infty = 91 \text{ dyn/cm}$ ;  $\sigma^* = 18 \text{ dyn/cm}$ ;  $\tau_r = 1 \times 10^{-4} \text{ s}$ ; and  $R = (1) 0.05$ , (2) 0.075, (3) 0.1, (4) 0.125, and (5) 0.15 cm.

charged and uncharged droplets, respectively, with the equilibrium structure of the near-surface layer of the liquid.

It is easy to see that the second and fourth coefficients of the dispersion equation are imaginary, and the consideration of dynamic surface tension leads to an increase in the order of the dispersion equation in comparison with the equation written without considering relaxation. This is due to the appearance of damping of capillary oscillations of the droplet because of the relaxation of surface energy (coefficient of surface tension) toward their equilibrium values during the restoration of the electrical double layer.

For numerical illustration, let us turn to naturally occurring droplets: clouds, fogs [15, 16]. Let us evaluate the characteristics of oscillations for the tenth mode ( $n = 10$ ), as the period of their oscillations is close to the characteristic relaxation time.

To find numerical solutions, let us take the following average physicochemical characteristics of water droplets:  $\sigma_0 = 73 \text{ dyn/cm}$ ,  $\rho_1 = 1 \text{ g/cm}^3$ ,  $Q = 2 \times 10^{-5} \text{ statC}$ ,  $\epsilon_{\text{ex}} = 1$ ,  $\sigma_\infty = 91 \text{ dyn/cm}$ ,  $\sigma^* = 18 \text{ dyn/cm}$ ,  $\rho_2 = 1.3 \times 10^{-3} \text{ g/cm}^3$ , and  $\tau_r = 1 \times 10^{-4} \text{ s}$ .

Analysis of the numerical solutions showed that the first root  $\omega_{10}^{(1)}$  determines purely aperiodic decay, while the second and third roots  $\omega_{10}^{(2)}$  and  $\omega_{10}^{(3)}$  describe slowly decaying oscillations of cloud droplets (in this case, the damping decrement is four orders of magnitude lower than the frequency of intrinsic oscillations), and very fast decaying oscillations of small raindrops ( $\eta_{10}^{(j)}$

is one to two orders of magnitude lower than  $\text{Re } \omega_{10}^{(j)}$ ). Simultaneously, an increase in the droplet size leads to an increase in the ratio  $|\eta_{10}^{(j)}|/\text{Re } \omega_{10}^{(j)}$ , so that the oscillations of large raindrops are aperiodic. From the obtained numerical solutions, it is evident that a decrease in droplet size, considering the effect of dynamic surface tension, leads to an increase in the natural frequencies of oscillations in comparison with the frequencies determined without considering relaxation (see [13]). From this, it can be concluded that the damping oscillations of cloud droplets that are induced by surface tension relaxation correspond to high frequencies  $\text{Re } \omega_{10} \tau_r \gg 1$ .

To illustrate the dependences of the characteristics of damping droplet oscillations, let us separate the real component  $\text{Re } \omega_n^{(j)}$  and the imaginary component  $\text{Im } \omega_n^{(j)}$  of the analytical expressions for the frequency. The solutions to Eq. (12), obtained using the MATHEMATICA software, have the form

$$\text{Re } \omega_n^{(1)} = 0; \quad \text{Re } \omega_n^{(2)} = -\frac{\sqrt{3}}{2} \alpha_0 + \frac{\sqrt{3}}{2} \alpha_1;$$

$$\text{Re } \omega_n^{(3)} = \frac{\sqrt{3}}{2} \alpha_0 - \frac{\sqrt{3}}{2} \alpha_1;$$

$$\text{Im } \omega_n^{(1)} \equiv \eta_n^{(1)} = -\left(\frac{\chi_1}{3} + \alpha_0 + \alpha_1\right);$$

$$\text{Im } \omega_n^{(2)} = \text{Im } \omega_n^{(3)} \equiv \eta_n^{(2)} = -\left(\frac{\chi_1}{3} + \frac{\alpha_0}{2} + \frac{\alpha_1}{2}\right);$$

$$\alpha_0 = \frac{\sqrt[3]{2\delta_0}}{3(\delta_1 + \delta_2)^{\frac{1}{3}}}; \quad \alpha_1 = \frac{(\delta_1 + \delta_2)^{\frac{1}{3}}}{3\sqrt[3]{2}};$$

