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Asymptotic Calculation of the Intensity of Dipole Electromagnetic Radiation of an Uncharged Drop Oscillating in the Electrostatic Field

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Abstract—An analysis is carried out on the electromagnetic radiation of an uncharged drop oscillating in an electrostatic field using nonlinear asymptotic calculations for two small parameters (the value of a dimensionless stationary deformation of an initially spherical drop and the amplitude of its capillary oscillations). In the external electrostatic field, on the top of the drop, electric charges with the opposite signs are formed, to which the "effective" charges located on the drop's symmetry axis are added relatively. Since the distance between those "effective" charges is about the length of the drop's radius, they form a dipole, which at the distances much longer than the sizes of the drop creates a similar electric field as the drop itself. During the oscillation of the drop surface, the dipole will also oscillate, which will generate the electromagnetic waves of the dipole type.

Keywords: uncharged drop, electrostatic field, oscillations, dipole electromagnetic radiation

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INTRODUCTION

The dipole radiation of the electromagnetic waves of oscillating charged drops, discovered on a small parameter in a quadratic asymptotic calculation, repeatedly became the object of a theoretical study [1, 2] in the context of the problems of radiolocation of clouds [3]. The very problem of the electromagnetic radiation of the oscillating charged cloud drops was first mentioned in [4] and specified in [5, 6]. However, as it turned out, the authors in [4–6] revealed just an extremely small quadrupole radiation detectable in linear calculations. The dipole radiation emitted by the charged drop, which is fixed in nonlinear asymptotic calculations of the second order of smallness, is far more intense (by 14–15 orders of magnitude).

In [7], by analogy with [4–6], in the first order of smallness by the amplitude of oscillations, the intensity of the quadrupole radiation of an uncharged drop oscillating in the external uniform electrostatic field is calculated. In this case, the radiation is caused by the accelerated motion of the charges induced by the external electrostatic field during the oscillations of the drop surface. Its intensity is extremely small as in [4–6], and this radiation cannot be fixed by means of radiolocation (the integral radiation emitted by the clouds is meant).

Further, in the nonlinear asymptotic calculations of higher orders of smallness than the first, we calculate the intensity of the dipole electromagnetic radiation emitted by the uncharged drop oscillating in the external electrostatic field.

STATEMENT OF THE PROBLEM

Let us consider the problem on the electromagnetic radiation of the uncharged drop of an ideal, incompressible, ideally conducting liquid with density ρ and surface tension coefficient σ oscillating in a uniform electrostatic field with intensity $\vec{E}_0$. Let us accept that the drop is in vacuum, and its volume is determined by that of a sphere with radius R. Under the effect of the external electrostatic field, in the drop, electrical charges are induced: negative charge on its half turned toward the field, and positive charge on the opposite half, which form the dipole moment [8], p. 19. The drop thus extends into a spheroid with $r = r(\theta)$, whose symmetry axis is collinear to the external field. The square of the eccentricity of the spheroid e^2 will be assumed to be a small parameter $e^2 \ll 1$.

A capillary wave motion will exist on the drop surface, this time generated by the thermal motion of the water molecules [9] and causing the distortion $\xi(\theta, \varphi, t)$ of the spheroidal form, which is equilibrium in the field. For simplicity, let us accept that this disturbance is axisymmetric (this simplification will not affect the general consideration). The ratio $\max|\xi(\theta,t)/R|$ is the second small parameter of the problem designated as ε. The induced charges will be distributed over the drop surface disturbed by the capillary wave motion. The equation of the drop surface will be as follows:

$$r(\theta,t) = r(\theta) + \xi(\theta,t). \tag{1}$$

All calculations of the problem will be performed in a spherical system of coordinates (r, θ, φ) with the beginning in the center of the drop masses in dimensionless variables, in which $R = \rho = \sigma = 1$.

The motion in the drop will be assumed as potential with potential $\psi(\vec{r},t)$, so that the field of rates of the wave flow in the liquid of the drop will be determined as $\vec{V}(\vec{r},t) = \nabla\psi(\vec{r},t)$. In dimensionless variables the potential $\psi(\vec{r},t)$ will have the same order of smallness as the amplitude of oscillations of the drop surface $\psi(\vec{r},t) \sim \xi(\theta,t) \sim \varepsilon$ (because the disturbance $\xi(\theta, t)$ is generated by the capillary wave motion). Also, we shall introduce the electric potential $\Phi(\vec{r},t)$ of the field of the charges induced in the drop.

The mathematical formulation of the problem on the electromagnetic radiation emitted by the uncharged drop oscillating in the external electrostatic field has the form

$$\Delta\psi(\vec{r},t) = 0; \tag{2}$$

$$\Delta\Phi(\vec{r},t) = 0; \tag{3}$$

$$r \to 0: \quad \psi(\vec{r},t) \to 0; \qquad r \to \infty: \quad \Phi(\vec{r},t) \to 0; \tag{4}$$

$$\begin{gathered} r = r(\theta) + \xi(\theta,t): \\ \frac{\partial\xi(\theta,t)}{\partial t} = \frac{\partial\psi(\vec{r},t)}{\partial r} \\ - \frac{1}{r^2}\frac{\partial\psi(\vec{r},t)}{\partial\theta}\left(\frac{\partial r(\theta)}{\partial\theta} + \frac{\partial\xi(\theta,t)}{\partial\theta}\right); \\ P(\vec{r},t) - P_{\text{atm}} + P_E(\vec{r},t) = P_\sigma(\vec{r},t); \\ \Phi(\vec{r},t) = \Phi_s(t), \end{gathered} \tag{5}$$

where (5) $\Phi_s(t)$ is the constant value of the electric potential of the drop along its surface; $P(\vec{r},t) = P_0 - \dfrac{\partial\psi(\vec{r},t)}{\partial t}$ is the hydrodynamic pressure; P_0 is the constant pressure inside the drop in the equilibrium state; P_{atm} is the constant pressure outside of the drop in the equilibrium state; $P_E = (\nabla\Phi)^2/8\pi$ is the electric field pressure; $P_\sigma = \operatorname{div}\vec{n}(\vec{r},t)$ is the capillary pressure; and $\vec{n}(\vec{r},t)$ is the unit normal vector to a disturbed surface of the drop:

$$\vec{n}(\vec{r},t) = \left.\frac{\nabla(r - r(\theta,t))}{|\nabla(r - r(\theta,t))|}\right|_{r=r(\theta,t)}. \tag{6}$$

In (6), $r(\theta, t)$ is determined by (1).

