Liquid Viscosity Effect on Drop Formation Regularities under Electrohydrodynamic Instability Realization

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Abstract—The peculiarities of a liquid viscosity effect on the liquid electrodispersion from the end of capillary through which the liquid is supplied to the discharged system or during the disintegration of a strongly charged drop were studied in an analytical way. It was shown that, upon the electrodispersion of the electroconducting liquid with a low-viscosity, the latter emitted highly charged droplets, initially unstable with respect to their own charge, breaking up into hundreds of even smaller and strongly charged ones, with a corona discharge being ignited around them. As a result, a fan-shaped glow appeared at the top of the liquid meniscus at the end of the capillary or at the top of the decaying charged drop. For the viscous electroconducting liquid, the series of successive disintegrations of the charged daughter droplets were immediately interrupted owing to viscosity damping effect of self-charge-resistant droplets, and no corona discharge glow was formed.

INTRODUCTION

Capillary electrodispersion in liquids is extensively used in science and engineering for the creation of ion-cluster-drop beams in liquid mass-spectrometry (to analyze heavy-volatile and organic substances). It is used as well in liquid-metal sources of ions, in liquid-metal epitaxy and lithography, for obtainment refractory powders, for reactive cosmic engineering, for fast scattering of dense aerodispersive systems, on creation of monodisperse drop flows in thermonuclear synthesis, in drop-jet print, macroparticle accelerators, for insecticides spraying, fuel-lubrication materials, lacquers, and paints. In addition, the phenomenon of the charged drops' dispersion in the external electric fields is used to interpret the geophysical phenomena, such as Saint Elmo's fire, waterspout lights, initiations of the lightning discharge and its gathering charges from separate clouds' drops to maintain its own existence. In particular, the charged drops' oscillations in the external electric fields induce radio radiation from the thunderstorm clouds [1].

Over the past dozens of years, attempts have constantly been made to develop classification of the observed modes of the electrodispersed liquid both on experimental [2–6] and theoretical bases [7]. By the present time, a few tens of modes were defined (and their number is still increasing), since the electrodispersion phenomenology varies during the change in any physicochemical property of the dispersed liquid, upon the change of the working liquid, or the change in the parameters of the device and the external environment. In this connection, it seems useful to investigate the peculiarities of the drops' formation at realization of the electrostatic instability.

At the beginning of the 20th century, at the onset of the gas discharges study, J. Zeleny carried out the first experiments [8–10] of examining the electric discharges from the liquid meniscus at the vertex of capillary through which the liquid was supplied to the discharge system. In a few years, the experiments of Zeleny on the electric charges from the liquid electrode were repeated by English [11] with the use of a more perfect equipment and having a more complete knowledge on the gas discharges. On the whole, he supported the conclusions of Zeleny.

In [8–11], the electric discharge from the liquid meniscus was accompanied by the emission of strongly (exceeding the Rayleigh limit [12, 13]) charged drops, around which the corona discharge ignited and the glow appeared. The glowing drops formed a luminous fan-region (if observed from sideways) round the meniscus top. That is why Zeleny referred to this phenomenon as a "fan" glowing. Later, in other experiments (performed in connection with Saint Elmo's fire study), the fan glow was also observed in the region of the strongly charged drops
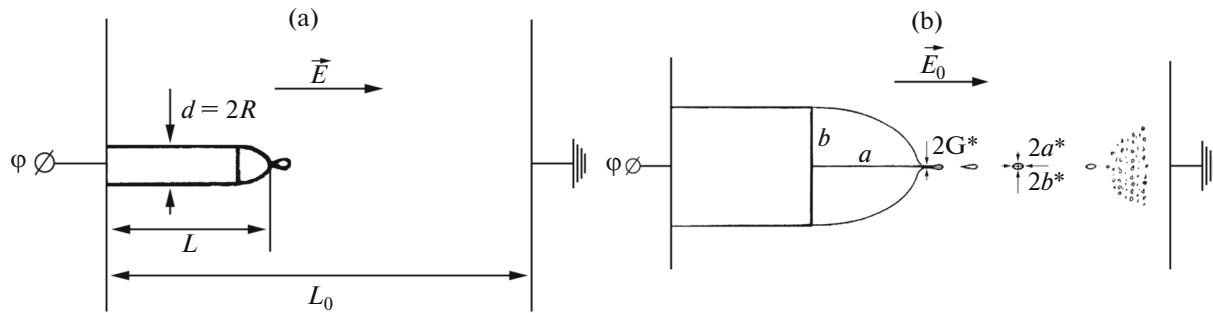
Fig. 1. Device scheme for electrodispersion of liquid: (a) General view of meniscus of a low viscous liquid at the capillary vertex, (b) separating a droplet in the external electrostatic field.

sunken upon weakly conducting objects [14, 15]. It was found that the phenomenology of the glow formation upon the liquid meniscus discharge depends on the liquid viscosity. Thus, it generates only at the electrodispersion of low viscous liquids (spiritus or water), starting directly from the meniscus vertex at the end of capillary, through which liquid is supplied to the discharge system. It can appear as well at the top of a strongly charged drop, at a distance of an order of diameter of the emitted daughter droplet [8, 10]. The fan-glow is formed directly at the meniscus vicinity (a drop) due to the electrostatic disintegration into several hundred daughter droplets of strongly charged (exceeding the Rayleigh limit [12, 13]) daughter droplets that, in turn, also disintegrate, and due to ignition of corona discharge glowing around them. For the viscous liquids, a thin jet of liquid is thrown from the meniscus vertex, which disintegrates into separate strongly charged droplets that, in turn, are divided into parts of comparable sizes, stable to the electrostatic disintegration (because of the strong damping effect of the liquid viscosity) [16], and no fan glowing appears. The length of the jet is determined by viscosity of the liquid. For instance, that for glycerol is an order of tens of the capillary diameters [9].