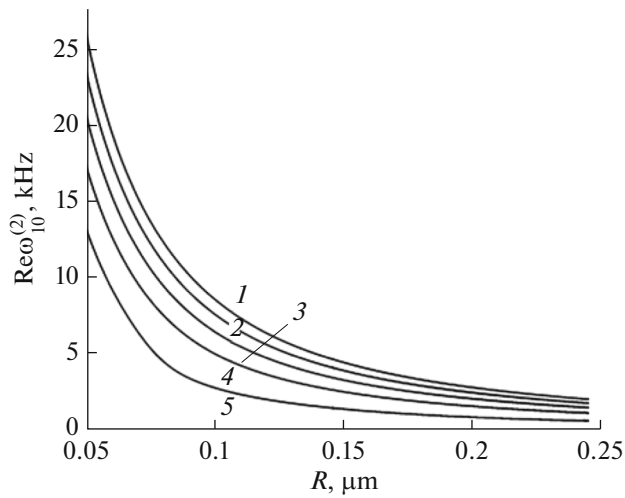
$$\delta_0 = \chi_1^2 - \chi_2; \quad \delta_1 = 2\chi_1^3 - 9\chi_1\chi_2 + 27\chi_3;$$

$$\delta_2 = 3\sqrt[3]{\chi_1^2\chi_2^2 - 4\chi_2^3 - 4\chi_1^3\chi_3 + 18\chi_1\chi_2\chi_3 - 27\chi_3^2};$$

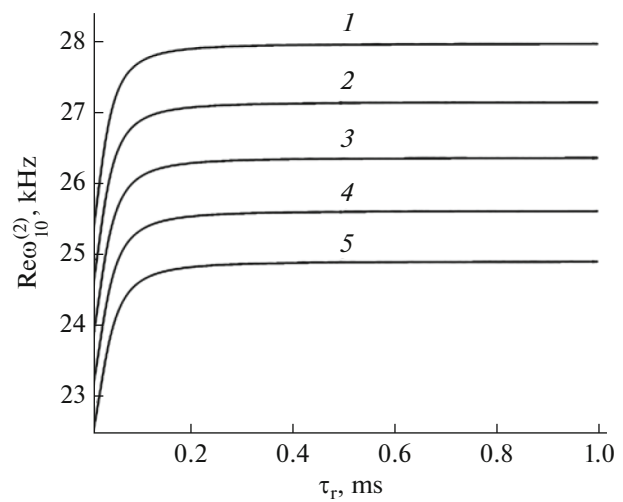
$$\chi_1 = \frac{1}{\tau_r}; \quad \chi_2 = \omega_{n0}^2 + \frac{\sigma^*}{\sigma_0} \omega_{n0}^2; \quad \chi_3 = \frac{\omega_{n0}^2}{\tau_r}.$$

Figures 1 and 2 depict the dependences of the real part  $\text{Re } \omega_{10}^{(2)}$  of the complex frequency on the uncompensated charge on the droplet (on the Rayleigh parameter  $W$ ) and its radius, respectively, according to (20). These dependences are qualitatively expectable from the general trends in electrohydrodynamics: the frequency is seen to decrease with increasing charge and droplet radius.

Figure 3 shows the dependences of  $\text{Re } \omega_{10}^{(2)}$  on the characteristic relaxation time  $\tau_r$  and radius. It can be observed that the dependences on  $\tau_r$  at all considered droplet radii are approximately linear for  $\tau_r < 0.1 \text{ ms}$  and  $\tau_r > 0.1 \text{ ms}$ , but with significantly different slopes: in the first case, the dependence of  $\text{Re } \omega_{10}^{(2)}$  on  $\tau_r$  is



**Fig. 2.** Dependences of the real component  $\text{Re } \omega_{10}^{(2)}$  of the complex frequency of oscillations of a charged raindrop on its radius  $R$  at the same parameters as in Fig. 1, except  $W =$  (1) 0.1, (2) 0.3, (3) 0.5, (4) 0.7, and (5) 0.9.