Let us add the integral conditions to the written-out system, namely, the invariability of the total volume of the drop (the consequence of the incompressibility of the liquid), the immobility of its center of mass, and the uncharged state of the drop:

$$\begin{gathered} \int_V r^2 dr \sin\theta d\theta d\varphi = \frac{4}{3}\pi; \\ \int_V \vec{r} \cdot r^2 dr \sin\theta d\theta d\varphi = 0; \end{gathered} \tag{7}$$

$$\begin{gathered} V = [0 \le r \le r(\theta) + \xi(\theta,t); \\ 0 \le \theta \le \pi, \;\; 0 \le \varphi \le 2\pi]; \;\; \frac{1}{4\pi}\oint_S (\vec{n},\nabla\Phi) dS = 0; \\ S = [r = r(\theta) + \xi(\theta,t), 0 \le \theta \le \pi, 0 \le \varphi \le 2\pi]. \end{gathered} \tag{8}$$

Let us expand the sought values by the orders of smallness of the dimensionless amplitude of oscillations ε:

$$\begin{gathered} \xi(\theta,t) = \xi^{(1)}(\theta,t) + O(\varepsilon^2); \\ \psi_j(\vec{r},t) = \psi_j^{(1)}(\vec{r},t) + O(\varepsilon^2); \\ P(\vec{r},t) = P^{(0)}(\vec{r}) + P^{(1)}(\vec{r},t) + O(\varepsilon^2); \\ P_\sigma(\vec{r},t) = P_\sigma^{(0)}(\vec{r}) + P_\sigma^{(1)}(\vec{r},t) + O(\varepsilon^2); \\ P_E(\vec{r},t) = P_E^{(0)}(\vec{r}) + P_E^{(1)}(\vec{r},t) + O(\varepsilon^2); \\ \Phi(\vec{r},t) = \Phi^{(0)}(\vec{r}) + \Phi^{(1)}(\vec{r},t) + O(\varepsilon^2), \end{gathered} \tag{9}$$

where $\Phi^{(0)}(\vec{r})$ is the electric potential in the vicinity of the equilibrium uncharged spheroid found in the external electrostatic field and $\Phi^{(1)}(\vec{r},t)$ is the electric potential of the induced charges of the disturbed spheroid. The upper index designates the order of smallness on ε.

Inserting expansion (9) into (2)–(8), we shall determine the problems of the zeroth and first orders.

THE PROBLEM OF THE ZEROTH ORDER

To find the equilibrium surface of the drop and the electric potential $\Phi^{(0)}(\vec{r})$in the vicinity of the undisturbed surface of the drop, we shall give the mathematical formulation of the problem of the zeroth order of smallness in ε:

$$\begin{gathered} \Delta\Phi^{(0)}(\vec{r}) = 0; \\ r \to \infty: \Phi^{(0)}(\vec{r}) \to 0; \quad r = r(\theta): \Phi^{(0)}(\vec{r}) = \text{const}; \\ P^{(0)} - P_{\text{atm}} + P_E^{(0)}(\vec{r}) = P_\sigma^{(0)}(\vec{r}); \\ \int_V r^2 dr \sin\theta d\theta d\varphi = \frac{4}{3}\pi; \;\; \int_V \vec{r} \cdot r^2 dr \sin\theta d\theta d\varphi = 0; \\ V = [0 \le r \le r(\theta), \;\; 0 \le \theta \le \pi, \;\; 0 \le \varphi \le 2\pi]; \\ \frac{1}{4\pi}\oint_S \left(\vec{n}_0(\vec{r}), \nabla\Phi^{(0)}(\vec{r})\right) dS = 0; \\ S = [r = r(\theta), 0 \le \theta \le \pi, 0 \le \varphi \le 2\pi], \end{gathered} \tag{10}$$

where $\vec{n}_0(\vec{r})$ is the unit normal vector to the undisturbed surface of a spheroidal drop, which is determined by ratio (6) on the surface of the undisturbed spheroid.

After solving problem (10), the same as was done in [7, 10], we shall obtain the expression for the form of the equilibrium surface of the drop that coincides with accuracy to the eccentricity squared e^2 with the equation of the extended spheroid in the following form:

$$r(\theta) \approx 1 + e^2 h(\theta) + O\left(e^4\right) \equiv 1 + \frac{1}{3} e^2 P_2(\mu) + O\left(e^4\right), \quad (11)$$

where the eccentricity is connected with the intensity of the electrostatic field by the ratio $e \equiv \sqrt{9E_0^2/16\pi}$.

The potential $\Phi^{(0)}(\vec{r})$ in an approximation linear in e^2 is easy to find. It can be found both by transition from the known expression [8, p. 41] for the electric potential of the extended conducting ellipsoid in a uniform external field (mentioned in [8], p. 41 in spheroidal coordinates) and also using a direct solution of a relative electrostatic problem (9) in a spherical system of coordinates by the perturbation method [11]:

$$\Phi^{(0)}(\vec{r}) = E_0 r P_1(\mu)\left(\frac{1}{r^3} - 1\right) + \frac{2}{5}\frac{1}{r^2} e^2 E_0 \left(P_1(\mu) + \frac{3}{2}\frac{1}{r^2} P_3(\mu)\right); \quad (12)$$
$$\mu \equiv \cos\theta,$$

where $P_n(\cos\theta)$ are Legendre polynomials [12].

The solution of the problems of the first order of smallness in ε. In the problem under study, there are two small parameters, namely, the eccentricity of the equilibrium surface of the drop e and the amplitude ε of its oscillations. It is also noteworthy that $E_0 \sim e$. In further calculations of the drop oscillations, we must take into account the terms with an order of smallness of $\sim\varepsilon$; the terms of $\sim e\varepsilon$ (which consider the interaction of disturbance ξ with the intensity of the electrostatic field E_0); the terms of $\sim e^2\varepsilon$, which consider the interaction of disturbance with the deviation of the equilibrium surface of the drop from a sphere; and the terms of $\sim E_0 e^2 \varepsilon$, which consider the interaction of disturbance with the field and deviation of the equilibrium surface of the drop from sphere.