In connection with the aforementioned by the example of the schematic device for the liquid electrodispersion (Fig. 1a), let us discuss the regularities of drop formation of light viscous and extremely viscous liquids.

PROBLEM FORMULATION

The electroconducting incompressible liquid with mass density ρ and coefficient of kinematic viscosity v is supplied to the discharge system through R capillary radius under P pressure. The length of capillary, together with the liquid meniscus at its vertex, is L. The volume liquid consumption is χ. The absolute temperature of the system T is accepted as constant. Between the capillary and counter-electrode the constant difference of potentials φ is maintained so that the electrostatic field intensity between the flat electrodes is $E_0 \approx \varphi/L_0$, and the liquid meniscus turns out to be superficially charged. Let us accept that the electric field intensity in the vicinity of the meniscus vertex (at the point of the droplet separation) will be $E \approx 3E_0$.

The droplet separates from the meniscus under the electrostatic field, which acts from the side of the external electrostatic field and the field of the meniscus charge, when the charge of the droplet is separated. The calculation scheme of the droplet separation regularities from the liquid meniscus is described in detail in [17, p. 53, problem 1]. The surface tension force (the Laplace force) in radius constriction keeps the drop (Fig. 1b) [18]. The calculations [13] show that the separated drop charge is higher than the critical one according to Rayleigh, and it disintegrates during the time of the period of the main mode oscillations of the thrown droplet, throwing about a hundred strongly charged daughter droplets smaller by two orders, which, in turn, disintegrate in accordance with the same law. As a result, a polydisperse in sizes and charges flare of electrodispersion, is formed (Fig. 2) for devices used for dispersion of paintwork materials, chemical poisons, and fuels. During disintegration, the strongly charged drop loses $\approx 23\%$ of its charge and approximately 0.05% of its mass and becomes stable with respect to its own charge [19]. However, because all emitted droplets pass similar accelerating difference of potentials and obtain similar energy, small droplets will have a higher rate compared to the large ones with the lesser charge than Rayleigh's. Therefore, the strongly charged small droplets will catch up the large droplets, whose charge is less than critical according to Rayleigh's, and they will additionally charge them and lead them to a new Rayleigh's disintegration (Fig. 3).

In a stationary mode of work of the device for the liquid electrodispersion, the amount of the liquid that flows into the meniscus per unit of time χ must equal the liquid consumption for the electrodispersion $4\pi r^3 N/3$, where r is the droplet radius and N is the number of droplets emitted from the meniscus vertex

per a unit of time. Under these conditions, the characteristic time between the separation of two sequential droplets $\Delta\tau_0$ will be determined as a ratio of the time unit to N. From a strict theoretical viewpoint, this time must be much more than a unit divided into the increment of instability development in γ constriction, which binds meniscus with the separating droplet. Here, $\Delta\tau_0 \gg 1/\gamma$.

To correctly comprehend the viscosity effect on the regularities of the liquid dispersion, we must take into account that, at a constant value of the liquid kinematic viscosity coefficient, it can manifest itself both as extremely and lightly viscous, depending on geometric sizes of the region it occupies. Let us consider that, during the motion of the liquid along the capillary, the viscosity effect (per friction force balance on the capillary and the flow inertia force) will depend on capillary radius R and liquid density ρ: for a thin capillary, the friction force will exceed the inertion force, while it is just the opposite for a thick capillary. Therefore, let us dedimensionalize the coefficient of the liquid kinematic viscosity ν to $R\,\rho$ and the coefficient of liquid surface tension σ. To the same dedimensionalizing (in the SGSE system of physical units), all of the rest physical values will be subjected, which will be met in the problem. The problem dimensionless viscosity is as follows:

$$\frac{\nu}{\nu_*} = \nu\sqrt{\frac{\rho}{\sigma R}}.$$

As is seen, the value of the dimensionless viscosity depends on ν, R, ρ, and σ. For the light viscous liquid, the drop separation occurs directly from the meniscus vertex (Fig. 1b). The droplet separates from the meniscus vertex, where the field intensity is maximum $E \approx 3E_0$, and it is connected by the constriction (in the form of cateonoid) with the meniscus [13, 18]. The constriction breaks at its most narrow place with the radius r_*, where $r_* < r$ [18].