**Fig. 3.** Dependences of the real component  $\text{Re } \omega_{10}^{(2)}$  of the complex frequency of oscillations on the characteristic relaxation time  $\tau_r$  at the same parameters as in Fig. 1, except  $R =$  (1) 0.5, (2) 0.51, (3) 0.52, (4) 0.53, and (5) 0.54 mm.

much stronger. That is, the dependence of  $\text{Re } \omega_{10}^{(2)}$  on the characteristic relaxation time  $\tau_r$  at short  $\tau_r$  is more significant. If one also takes into account the disruption of the order of water molecules in the near-surface layer under short-term force influence and the increase in the coefficient of surface tension of water in this region, everything becomes clear.

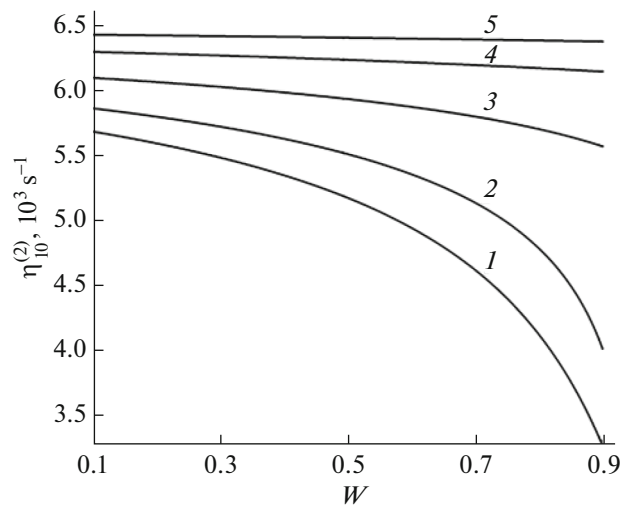
Figures 4 and 5 present the dependences of the damping decrement caused by the relaxation of surface tension on the charge (Rayleigh parameter) and droplet radius. The decrease in the damping decrement with an increase in charge to values close to critical for the droplet instability relative to its own charge is caused by the decrease in the oscillation frequency as the droplet charge approaches the critical value [13]. The growth of the damping decrement with an increase in droplet radius and a decrease in droplet charge, with the dependence  $\eta_{10}^{(2)}(R, Q)$  approaching a slowly increasing asymptote, is likely to be a result of the combination of two trends:  $\eta_{10}^{(2)}$  increases with increasing  $R$ , but decreases with decreasing  $Q$ .

Figure 6 shows the dependence of the damping decrement  $\eta_{10}^{(2)}$  of capillary oscillations of a droplet with radius  $R = 1.1$  mm due to the relaxation of surface tension on the characteristic relaxation time  $\tau_r$ . One can see that, with a decrease in  $\tau_r$ , the damping decrement rapidly increases, corresponding to an increased role of disrupting the order of water molecules in its near-surface layer at small  $\tau_r$ .

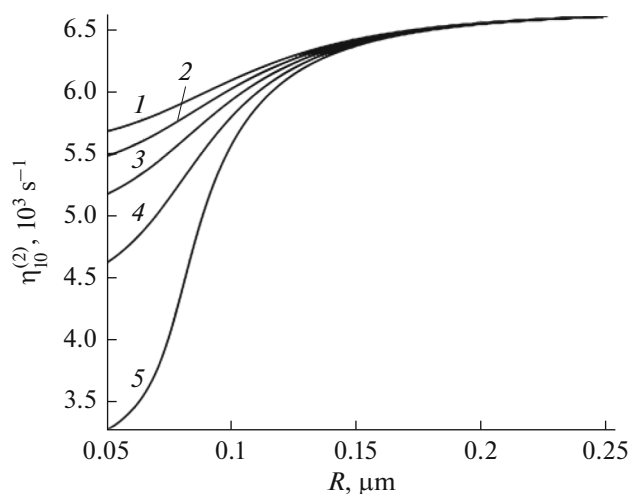
Figures 7 and 8 illustrate the time dependences of the amplitude  $M_{10}(t)$  of the disturbance of the equilibrium spherical shape of a charged droplet with a radius of  $R = 1.283$  mm and a charge of  $Q = 7 \times 10^{-4}$  statC

during damped capillary oscillations due to the relaxation of surface tension.