Because of the linearity of Eqs. (2) and (3), both the hydrodynamic potential $\psi(\vec{r},t)$ and each of the components of the electric potential $\Phi(\vec{r},t)$ must satisfy them. The solutions of (2) and (3) for $\psi(\vec{r},t)$ and $\Phi^{(1)}(\vec{r},t)$ that meet the requirements of the boundary conditions of (4) and the disturbance of the equilibrium form of the drop surface $\xi(\theta, t)$ will be written as follows:

$$\psi(r,\theta,t) = \varepsilon \sum_{n=0}^{\infty} D_n(t) r^n P_n(\mu); \quad (13)$$

$$\Phi^{(1)}(r,\theta,t) = \varepsilon \sum_{n=0}^{\infty} F_n(t) r^{-(n+1)} P_n(\mu); \quad (14)$$

$$\xi(\theta,t) = \varepsilon \sum_{n=0}^{\infty} M_n(t) P_n(\mu), \quad (15)$$

where the order of smallness is indicated by the degree of ε.

The solution of the problem for the electric potential (in the first order of smallness in ε). Let us choose the boundary problem of the first order in ε to determine the electric potential $\Phi^{(1)}(\vec{r},t)$. The system of equations to determine coefficient F_n in solution (14) is obtained from (4) and (5) by grouping the terms $\sim\varepsilon$:

$$r = 1;$$
$$\Phi^{(1)} + e^2 \frac{\partial \Phi^{(1)}}{\partial r} P_2(\mu) - 3E_0\left(1 + e^2\left((P_1(\mu))^2 - \frac{3}{5}\right)\right) \times P_1(\mu)\xi(\theta,t) = \Phi_s^{(1)}\delta_{n,0};$$
$$\int_0^{\pi}\left(\frac{\partial \Phi^{(1)}}{\partial r} + \frac{1}{3}e^2\left(\left(2\frac{\partial \Phi^{(1)}}{\partial r} + \frac{\partial^2 \Phi^{(1)}}{\partial r^2}\right)P_2(\mu) - \frac{\partial \Phi^{(1)}}{\partial \theta}\frac{\partial P_2(\mu)}{\partial \theta} + E_0\left(\frac{18}{5}(4P_3(\mu) + P_1(\mu))\xi(\theta,t) - 9(P_1(\mu))^2 \frac{\partial P_1(\mu)}{\partial \theta}\frac{\partial \xi(\theta,t)}{\partial \theta}\right)\right)\right)\sin\theta d\theta = 0. \quad (16)$$

Here $\Phi_s^{(1)}$ is the correction of the first order of smallness to the potential value of the surface of the drop.

Inserting the expansions (14) and(15) into (16), we get the expressions for coefficients $F_n(t)$ in the following form:

$$F_0(t) = -\frac{6}{35}e^2 M_3(t); \quad \Phi_s^{(1)} = -\frac{6}{35}e^2 M_3(t);$$

$$F_n(t) = E_0\left(3\left(\mu_{n-1}^{+} M_{n-1}(t) + \mu_{n+1}^{-} M_{n+1}(t)\right) + e^2\left(M_{n-3}(t) l_1 + M_{n-1}(t) l_2 + M_{n+1}(t) l_3 + M_{n+3}(t) l_4\right)\right);$$
$$(n \ge 1);$$

$$\mu_n^{+} = \frac{n+1}{2n+1}; \quad \mu_n^{-} = \frac{n}{2n+1}.$$

Finally, inserting the obtained expression into (14), we find the expansion for the electric potential $\Phi^{(1)}(\vec{r},t)$ in the following form:

$$\Phi^{(1)}(\vec{r},t) = E_0\varepsilon\Bigg(-\frac{6}{35}\frac{e^2M_3(t)}{r} + \sum_{n=1}^{\infty}\Big[3\big(\mu_{n-1}^{+}M_{n-1}(t) + \mu_{n+1}^{-}M_{n+1}(t)\big) + e^2\big(M_{n-3}(t)l_1 + M_{n-1}(t)l_2 + M_{n+1}(t)l_3 + M_{n+3}(t)l_4\big)\Big]\Bigg)r^{-(n+1)}P_n(\mu);$$

$$l_1 = \frac{3n(n-2)(n-1)(n+1)}{2(2n-5)(2n-3)(2n-1)}; \qquad (17)$$

$$l_2 = \frac{n\left(50n^4 + n^3 - 103n^2 + 39n - 27\right)}{10\left(4n^2 - 9\right)(2n-1)^2};$$

$$l_3 = \frac{n(n+1)\left(50n^4 + 321n^3 + 621n^2 + 278n - 180\right)}{10\left(4n^2 + 8n - 5\right)(2n+3)^2};$$

$$l_4 = \frac{3(n+5)(n+3)(n+2)(n+1)}{2(2n+7)(2n+5)(2n+3)}.$$

The solution of hydrodynamic part of the problem (in the first order of smallness in ε). The derivation of evolutionary equation. In the first order of smallness in ε, to determine the coefficients D_n and M_n in solutions of (13) and (15) from (5) and (7), we obtain

$$r = 1;\quad \int_0^{\pi}\left(1 + 2e^2h(\theta)\right)\xi(\theta,t)\sin\theta d\theta = 0;$$

$$\int_0^{\pi}\left(1 + 3e^2h(\theta)\right)\xi(\theta,t)\cos\theta\sin\theta d\theta = 0;$$

$$\frac{\partial\xi(\theta,t)}{\partial t} = \frac{\partial\psi(r,\theta,t)}{\partial r} + e^2\left(\frac{\partial^2\psi(r,\theta,t)}{\partial r^2}h(\theta) - \frac{\partial\psi(r,\theta,t)}{\partial\theta}\frac{\partial h(\theta)}{\partial\theta}\right);$$

$$P^{(1)} + P_E^{(1)} = P_\sigma^{(1)};$$

$$P^{(1)} = -\left(\frac{\partial\psi}{\partial t} + e^2\frac{\partial^2\psi}{\partial r\partial t}h(\theta)\right);$$