For the viscous liquids, all other things being equal, the unsteady development increment in the constriction will be substantially smaller, and the instability development time, respectively, greater [20, 21]. As to the liquid, it will keep on flowing, and a protrusion will form at the meniscus vertex, behind which the $2r$ thick jet will come out of the meniscus under the effect of the internal pressure in the liquid and due to the external electrostatic field. The separation of the droplet will occur from the vertex of the jet, where the electric field intensity will increase compared to $3E_0$ due to a charge induced by the $3E_0$ field at the jet vertex (the longer the jet, the higher the induced charge value). The vertex of the jet will be separated under the external electrostatic field with the charge on it due to the development of capillary waves instability in the charged jet, as is schematically shown in Fig. 4.
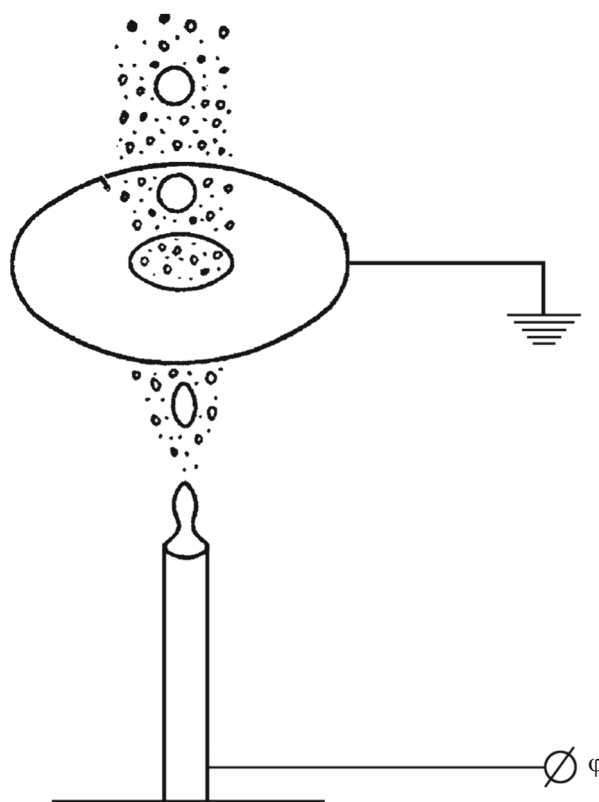

Fig. 2. General scheme of the device for electrodispersion used in practice.

Fig. 3. Scheme of additional charging of slow big drops by small, fast, and strongly charged.
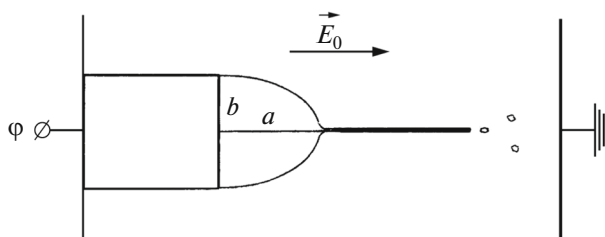

Fig. 4. General view of meniscus of a high viscous liquid at the end of capillary, separating a jet in the external electrostatic field.

Thus, the effect of the liquid viscosity per an increment value of the axisymmetric jet instability must be considered. In other words, the problem is to be solved on the instability of the charged axisymmetric jet of the viscous incompressible liquid.

PROBLEM DEFINITION

A continuous cylindric jet of R radius of viscous incompressible electroconducting liquid with density ρ, coefficient of kinematic viscosity v, and coefficient of surface tension σ moves along the symmetry axis at a rate of $\vec{U}_0$.

We shall assume that the jet is maintained at a constant electric potential Φ_* and the electric charge is distributed across its surface with a superficial charge density χ_0. Let us pass into the inertial system of coordinates, which moves together with the jet at its rate $\vec{U}_0$. In such a system of calculation, the field of rates of the liquid flow in the jet $\vec{U}(\vec{r},t)$ is determined by thermal waves [22]. The capillary waves' amplitude on the jet surface will be a small value, on the order of magnitude of tens of shares of angstrom for any liquids: from cryogenic to liquid metals. Indeed, in a liquid, there is always a thermal (generated by the thermal motion of its motion) amplitude capillary wave motion $\sim\sqrt{\kappa T/\sigma}$ where κ is the Boltzmann constant and T is the absolute temperature [22]. The surface charge density on the jet disturbed by the capillary wave motion of the thermal amplitude will be $\chi(\varphi, z, t)$.

The calculations will be performed using the cylindrical system of coordinates r, φ, z, whose ort $\vec{n}_z$ is oriented along the symmetry axis of the jet (its surface is not disturbed by the capillary wave motion).

The equation of the jet surface disturbed by the capillary wave motion shall be written as follows:

$$r(\varphi,z,t) = R + \xi(\varphi,z,t), \quad |\xi(\varphi,z,t)| \ll R.$$

We shall solve the problem on the stability of axisymmetrical waves on the charged jet.