Figure 7 demonstrates that the shape of the curves and the damping rate strongly depend on the smallness of the assumed value of the characteristic relaxation time  $\tau_r$ . They vary from very rapid damping for curve 1 ( $\tau_r = 10^{-5}$  s) to rather slow damping ( $\tau_r = 10^{-3}$  s). Figure 8 shows the curves from the small vicinity of the point  $|\eta_{10}^{(2)}|/\text{Re } \omega_{10}^{(2)} = 1$  ( $\tau_r = 10^{-4}$  s) where damping is fast. Curves 2 in Figs. 1 and 2 are identical.



**Fig. 4.** Dependences of the damping decrement  $\eta_{10}^{(2)}$  of capillary oscillations of a charged raindrop on the Rayleigh parameter  $W$  at the same parameters as in Fig. 1, except  $R =$  (1) 0.05, (2) 0.075, (3) 0.1, (4) 0.125, and (5) 0.15 cm.



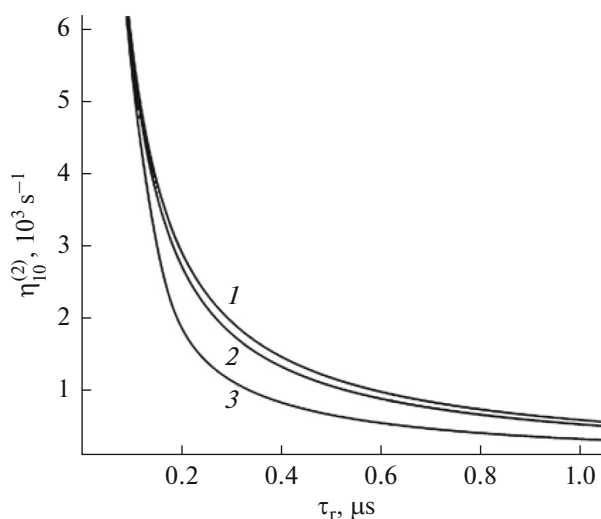
**Fig. 5.** Dependences of the damping decrement  $\eta_{10}^{(2)}$  of capillary oscillations of a charged raindrop on its radius  $R$  at the same parameters as in Fig. 1, except  $W = (1) 0.1, (2) 0.3, (3) 0.5, (4) 0.7, \text{ and } (5) 0.9$ .

Figure 9 presents the dependence of the amplitude  $M_{10}(t)$  of the disturbance of the equilibrium shape of a charged droplet with a radius of  $R = 1.283$  mm and a charge of  $Q = 7 \times 10^{-4}$  statC on the characteristic relaxation time  $\tau_r$ , calculated at various moments of real time.

Let us intersect the curves in Fig. 9 by vertical lines corresponding to specific values of  $\tau_r$  to obtain a representation of the change in the oscillation amplitude over real time at the given values of  $\tau_r$ . It is seen that, at short  $\tau_r$ , the amplitude of capillary oscillations drops to zero within half a period of oscillation.