$$P_E^{(1)} = -\frac{3E_0}{4\pi}\left(6E_0\xi(\theta,t)P_1(\mu) + \frac{\partial\Phi^{(1)}}{\partial r}\right)P_1(\mu);$$

$$P_\sigma^{(1)} = -(2 + L_\theta)\xi(\theta,t) + 2e^2\left(\xi(\theta,t)L_\theta h(\theta) + h(\theta)(2 + L_\theta)\xi(\theta,t)\right);$$

$$L_\theta \equiv \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right),$$

these expressions allow us to get the ratios between the coefficients $M_n(t)$ and $D_n(t)$:

$$M_0(t) = -\frac{2}{3}e^2M_2(t);\quad M_1(t) = -\frac{3}{5}e^2M_3(t);$$

$$D_n(t) = \frac{1}{n}\Bigg(\frac{\partial M_n(t)}{\partial t}\left(1 - \frac{1}{3n}e^2\left(n(n-1)K_{2,n,n} - \alpha_{2,n,n}\right)\right) - \frac{e^2}{3}\Bigg(\frac{\partial M_{n-2}(t)}{\partial t}\frac{\left((n-2)(n-3)K_{2,n-2,n} - \alpha_{2,n-2,n}\right)}{(n-2)} + \frac{\partial M_{n+2}(t)}{\partial t}\frac{\left((n+2)(n+1)K_{2,n+2,n} - \alpha_{2,n+2,n}\right)}{(n+2)}\Bigg)\Bigg);$$

$$(n \ge 0);$$

$$K_{m,k,n} = \left[C_{k0,m0}^{n0}\right]^2;$$

$$\alpha_{m,k,n} = -\sqrt{m(m+1)k(k+1)}C_{m0,k0}^{n0}C_{m-1,k1}^{n0};$$

$C_{m_k,l_p}^{n_j}$ are the Clebsch–Gordan coefficients [12], different from zero only when the indices satisfy the relations $|m - k| \le n \le m + k$, and $m + k + n$ is even.

In addition to the relations between the coefficients, we shall also obtain a nonuniform differential equation to find the coefficient $M_n(t)$ at $n \ge 0$:

$$\frac{\partial^2}{\partial t^2}M_n(t) + \omega_n^2M_n(t) = e^2\left(\frac{\partial^2}{\partial t^2}M_{n-2}(t)\chi_1 + M_{n-2}(t)\chi_2 + \frac{\partial^2}{\partial t^2}M_{n+2}(t)\chi_3 + M_{n+2}(t)\chi_4\right);$$

$$\chi_1 = -\frac{n(n-1)}{2(2n-1)(2n-3)};\quad \chi_2 = \frac{n^2(n-1)^2(n+2)}{(2n-1)(2n-3)};$$

$$\chi_3 = -\frac{(n+1)(n+4)}{2(2n+3)(2n+5)};$$

$$\chi_4 = \frac{n(n+1)(n+2)\left(n^2 + 9n + 10\right)}{(2n+3)(2n+5)}.$$

Getting rid of the nonuniformity, setting to zero the right part of the evolutionary equation, and accepting that the amplitudes $M_n(t)$ depend on time t harmonically $\sim\exp(i\omega_nt)$, we shall obtain the expression for the frequency of oscillations of the nth mode (written in dimensional form)

$$\omega_n^2 = \frac{\sigma}{\rho R^3}n(n-1)(n+2) \times\left[1 - e^2\frac{\left(2n^5 + 23n^4 + 21n^3 - 17n^2 - 7n - 2\right)}{(n-1)(n+2)(2n-1)(2n+1)(2n+3)}\right], \qquad (18)$$

which qualitatively coincides with that obtained in [13] but having asymptotics more adequate to the real situation; ω_n is the frequency of intrinsic oscillations of the surface of the uncharged spheroidal drop in the uniform electrostatic field.

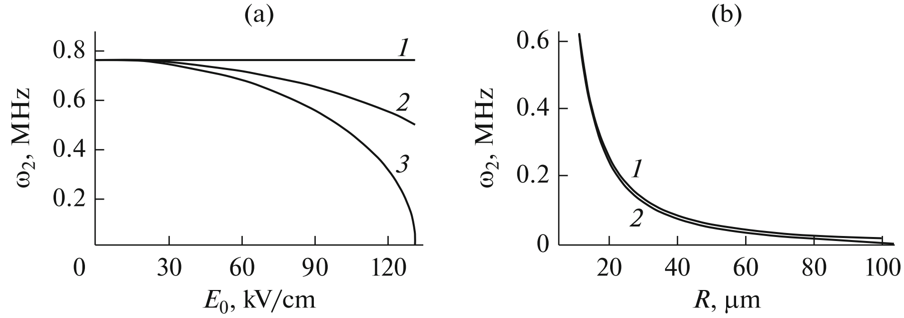

Fig. 1. Dependences of the frequency of electromagnetic radiation (a) of the main mode of the uncharged spherical drop (curve *1*), of the frequency of electromagnetic radiation of the spheroidal drop (curve *2*), and of the artificially created frequency of electromagnetic radiation of the spheroidal drop (the numerical coefficient at the eccentricity is 1.74) (curve *3*) which oscillates on the main mode in the external field on the electrostatic field intensity calculated at $j = 2$, $\mu = 0.1$, $\sigma = 73$ dyne/cm, $\rho = 1$ g/cm^3, $R = 10$ µm; (b) of the unit uncharged spheroidal drop (curve *1*) and modified frequency (curve *2*) oscillating on the main mode on the radius of the equal-sized drop calculated at the values of other physical values (see Fig. 1a) and $E_0 = 40$ kV/cm.

In Fig. 1a, straight line *1* designates the frequency of the main mode of the uncharged spherical drop, and curve *2* shows the dependence of the frequency of oscillations of the main mode of the spheroidal drop on the intensity of the electrostatic field (the dependence is realized through the eccentricity and is determined by the dimensional expression $e^2 \equiv 9E_0^2 R/16\pi\sigma$ [14]). Curve *3* denotes the frequency of the main mode of the uncharged spheroidal drop artificially adjusted to the real situation: it is known that, at $\left(E_0^2 R/\sigma\right) \geq 2.62$, the frequency goes to zero, and the drop is subjected to instability with respect to the polarization charge [15]. To turn the frequency squared (18) to zero, let us place the adjustable parameter 1.74 in front of e^2 in square brackets. This approach is certainly rough for the theoretical study, but it allows us to bring into agreement the calculation and experimental data [15]. The precise calculation is carried out in a linear approximation in e^2, and its results are applicable just at small e^2 (where $E_0 \approx 50$–60 kV/cm).