The mathematic formulation of the problem is as follows:

$$\frac{d\vec{U}(\vec{r},t)}{dt} = -\frac{1}{\rho}\nabla P + v\Delta\vec{U}(\vec{r},t),$$

$$\nabla\vec{U}(\vec{r},t) = 0, \quad \Delta\Phi(\vec{r},t) = 0;$$

$$r \to 0: |\vec{U}(\vec{r},t)| < \infty,$$

$$r \to \infty: |\nabla\Phi(\vec{r},t)| \to 0;$$

$$r = R + \xi:$$

$$-\frac{\partial\xi(\varphi,z,t)}{\partial t} + \vec{U}(\vec{r},t)\times\nabla[r - (R + \xi(\varphi,z,t))] = 0;$$

$$\frac{\partial U_\varphi(\vec{r},t)}{\partial r} + \frac{1}{r}\frac{\partial U_r(\vec{r},t)}{\partial\varphi} - \frac{1}{r}U_\varphi(\vec{r},t) = 0;$$

$$\frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z} = 0;$$

$$-P(\vec{r},t) - P_0 + 2v\frac{\partial U_r(\vec{r},t)}{\partial r} - P_q(\vec{r},t) + P_\sigma(\vec{r},t) = 0;$$

$$\Phi(\vec{r},t) = \Phi_*. \tag{1}$$

In Eq. (1), $U_r(\vec{r},t)$, $U_\varphi(\vec{r},t)$, and $U_z(\vec{r},t)$ are the field of rates components; $P(\vec{r},t)$ and P_0 are the hydrodynamic and atmospheric pressures; $P_q(\vec{r},t)$ and $P_\sigma(\vec{r},t)$ are the pressures of the electric field and the surface tension forces; and $\Phi(\vec{r},t)$ is the electric potential.

LINEARIZATION OF THE PROBLEM

We shall solve the formulated problem using the dimensionless alternatives, in which $R = \rho = \sigma = 1$, in the linear approximation according to $|\xi(\varphi,z,t)|/R$. Problem (1), if all of the previous physical values are designated as earlier, will be written as follows:

$$r(\varphi,z,t) = 1 + \xi(\varphi,z,t), \quad |\xi| \ll R,$$

$$r \le 1:$$

$$\frac{\partial\vec{U}(\vec{r},t)}{\partial t} = -\frac{1}{\rho}\nabla P(\vec{r},t) + v\times\Delta\vec{U}(\vec{r},t),$$

$$\nabla\vec{U}(\vec{r},t) = 0,$$

$$r \ge 1: \Delta\phi(\vec{r},t) = 0,$$

$$r \to 0: |\vec{U}(\vec{r},t)| < \infty, \quad r \to \infty: |\nabla\phi(\vec{r},t)| \to 0;$$

$$r = 1:$$

$$-\frac{\partial\xi(\varphi,z,t)}{\partial t} + \vec{U}_r(\vec{r},t) = 0,$$

$$\frac{\partial U_\varphi(\vec{r},t)}{\partial r} + \frac{\partial U_r(\vec{r},t)}{\partial\varphi} - U_\varphi(\vec{r},t) = 0,$$

$$\frac{\partial U_z(\vec{r},t)}{\partial r} + \frac{\partial U_r(\vec{r},t)}{\partial z} = 0,$$

$$-p(\vec{r},t) + 2v\frac{\partial U_r(\vec{r},t)}{\partial r} - p_q(\vec{r},t) + p_\sigma(\vec{r},t) = 0,$$

$$\phi(\vec{r},t) - 4\pi\chi_0\xi(\varphi,z,t) = 0. \tag{2}$$

In Eq. (2), $\phi(\vec{r},t)$, $p(\vec{r},t)$, $p_q(\vec{r},t)$, and $p_\sigma(\vec{r},t)$ corrections—induced by the capillary waves on the jet surface—of the first order of smallness on small parameter $|\xi(\varphi,z,t)|/R$ to the electric potential, hydrodynamic pressure, and pressures of the electric and capillary forces, respectively.

Let us make an expansion in a small parameter of the expressions for the Laplace pressure $P_\sigma(\vec{r},t) = \mathrm{div}\vec{n}(\vec{r},t)$ (where $\vec{n}(\vec{r},t)$ is the ort of normal to the jet surface) and the pressure of the electric field $P_q = [\nabla(\Phi + \phi)]^2/8\pi$ and, for the values of the first order of smallness $p_q(\vec{r},t)$ and $p_\sigma(\vec{r},t)$ that enter into (2), we shall obtain the following ratios:

$$p_\sigma(\vec{r},t) = -\left(\xi(\varphi,z,t) + \frac{\partial^2\xi(\varphi,z,t)}{\partial\varphi^2} + \frac{\partial^2\xi(\varphi,z,t)}{\partial z^2}\right),$$

$$p_q(\vec{r},t) = -4\pi\chi_0^2\xi(\varphi,z,t) - \chi_0\frac{\partial\phi(\varphi,z,t)}{\partial r}.$$

SCALARIZATION OF LINEAR PROBLEM

We shall solve the system of equations (2) and (3) using the method of operator scalarization [23], decomposing the fields of rates $\vec{U}(\vec{r},t)$ for a sum of three orthogonal vector fields using vector differential operators $\widehat{N}_i$:

$$\vec{U}(\vec{r},t) = \sum_{i=1}^{3}\widehat{N}_i\psi_i(\vec{r},t),\ (i=1,2,3);\ \text{if}\ \widehat{N}_j^+\widehat{N}_i = 0,$$

$$\Delta\widehat{N}_i = \widehat{N}_i\Delta;\ (\text{wherein } i \neq j;\ i,j = 1,2,3). \quad (3)$$

In Eq. (3), $\psi_i(\vec{r},t)$ are the sought scalar functions; $\widehat{N}_j^+$ are the operators that are hermite conjugated to operators $\widehat{N}_j$.