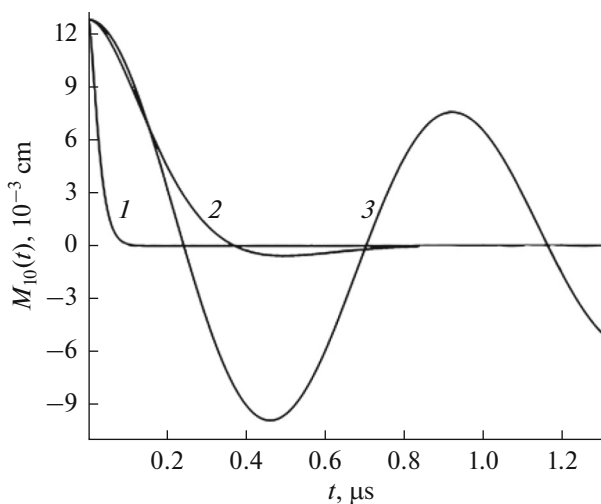
#### ELECTRICAL DOUBLE LAYER, ITS DESTRUCTION AND RESTORATION

Based on the above graphs (see, e.g., Figs. 7–9), one can see the following picture of the time evolution of the boundary electrical double layer. The electrical double layer at the surface of a polar liquid forms spontaneously with the appearance of the spontaneous orientation of near-surface dipoles with negatively charged ends directed outward of the liquid because this process is accompanied by a decrease in the free energy of the liquid surface (it was experimentally determined that the coefficient of surface tension of the liquid decreases [4, 8]). Positively charged ends of dipoles attract negative ions from the bulk of the liquid and induce additional orientation of dipole molecules, resulting in a stable structure rigidly attached to the surface and diffuse on the side of the liquid. During a short-term (when the interaction time is approximately equal to the characteristic relaxation time) force impact on the liquid surface, this electrical double layer is disrupted, this disruption is contrib-



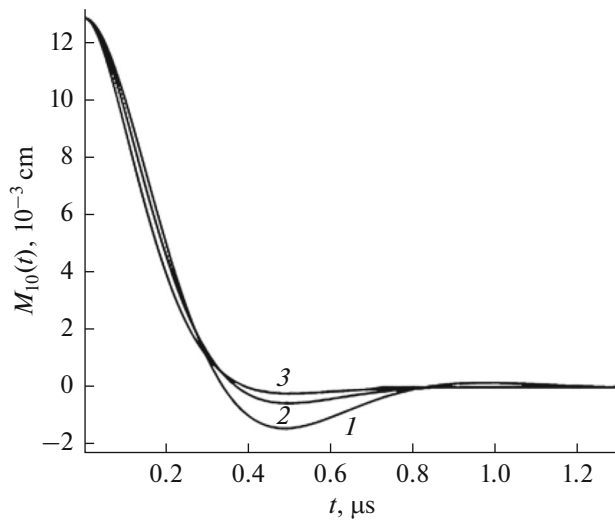
**Fig. 6.** Dependences of the damping decrement  $\eta_{10}^{(2)}$  of capillary oscillations of a droplet with a radius of  $R = 1.1$  mm on the characteristic relaxation time  $\tau_r$  at the same parameters as in Fig. 1, except  $W = (1) 0.1, (2) 0.5, \text{ and } (3) 0.9$ .

uted by the mechanical energy of the wave, the free energy of a unit surface increases, and so does the coefficient of surface tension (which is determined by the surface density of free energy [17, p. 351]). An example of a force impact on the liquid surface can be a high-frequency capillary wave running along the surface (or standing, as in the case of a droplet), disrupting the order of dipoles at the liquid surface [9, p. 57].



**Fig. 7.** Time dependences of the amplitude  $M_{10}(t)$  of the disturbance of the equilibrium shape of a charged droplet with a radius of  $R = 1.283$  mm and a charge of  $Q = 7 \times 10^{-4}$  statC at  $n = 10; \epsilon_{ex} = 1; \rho_1 = 1 \text{ g/cm}^3; \rho_2 = 1.3 \times 10^{-3} \text{ g/cm}^3; \sigma_0 = 73 \text{ dyn/cm}; \sigma_\infty = 91 \text{ dyn/cm}; \sigma^* = 18 \text{ dyn/cm}; \text{ and } \tau_r = (1) 1 \times 10^{-5}, (2) 1 \times 10^{-4}, \text{ and } (3) 1 \times 10^{-3} \text{ s}$ .