If the frequency of oscillations of the uncharged spheroidal drop in a uniform electrostatic field depends on the radius of the equal spherical drop in the region of $E_0 = 40$ kV/cm, then dependences *2* and *3* will have a similar appearance, qualitatively and quantitatively, as is seen from Fig. 1b. If we plot similar dependences at $E_0 = 1$ kV/cm, they will coincide within the limits of thickness of the line.

The solution of a nonuniform evolutionary equation are the harmonic functions of time t with the coefficients

$$M_n(t) = a_n \exp\left(i\left(\omega_n t + b_n\right)\right) + \text{c.c.}; \quad (n \geq 0),$$

where a_n and b_n are real constants which are determined from the initial conditions, and "c.c." are the terms that are complex conjugate to the written ones.

Thus, on the basis of (1), (11), and (15) for the form of surface of the oscillating uncharged drop in the external uniform electrostatic field, we obtain the following analytic expression:

$$r(\vec{r}, t) = 1 + e^2 h(\theta) + \varepsilon \sum_{n=0}^{\infty} M_n(t) P_n(\mu). \qquad (19)$$

Thus, the form of the disturbed surface of a spheroidal drop in the external field is written in the first order of smallness in the dimensionless amplitude of oscillations ε and in the linear approximation in the eccentricity squared e^2.

VALUES OF POLARIZATION CHARGES

The values of polarization charges of each of the halves of the disturbed surface of the drop $r(\theta, t)$ are determined by the following equations:

$$\begin{gathered} q_+ = \int_{S_1} dq_+ = \int_{S_1} \nu(\theta, t)\, dS_1; \\ S_1 \equiv \left[r = r(\theta, t); 0 \leq \theta \leq \frac{\pi}{2}; 0 \leq \varphi \leq 2\pi \right]; \\ q_- = \int_{S_2} dq_- = \int_{S_2} \nu(\theta, t)\, dS_2; \\ S_2 \equiv \left[r = r(\theta, t); \frac{\pi}{2} \leq \theta \leq \pi; 0 \leq \varphi \leq 2\pi \right]. \end{gathered} \qquad (20)$$

Here q_+ is the positive and q_- is the negative polarizing charges, and $r(\theta, t)$ is determined by (1).

Let us study the positively charged half of the drop, writing the induced charge in (20) and expressing it through the surface charge density $\nu = \nu(\vec{r}, t) = \nu(\vec{r}, t) = -\left(\vec{n}(\vec{r}, t), \nabla\Phi(\vec{r}, t)\right)/4\pi$ on the disturbed surface of the drop $r(\theta, t)$ in the following form:

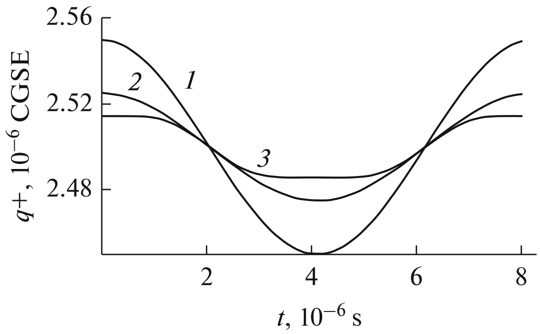

Fig. 2. Dependence of the value of the positive polarization charge of drop q_+ on time calculated at $E_0 = 1000$ V/cm ($\sim 8 \times 10^{-3} E_{0cr}$) and other physical values indicated in Fig. 1. Curve *1* corresponds to the initial excitation of the equilibrium form of the drop surface in the linear approximation in ε of the form $\varepsilon P_2(\mu)$; curve *2*, $\varepsilon[P_2(\mu) + P_3(\mu)]/2$; curve *3*, $\varepsilon[P_2(\mu) + P_3(\mu) + P_4(\mu)]/3$.

$$q_+ = \int_{S_1} \frac{\nu(\vec{r},t)}{(\vec{n}(\vec{r},t),\vec{e}_r)} r^2 \sin\theta d\theta d\varphi$$
$$= -\frac{1}{4\pi}\int_{S_1} \frac{(\vec{n}(\vec{r},t),\nabla\Phi(\vec{r},t))}{(\vec{n}(\vec{r},t),\vec{e}_r)} r^2 \bigg|_{r=r(\theta,t)} \sin\theta d\theta d\varphi; \tag{21}$$

we take into account that $\Phi(\vec{r},t) \equiv \Phi^{(0)}(\vec{r}) + \Phi^{(1)}(\vec{r},t)$.

Inserting into (21) the expansion for the electric potential from (12) and the normal vector for the disturbed surface of the drop to the accuracy of the terms ~ε taking into account (14) and (17), we find the value of the positive charge on the disturbed surface of the drop $r(\theta, t)$:

$$q_+(t) = \frac{3}{4}E_0$$
$$\times\left(1 + \frac{1}{15}e^2 + 2\varepsilon\sum_{n=0}^{\infty} M_n(t)\left(G_1(n) + e^2 G_2(n)\right)\right). \tag{21a}$$

The expressions for the coefficients $G_1(n)$ and $G_2(n)$ will be placed in Appendix 1 (because of space limitation). It is seen that part of expression (26), which depends on time, contains the terms ~ ε and ~εe^2. It is also noteworthy that $E_0 \sim e$.

Figure 2 illustrates the dependences of q_+ on time according to (21a). As is seen, the value of the induced charge periodically changes with time. In the calculations, for precision, the eccentricity of the uncharged drop in the external electrostatic field was accepted as the definite relation $e \equiv \sqrt{9E_0^2/16\pi}$ [14].

Similarly, for the second half of the spheroidal drop, the value of the negative polarization charge will differ from (21a) just by the sign.