Since the equilibrium form of the jet is axisymmetric, to select $\widehat{N}_i$ is convenient in the form of:

$$\widehat{N}_1 \equiv \nabla,\ \ \widehat{N}_2 \equiv \nabla\times\vec{e}_z,\ \ \widehat{N}_3 \equiv \nabla\times(\nabla\times\vec{e}_z).$$

The components of the fields of rate $\vec{U}(\vec{r},t)$ will be expressed via the scalar functions $\psi_i(\vec{r},t)$:

$$U_r = \frac{\partial\psi_1}{\partial r} + \frac{1}{r}\frac{\partial\psi_2}{\partial\varphi} + \frac{\partial^2\psi_3}{\partial z\partial\varphi},$$

$$U_\varphi = \frac{1}{r}\frac{\partial\psi_1}{\partial\varphi} - \frac{\partial\psi_2}{\partial r} + \frac{1}{r}\frac{\partial^2\psi_3}{\partial z\partial\varphi},$$

$$U_z = \frac{\partial\psi_1}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi_3}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi_3}{\partial\varphi^2}\right]. \quad (4)$$

To substitute decomposition $\vec{U}(\vec{r},t)$ (3) into system (2) and using the properties of orthogonality and commutativity of operators $\widehat{N}_i$ (3), we shall obtain the system of scalar equations with respect to the sought functions $\psi_i(\vec{r},t)$:

$$\nabla\psi_1(\vec{r},t) = 0,$$

$$\Delta\psi_i(\vec{r},t) - \frac{1}{\nu}\frac{\partial\psi_i(\vec{r},t)}{\partial t} = 0,\ \ (i = 2,3), \quad (5)$$

$$p(\vec{r},t) = -\frac{\partial\psi_1(\vec{r},t)}{\partial t}. \quad (6)$$

Using Eqs. (4) and (6) and the boundary conditions in Eq. (2), let us rewrite them as the boundary conditions for

$\psi_i(\vec{r},t)$ and $\xi(\varphi,z,t)$:

$$r = 1:$$

$$\frac{\partial\xi(\varphi,z,t)}{\partial t} - \left[\frac{\partial\psi_1(\vec{r},t)}{\partial t} + \frac{1}{r}\frac{\partial\psi_2(\vec{r},t)}{\partial\varphi} + \frac{\partial^2\psi_3(\vec{r},t)}{\partial z\partial r}\right] = 0,$$

$$2\frac{\partial}{\partial\varphi}\left\{\frac{\partial\psi_1(\vec{r},t)}{\partial r} - \psi_1(\vec{r},t)\right\}$$

$$-\left\{\frac{\partial^2\psi_2(\vec{r},t)}{\partial r^2} - \frac{\partial\psi_2(\vec{r},t)}{\partial r} - \frac{\partial^2\psi_2(\vec{r},t)}{\partial\varphi^2}\right\}$$

$$+ 2\frac{\partial^2}{\partial z\partial\varphi}\left\{\frac{\partial\psi_3(\vec{r},t)}{\partial r} - \psi_3(\vec{r},t)\right\} = 0,$$

$$\frac{\partial}{\partial r}\left\{\frac{\partial\psi_1(\vec{r},t)}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi_3(\vec{r},t)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi_3(\vec{r},t)}{\partial\varphi^2}\right]\right\}$$

$$+\frac{\partial}{\partial z}\left\{\frac{\partial\psi_1(\vec{r},t)}{\partial r} + \frac{1}{r}\frac{\partial\psi_2(\vec{r},t)}{\partial\varphi} + \frac{\partial^2\psi_3(\vec{r},t)}{\partial z\partial\varphi^2}\right\} = 0,$$

$$\frac{\partial\psi_1(\vec{r},t)}{\partial t} + 2\nu\frac{\partial}{\partial r}$$

$$\times\left\{\frac{\partial\psi_1(\vec{r},t)}{\partial r} + \frac{1}{r}\frac{\partial\psi_2(\vec{r},t)}{\partial\varphi} + \frac{\partial^2\psi_3(\vec{r},t)}{\partial z\partial r}\right\}$$

$$+ 4\pi\chi_0^2\xi(\varphi,z,t) + \chi_0\frac{\partial\phi(\vec{r},t)}{\partial r}$$

$$-\left\{\xi(\varphi,z,t) + \frac{\partial^2\xi(\varphi,z,t)}{\partial\varphi^2} + \frac{\partial^2\xi(\varphi,z,t)}{\partial z^2}\right\} = 0. \quad (7)$$

Since $\xi(\varphi,z,t)$, $\phi(\vec{r},t)$, and $\psi_i(\vec{r},t)$ describe small deviations from the equilibrium state, then, to follow their evolution in time, assume that their time dependence is determined by the following exponent:

$$\xi(\varphi,z,t),\ \ \phi(\vec{r},t),\ \ \psi_i(\vec{r},t) \sim \exp(st),$$

where S is the complex frequency in a general case.