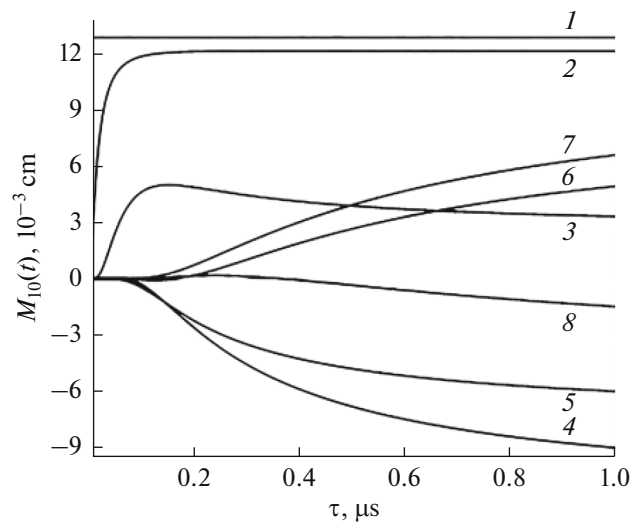
**Fig. 8.** Time dependences of the amplitude  $M_{10}(t)$  of the disturbance of the equilibrium shape of a charged droplet with a radius of  $R = 1.283$  mm and a charge of  $Q = 7 \times 10^{-4}$  statC during damping capillary oscillations at the same parameters as in Fig. 1, except (1)  $|\eta_{10}^{(2)}|/\text{Re } \omega_{10}^{(2)} = 0.7$  at  $\tau_r = 1.366 \times 10^{-4}$  s, (2)  $|\eta_{10}^{(2)}|/\text{Re } \omega_{10}^{(2)} = 1$  at  $\tau_r = 1 \times 10^{-4}$  s, and (3)  $|\eta_{10}^{(2)}|/\text{Re } \omega_{10}^{(2)} = 1.3$  at  $\tau_r = 0.791 \times 10^{-4}$  s.

Within the time  $\tau_r$ , the order of dipoles (equilibrium structure of the near-surface layer of the liquid) is restored. Then the described process resumes.

#### ACOUSTIC AND ELECTROMAGNETIC RADIATION OF AN OSCILLATING CHARGED DROPLET

When discussing the radiation of an oscillating droplet, it should be noted that acoustic radiation can change only with a change in the coefficient of surface tension of the liquid, approximately proportional to  $\sqrt{\sigma}$ . As for electromagnetic radiation, the issue is somewhat more complex. The droplet is charged, and the liquid (at least in the near-surface layer) consists of polar molecules creating an electric field around them. When the order in the orientation of molecules is disrupted, the electrostatic field created by them disappears, leading to the generation of an electromagnetic pulse. The question arises about estimating the intensity of such radiation at least in order of magnitude. The only thing that can be said about such radiation is that it should be quadrupolar because the parallel orientation of water dipoles on its surface is due to the interaction of water quadrupoles with dipoles [3], and its frequency should be on the order of  $\tau_r^{-1}$ .

The intensity of regular electromagnetic radiation due to oscillations of a charged droplet has long been calculated [18, 19].



**Fig. 9.** Dependences of the amplitude  $M_{10}(t)$  of the disturbance of the equilibrium shape of a charged droplet with a radius of  $R = 1.283$  mm and a charge of  $Q = 7 \times 10^{-4}$  statC on the characteristic relaxation time  $\tau_r$ , at the same parameters as in Fig. 7, except  $t =$  (1) 0, (2) 0.05, (3) 0.2, (4) 0.4, (5) 0.6, (6) 0.8, (7) 1, and (8) 1.2 ms.

#### CONCLUSIONS

The coefficient of dynamic surface tension may depend on the frequency of external influences on the water surface. It was found that the orientation of dipoles in the near-surface layer of the liquid weakly responds to force impacts with frequencies  $\omega \ll \tau_r^{-1}$ , but when subjected to a frequency  $\omega \sim \tau_r^{-1}$ , the ordered orientation of water dipoles in the near-surface layer is disrupted, leading to an increase in the coefficient of surface tension of the liquid.

#### FUNDING

This work was supported by the Russian Science Foundation (project no. 19-19-00598 "Hydrodynamics and Energy of Droplets and Droplet Streams: Formation, Motion, Breakup, and Interaction with Contact Surface," <https://rscf.ru/project/19-19-00598/>).

#### CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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*Translated by M. Baznat*

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