THE MODEL OF THE OSCILLATING DIPOLE

Thus, the uncharged drop polarized in the external electrostatic field can be represented as a system of two polarized charges equal in magnitudes but opposite in signs (which change in value during the oscillations of the surface of the drop), shifted for some distance smaller than the drop diameter relative to each other. It is reasonable to introduce into the study the "effective" centers with the positive and negative charges, which are determined by the following relations:

$$\vec{R}_{q_\pm} = \frac{1}{q_\pm}\int_{S_{1,2}} \vec{r}\, dq_\pm.$$

These centers will be located along the symmetry axis of the drop (because of the symmetry of oscillations), and during the oscillations of its surface, they will also oscillate. Note that their oscillations will occur in antiphase relative to each other (the centers will come close to each other and move away from one another). In other words, we shall get the "effective" dipole [8]

$$\vec{d}(t) = 2q_+(t)\cdot\vec{R}_{q_+}(t), \tag{22}$$

which will oscillate and simultaneously radiate electromagnetic waves.

The intensity of the dipole is determined by the known expression [16], p. 213

$$I = \frac{2}{3c^3}\left(\frac{d^2\vec{d}(t)}{dt^2}\right)^2. \tag{23}$$

According to (23), the analytic expression for the power of the electromagnetic radiation of a unit drop with consideration of (22) is easy to obtain. For this, we need only the analytic expression for the vector of displacement of the center of the induced charge (e.g., positive) of the spheroidal drop $\vec{R}_{q^+}$. The general form of it is as follows:

$$\vec{R}_{q_+}(t) = \frac{1}{q_+}\int_{S_1} \vec{r}\, dq_+ = \frac{1}{q_+}\int_{S_1} r\vec{e}_r dq_+;$$

$$S_1 \equiv \left[r = r(\theta,t); 0 \le \theta \le \frac{\pi}{2}; 0 \le \varphi \le 2\pi\right].$$

The radial unit vector $\vec{e}_r$ of a spherical system of coordinates is related to those of the Cartesian system of coordinates:

$$\vec{e}_r = \vec{e}_x \sin\theta\cos\varphi + \vec{e}_y \sin\theta\sin\varphi + \vec{e}_z\cos\theta. \tag{24}$$

Since the vector of intensity of the electric field is directed along the z axis, the displacement of the center of the charge of the drop in the plane x, y does not occur:

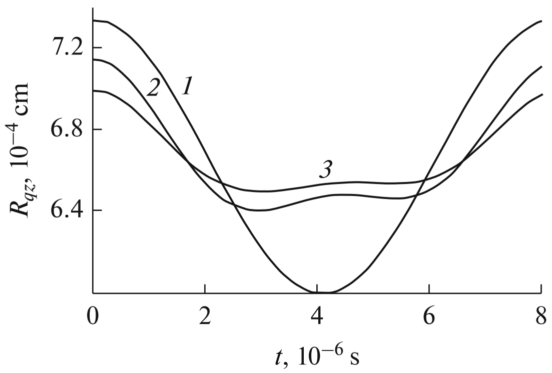

Fig. 3. Dependence of displacement of the center of positive polarization charge of the drop R_{qz} on time and physical values accepted in Fig. 2. Curves *1—3* correspond to the same accepted values as in Fig. *2*.

$$R_{qx} = R_{qy} = 0.$$

Taking into account (24), we shall write the projection of the displacement vector of the center of a positive polarization charge along the z axis in the form

$$R_{qz}(t) = -\frac{1}{4\pi q_+} \times \int_{S_1} \frac{(\vec{n}(t), \nabla\Phi(t))}{(\vec{n}(t), \vec{e}_r)} r^3 \bigg|_{r=r(\theta,t)} \cos\theta \sin\theta d\theta d\varphi. \quad (25)$$

After the integration of a half spheroid S_1, it is easy to find

$$R_{qz} = \frac{1}{2q_+} E_0 \times \left(1 + \frac{2}{5}e^2 + 3\varepsilon \sum_{n=0}^{\infty} M_n(t)\left(G_3(n) + e^2 G_4(n)\right)\right).$$

Inserting into this expression the value of the induced charge (21a), we shall obtain a resultant expression for the displacement of the center of the positive induced charge of the uncharged spheroidal drop along the z axis in the first order of smallness with respect to the disturbance of the surface and the eccentricity squared:

$$R_{qz}(t) = \frac{2}{3}\left(1 + \frac{1}{3}e^2 + \varepsilon \sum_{n=0}^{\infty} M_n(t)\left(3G_3(n) - 2G_1(n) - e^2\left(\frac{8}{15}G_1(n) + 2G_2(n) + \frac{1}{5}G_3(n) - 3G_4(n)\right)\right)\right). \quad (26)$$

Coefficients $G_3(n)$ and $G_4(n)$ are shown in Appendix 2. It is seen that the part of (26) that depends on time contains the terms $\sim\varepsilon$ and $\sim\varepsilon e^2$.

Figure 3 shows the dependences of $R_{qz}(t)$ for the positive induced charge according to (26). It is seen that R_{qz} periodically changes with time.

Similarly, for the second half of the spheroidal drop, we shall obtain the solution for the displacement of the center of the negative polarization charge, which differs just by the sign.

In order to obtain numerical estimates of the radiation intensity, let us preset the initial conditions in the form of the initial deformation of the equilibrium spheroidal form of the drop and the equality to zero of the initial speed of movement of the surface:

$$t = 0:\ \xi(\theta) = \varepsilon \sum_{j \in \Xi} h_j P_j(\mu); \quad \sum_{j \in \Xi} h_j = 1; \quad \frac{\partial \xi(\theta)}{\partial t} = 0, \quad (27)$$

where h_j are the coefficients determining the partial contribution of the jth oscillation mode in the total initial disturbance and Ξ is the set of values of the numbers of the initially disturbed oscillation modes.

Satisfying the initial conditions of (27), for the real constants of a_n and b_n, we shall get the following values:

$$a_n = \frac{1}{2}h_j\left(\delta_{j,n} + e^2\left(-\delta_{j,n-2}\frac{n(n-1)\left(3n^2 - 3n + 2\right)}{4(2n-1)(2n-3)(3n-4)} + \delta_{j,n+2}\frac{(n+1)\left(3n^3 + 27n^2 + 44n + 16\right)}{4(2n+3)(2n+5)(3n+2)}\right)\right);$$

$$b_n = 0;\ (j \in \Xi\ \ n = 0, 1, 2, \ldots),$$

where $\delta_{j,n}$ is the Kronecker symbol.