DISPERSION EQUATION DERIVATION

We shall seek solutions of (4) and (5) in the cylindrical system of coordinates, which satisfy the above conditions of restriction, in the following form:

$$\psi_1(\vec{r},t) = \int_0^\infty C^{(1)}I_0(kr)\exp(ikz)\exp(st)\,dk,$$

$$\psi_i(\vec{r},t) = \int_0^\infty C^{(i)}I_0(lr)\exp(ikz)\exp(st)\,dk,$$

$$(i = 2;3);$$

$$\phi(\vec{r},t) = \int_0^\infty C^{(4)}I_0(kr)\exp(ikz)\exp(st)\,dk. \quad (8)$$
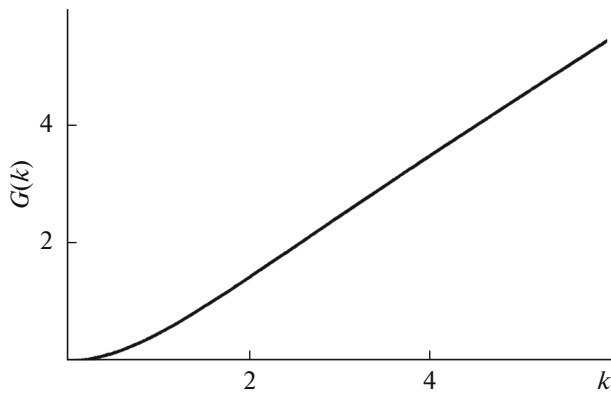
We shall also insert $\xi(z,\varphi,t)$:

Fig. 5. Dependence of dimensionless multiplier $G(k)$ on dimensionless wave number k.

$$\xi(z,\varphi,t) = \int_0^\infty D\exp(ikz)\exp(st)\,dk. \quad (9)$$

In Eqs. (8)–(9), k is the wave number; $l^2 \equiv k^2 + s/\nu$; $I_m(x)$ and $K_m(x)$ are modified functions of Bessel of the first and second type in [24]; $C^{(i)}$, where $i = 1, 2, 3, 4$, and D are the coefficients of expansions that depend on k.

$$\int_0^{2\pi} \exp[i(k_1 - k_2)z]\,dz = \delta(k_1 - k_2), \quad (10)$$

where $\delta(k_1 - k_2)$ is the Diraque delta-function [25, p. 902]; it is simple to obtain connection with coefficients

$C^{(4)}$ and D:

$$C^{(4)} = \frac{4\pi\chi_0}{K_0(k)}D. \quad (11)$$

Substituting solutions of (8) with account for (9) and (11) in the boundary conditions of (7) and using the ratios of (10), we shall obtain the equation system with respect to the unknown coefficients D and $C^{(i)}$, $(i = 1, 2, 3)$:

$$Ds - C^{(1)}kI_0'(k) - C^{(3)}iklI_0'(l) = 0, \quad (12)$$

$$C^{(2)}\left(I_0'(l) - l \times I_0''(l)\right) = 0,$$

$$C^{(1)}2ik^2I_0'(k)$$
$$- C^{(3)}\left(l^3I_0'''(l) + l^2I_0''(l) + l(k^2 - 1)I_0'(l)\right) = 0,$$

$$D\left\{4\pi\chi_0^2\left(1 + \frac{kK_0'(k)}{K_0(k)}\right) + k^2 - 1\right\}$$
$$+ C^{(1)}\left(sI_0(k) + 2\nu k^2I_0''(k)\right) + C^{(3)}2\nu ikl^2I_0''(l) = 0.$$

The dashes above the Bessel functions denote the derivatives on the argument, which can be expressed via the Bessel functions of the same and neighboring orders.

The system of uniform linear equations (12) has a nontrivial solution if its determinator is equal to zero $\det[a_{ij}] = 0$, where the elements a_{ij} are determined by the following ratios:

$$a_{11} = s,$$

$$a_{21} = a_{31} = 0, \quad a_{41} = k^2 - 1 + 4\pi\chi_0^2\left\{1 - k\frac{K_1(k)}{K_0(k)}\right\},$$

$$a_{12} = -kI_1(k), \quad a_{22} = 0, \quad a_{32} = 2ik^2I_1(k),$$

$$a_{42} = \left[s + 2\nu k^2\right]I_0(k) - 2\nu kI_1(k),$$

$$a_{13} = 0, \quad a_{23} = 2lI_1(l) - l^2I_0(l), \quad a_{33} = 0,$$

$$a_{43} = 0, \quad a_{14} = -iklI_1(l), \quad a_{24} = 0,$$

$$a_{34} = -(l^2 + k^2)lI_1(l),$$

$$a_{44} = 2\nu ik\left(l^2I_0(l) - lI_1(l)\right). \quad (13)$$

Revealing the determinator of the fourth order with the elements (13), we shall obtain the dispersion equation, which connects the complex frequencies s of axisymmetric waves on the jet surface with a wave number k:

$$s^2 - \frac{2\nu k^2 s}{I_0(k)}\left\{sI_1'(k) - \frac{2kl}{l^2 + k^2}\frac{I_1(k)}{I_1(l)}I_1'(l)\right\} = \omega_0^2(k);$$

$$\omega_0^2(k) = G(k)\left(1 - k^2 - w(1 - H(k))\right)\frac{l^2 - k^2}{l^2 + k^2};$$

$$G(k) \equiv k\frac{I_1(k)}{I_0(k)}; \quad H(k) \equiv k\frac{K_1(k)}{K_0(k)}; \quad (14)$$

$$w \equiv 4\pi\chi_0^2.$$

The graphs of functions $G(k)$, $H(k)$, and $\omega_0(k)$ are shown in Figs. 5–7.