As a result, we shall write the amplitudes of the first order of smallness in the expression for the form of the surface of the oscillating drop as follows:

$$M_n(t) = h_n \delta_{j,n}\cos(\omega_n t) + e^2\left(h_{n-2}\delta_{j,n-2}\frac{n(n-1)\left(3n^2 - 3n + 2\right)}{4(2n-1)(2n-3)(3n-4)} \times (\cos(\omega_{n-2}t) - \cos(\omega_n t)) - h_{n+2}\delta_{j,n+2}\frac{(n+1)\left(3n^3 + 27n^2 + 44n + 16\right)}{4(2n+3)(2n+5)(3n+2)} \times (\cos(\omega_{n+2}t) - \cos(\omega_n t))\right);$$

$$(j \in \Xi;\ \ n \geq 0).$$

Using the dimensional variables in the obtained expression, we shall derive on the basis of (23) the resultant expression for the intensity of the electromagnetic radiation of the uncharged spheroidal drop that oscillates in the external electrostatic field.

$$I = \frac{E_0^2 R^6 \varepsilon^2}{3c^3}\left(1 + \frac{1}{15}e^2 + 2\varepsilon\sum_{j\in\Xi} h_j\left(G_1(j) + e^2\left(G_1^+(j) + G_2^+(j) + G_3^+(j)\right)\right)\right)^2$$
$$\times\left(\sum_{j\in\Xi} h_j\left(\omega_j^2\left((3G_3(j) - 2G_1(j)) + e^2\left(-\frac{8}{15}G_1(j) - \frac{1}{5}G_3(j) - 2G_1^+(j) + 3G_1^-(j)\right)\right)\right.\right.$$
$$\left.\left. + e^2\left(\omega_{j+2}^2\left(2G_3^+(j) - 3G_3^-(j)\right) + \omega_{j-2}^2\left(3G_2^-(j) - 2G_2^+(j)\right)\right)\right)\right)^2. \quad (28)$$

The expressions for the coefficients $G_1^\pm(j)$, $G_2^\pm(j)$, and $G_3^\pm(j)$ are in Appendix 3.

It is easy to see from (28) that the expression for the intensity is substantially nonlinear, namely, there are two small parameters in the problem, ε and e^2 (note that $E_0 \sim e$). As a result, (28) contains the products of small parameters up to $\sim\varepsilon^5 e^{10}$.

Using (28) makes it possible to estimate the value of intensity of the electromagnetic radiation of various liquid-drop systems of artificial and natural origin, e.g., convective clouds.

A possible source of the electromagnetic radiation is connected with oscillations of the main mode of the finite amplitude of droplets that are met most often in clouds from 3 to 30 μm. Concentration n of such drops in a cloud is ~ 10 cm^{-3} [17], and the oscillations of the main mode are connected with the motion of drops relative to the medium [18]. In addition, a high amplitude of oscillations of the cloud drops can be caused for different reasons: coagulation; disintegration into smaller drops as a result of collisions or realization of electrostatic instability; hydrodynamic and electric interaction with the drops flowing nearby; aerodynamic interaction with a developed small-scale turbulence typical of the thunderstorm clouds. The amplitudes of oscillations of cloud drops can reach tens of percents of the drop radius according to the data of the field studies [19, 20].

For the possible source of the electromagnetic radiation connected with the oscillations of small uncharged drops in the electrostatic field, let us estimate the intensity of the background dipole electromagnetic radiation when the displacement of the centers of polarization charges results from the mode excitation $j = 2$. For the numerical estimations at $j = 2$, $\varepsilon = 0.1$, $h_2 = 1$, $\sigma = 73$ dyne/cm, $\rho = 1$ g/cm^3, $R = 10$ μm, and $E_0 = 50$ V/cm. Then, it is easy to get $I \sim 1 \times 10^{-28}$ μV from (28) at a frequency of ≈100 kHz.

It is noteworthy that contrary to the results obtained for the intensity of the dipole radiation of the oscillating charged drop [2], in the situation of the uncharged drop oscillating in the uniform electrostatic field discussed in this work, the dependence of the intensity on the drop radius is absent (which is also seen from the analytic expression (28) if we take into account the dependence of frequency on the radius). Upon variation of the drop radius, only the radiation frequency changes.

If we accept that all drops oscillate in-phase, the integral intensity of the electromagnetic radiation from the cloud 10 km in diameter will be N times higher than that of a single drop (N is the number of drops in a cloud). In this case $N \approx 5 \times 10^{20}$ is more; hence, the radiation intensity from the cloud is $I_{\text{in}} \sim 5 \times 10^{-8}$ μV. This radiation can be dependably registered by the radio-receiving equipment [21, p. 24], see also [3, 6]. The in-phase oscillations of a single drop can appear in a thunderstorm cloud at an abrupt change in the intensity of the in-cloud electric field, which occurs at a lightning discharge [6, 22]. If the oscillation phases are independent of each other, the integral intensity will be $\sqrt{N}$ times higher than the intensity of radiation of a single drop (N is a number of drops in a cloud) [6].

Figure 4a illustrates the dependences of the intensity of the electromagnetic radiation of a single uncharged drop on the intensity of external electric field calculated using (28): curve *1* is the frequency of oscillations in the expression for the intensity (28) determined using (18); curve *2* is the frequency of oscillations determined using the modified expression which goes to zero at a critical intensity of the field (curve *3* in Fig. 1). Figure 4 demonstrates that, in the region of small values of intensity, with an increase in the latter, the radiation intensity rapidly increases. Curve *1* goes somewhat beyond the region of its applicability, which is determined by condition $e^2 \ll 1$, but its extrapolation is presented for illustrative purposes. In the area of strong fields, if we determine the frequency using the modified expression and curve *2* is valid, with an increase in the intensity, the radiation intensity reaches its maximum (at $E_0 \sim 80$ kV/cm), and then it starts to decrease (see curve *2*, Fig. 4a), since at a critical value of the field the oscillation frequency approaches zero.