In the absence of the electric charge on the jet ($w = 0$), this equation coincides with the dispersion equation for the uncharged jet of a viscous incompressible liquid [21, p. 631].

For the ideal liquid ($\nu = 0$), the term of equation (14) proportional to viscosity is out, and equation (14) is simplified:

$$s^2 = \omega_0^2(k); \quad \omega_0^2(k) = G(k)\left(1 - k^2 - w(1 - H(k))\right);$$

and its solutions are written as follows:

$$s_{1,2} = \pm\sqrt{\omega_0^2(k)}. \quad (15)$$

When $\omega_0^2(k)$ is negative, solutions (15) describe the frequencies of the capillary waves that run along the

jet. If $\omega_0^2(k) > 0$, then (15) determine their increment of the amplitude increasing γ (at $s_1 = \sqrt{\omega_0^2(k)} > 0$) and their decrement decreasing of amplitude η (at $s_2 = \sqrt{\omega_0^2(k)} < 0$).

In the presence of viscosity in a liquid, its damping effect decreases the frequencies of the capillary waves or their increments and increases the decrement.

Let us consider the case of a low-viscous liquid, when condition $l \gg k$ is performed (note that $l^2 \equiv k^2 + s/\nu$, which means that $(s/\nu) \gg k^2$) in this case; then, for the long waves on the jet, whose length is much larger than its radius ($k \ll 1$), the ratios of [21, p. 636] are fulfilled:

$$I_0(k) \approx 1; \quad I_1(k) \approx \frac{k}{2}; \quad I_1(l) \approx \frac{l}{2};$$

$$I_1'(l) = I_1'(k) \approx \frac{1}{2};$$

and equation (14) takes the form of

$$s^2 + 3\nu k^2 s - \frac{k^2(1 - k^2 - w(1 - H(k)))}{2} = 0; \quad (15a)$$

and its solutions:

$$(s_{1,2})_{sv} = -\frac{3\nu k^2}{2}$$
$$\pm \sqrt{\frac{9\nu^2 k^4}{4} + \frac{k^2(1 - k^2 - w(1 - H(k)))}{2}}. \quad (16)$$

The graphs of dependences of dimensionless increments of instability of capillary waves on the jet on dimensionless wave number for various small values of the coefficient of kinematic viscosity of a liquid (0.125; 0.2; 0.3) calculated using (16) are shown in Fig. 8 (the curves go from top downward with viscosity increase). The value of the coefficient of the kinematic viscosity $v = 0.125$ is relevant to water. It is easy to see that the increase in viscosity reduces the increments and shifts the position of a maximum increment towards longer waves.

At a high viscosity of the liquid, when the damping effect decreases the frequency of the periodic motions of the liquid and their increments of instability and increase in decrements, when $l^2 = k^2 + (s/\nu) \approx k^2$ (or $(s/\nu) \to 0$ at v increase), equation (14) reduces to

$$s = \frac{1}{4\nu}(1 - k^2 - w(1 - H(k))). \quad (17)$$

The graphs of dependences of a dimensionless increment of instability of the capillary waves on the jet on the dimensionless wave number for various high values of the kinematic viscosity of a liquid (100; 150; 200) calculated using (17) are shown in Fig. 9 (the curves go from top downwards with the viscosity increase). The value of the coefficient of the kinematic
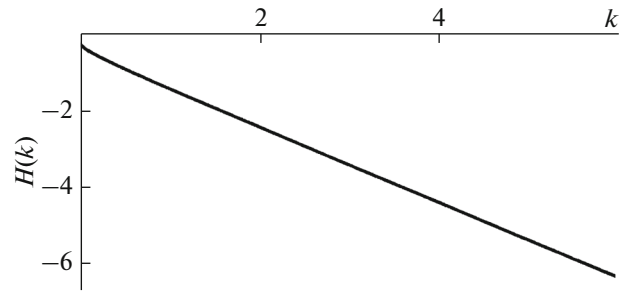

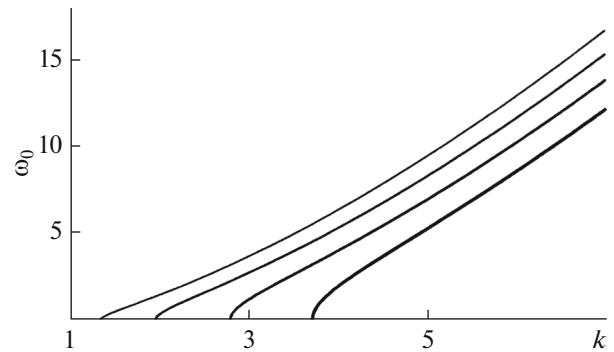**Fig. 6.** Dependence of dimensionless multiplier $H(k)$ on dimensionless wave number k.

Fig. 7. Dependence on dimensionless wave number k of dimensionless oscillation frequency of ideal liquid $\omega_0(k)$ calculated at $w = 1; 2; 3; 4$. Various thickness curves are located from the top downwards with increase in charge parameter w.
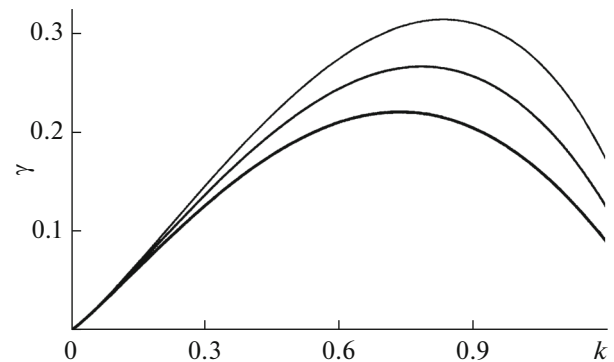

Fig. 8. Dependences on dimensionless increment of axisymmetric waves on the jet of a low viscous liquid $\gamma(k)$ on dimensionless wave number k calculated at $w = 1$ and various values of dimensionless coefficient of kinematic viscosity $v = 0.125; 0.2; 0.3$. Various thickness curves are located from top downwards with increase in viscosity.

viscosity $v = 100$ is relevant to glycerol. It is easy to see that the viscosity increase decreases the increments. The position of the maximum increment in the indicated region of the values of coefficients of kinematic viscosity almost remains unchanged.
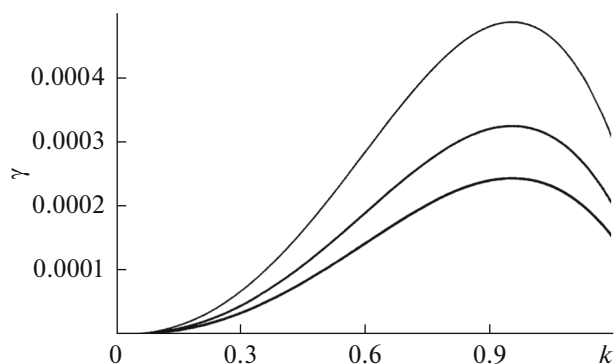
Fig. 9. Dependence of dimensionless increment of axisymmetric waves on the jet of high viscous liquid $\gamma(k)$ on the dimensionless wavenumber k, calculated at $w = 1$ and various values of the dimensionless coefficient of kinematic viscosity $v = 100; 150; 200$. Various thickness curves are located from the top downwards with increase in viscosity.

Comparison of Figs. 8 and 9 shows that the change in the liquid viscosity approximately by a thousand times (just like for transition from water to glycerol) also leads to a decrease in dimensionless increment of instability by approximately a thousand times, which explains the difference in phenomenology of the electrostatic decay of low- and high viscous liquid.

SHORT AND CONTINUOUS LONG JETS

It is noteworthy that calculations performed in this work were meant for the jets of continuous length, whereas real jets have a finite length. If a jet of a high viscous liquid is more or less long and it can be modeled as continuous long (neglecting the edge effects), then the jet of a low viscous liquid prior to the disintegration passes a negligibly small distance and it cannot be considered as continuous long. However, it should be taken into account that there are several model approaches in problems concerning jets. In one developed by Rayleigh [26] at the end of the 19th century, later improved by Basset [27] and Weber [28], a continuous long jet is described with capillary waves running along it with a real wave number, and their frequencies in a general case are complex, and occurrence of the imaginary part of the frequency means the wave instability. The greater part of the theoretic studies of the jet instabilities is performed in the framework of this very approach. In the second approach, which appeared in the middle of the 20th century [29−30], the wave number is considered to be complex and frequency as real. As a matter of fact, the second approach was closer to reality, for real jets are flowing from the capillary (pipe) and decay at the finite distance from the end of the capillary, when the wave number becomes imaginary.

It is noteworthy that all experiments are carried out using the jet with a finite length, and the majority of theoretical researches are performed for the continuous long jets (see, e.g., reviews [31, 32]). The results of these experimental and theoretical studies performed using different approaches can be compared with each other, and this comparison is fruitful. The reason, as shown in [30], is that the dispersion equations obtained above in particular approaches are formed from each other by a simple linear transformation.

CONCLUSIONS

It was shown that the electrodispersion of low- and high-viscous electroconducting liquids is realized using different physical scenarios, although the imitated drops in both cases carry a charge, more than critical in a sense of realization of the electrostatic instability. For the low viscous liquids, the separated drop throws two hundred daughter smaller by two orders strongly charged droplets, each of which, in turn, is unstable with respect to its own charge. Owing to the presence of high intensity of the electrostatic field of the intrinsic charge, the corona discharge ignites near each of the droplets. As a result, at the drop vertex of a low-viscous liquid, a fan glow appears.

By another scenario, the electrostatic instability of a high-viscous liquid of a charged drop is realized: because of the damping effect of a liquid viscosity, such a drop separates a finite number of daughter droplets, each of which disintegrates into two daughter droplets that are steady to their own charge and the fan glow does not occur. Hence, the decay phenomenology of the low- and high-viscous liquids is different.

All the aforementioned relates to the electroconducting liquid, when the relaxation maxwell time of the electric charge is far less than the characteristic time of separation of the emitted drop from the meniscus. For the low conducting liquids, the emitted drops will carry a charge less than critical to realize the electrostatic instability (because of low electroconductivity), and the fan glow at the meniscus vertex does not occur even for the low viscous liquids.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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