Figure 4 shows the curves which refer (for illustrative purposes) to the critical (for realization of the

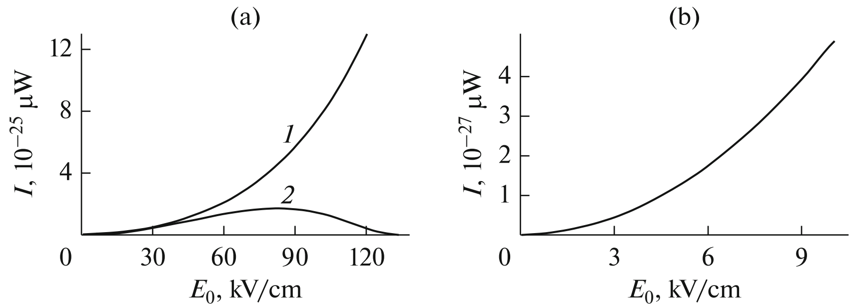

Fig. 4. Dependences of intensity of electromagnetic radiation of a unit uncharged drop oscillating in an electrostatic field (a) calculated according to (28) (curve *1*) and intensities of electromagnetic radiation of the unit uncharged drop with the modified frequency (the coefficient at eccentricity is 1.74) (curve *2*) calculated using the same values as in Fig. 1; (b) the same dependence, but calculated in the region of small values of intensity.

electrical instability of the drop) values of the field intensity. In reality, it seems reasonable to study uncharged cloud drops only in weak fields [23]. However, we can also take into account that the presence of weak charges on drops will not significantly strengthen the intensity of their electromagnetic radiation and will expand the study into the region of strong fields.

Figure 5 shows the dependence of the radiation intensity on the density of the liquid and the value of its surface tension coefficient (it should be emphasized that according to (28) the radiation intensity depends on the density of the liquid and the value of the surface tension coefficient not only through the frequency but also through the eccentricity).

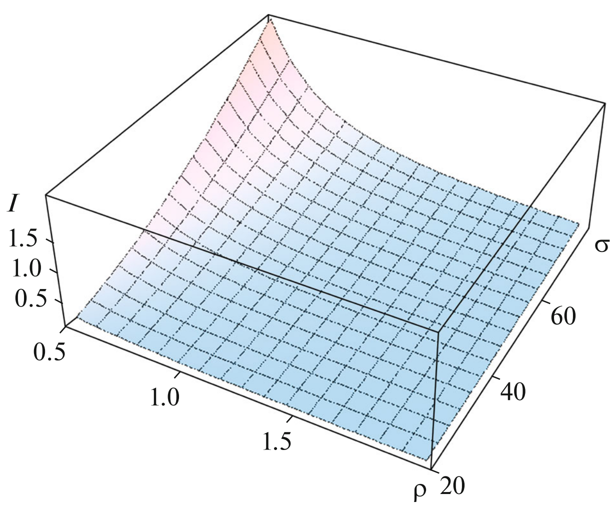

Fig. 5. Dependence of radiation intensity (the units of measurement are 10^{-28} μV) of a unit uncharged drop on the value of surface tension coefficient σ (the units of measurement are dyne/cm) and density of liquid ρ (the units of measurement are g/cm^3) calculated at E_0 = 1000 V/cm and the same other values.

CONCLUSIONS

The electromagnetic radiation of an uncharged drop of a conducting liquid oscillating in an electrostatic field is determined by the radiation of the charges that move at a growing rate and that are induced in the drop by the external electrostatic field. Comparing the charges with opposite signs, which are induced at the opposite halves of the drop, and the "effective" charges at the drop axis, we obtain an oscillating dipole, whose moment is modified during oscillations. Using the dipole radiation, we can model the radiation of the uncharged drop that oscillates in the electrostatic field. The radiation itself is revealed in the nonlinear asymptotic calculations, and it is dipole unlike quadrupole radiation, which is detected in the linear calculations and is by 10^{14}–10^{15} times less intense than the dipole one, and which actually does not affect the total intensity.

APPENDIX 1

The analytic expressions for the coefficients $G_1(n)$ and $G_2(n)$ in (21a):

$$G_1(n) = \frac{1}{(2n+1)}\left(n^2 F_{n-1} + (n+1)(n+2)F_{n+1}\right);$$

$$G_2(n) = -\frac{2}{35}\delta_{n,3} + \frac{1}{(2n+1)}\left(\frac{5n(n-1)(n-2)^2}{2(2n-3)(2n-1)}F_{n-3} + \frac{n^2\left(146n^3+73n^2-246n-153\right)}{30(2n-3)(2n+1)(2n+3)}F_{n-1} + \frac{(n+1)(n+2)\left(26n^3+65n^2+46n-20\right)}{30(2n-1)(2n+1)(2n+5)}F_{n+1} + \frac{(n+1)(n+2)(n+3)(n+4)}{2(2n+3)(2n+5)}F_{n+3}\right);$$

$$F_n = \int_0^{\pi/2} P_n(\mu)\sin\theta d\theta = \begin{cases} 1\ (n=0); \\ \dfrac{(-1)^{\frac{n-1}{2}}(n-1)!}{2^{n-1}(n+1)\left(\left(\frac{n-1}{2}\right)!\right)^2}\ (n = 2k+1); \\ 0\ (n = 2k). \end{cases}$$

APPENDIX 2

The expressions for the coefficients $G_3(n)$ and $G_4(n)$ in (26):

$$G_3(n) = \frac{(n+1)}{(2n+1)}\left(\frac{n(n-1)}{(2n-1)}F_{n-2} + \frac{(4n^3+10n^2+2n-3)}{(2n-1)(2n+3)}F_n + \frac{(n+2)(n+3)}{(2n+3)}F_{n+2}\right);$$

$$G_4(n) = -\frac{1}{35}\delta_{n,3} + k_1F_{n-4} + k_2F_{n-2} + k_3F_n + k_4F_{n+2} + k_5F_{n+4};$$

$$k_1 = \frac{3n(n-1)^2(n-2)(n-3)}{2(2n-5)(2n-3)(2n-1)(2n+1)};$$

$$k_2 = \frac{n(n-1)\left(376n^5 - 736n^4 - 1198n^3 + 154n^2 + 777n - 45\right)}{15(2n-5)(2n-3)(2n-1)(2n+1)(2n+3)(2n+1)};$$

$$k_3 = \frac{2(n+1)\left(136n^6 + 484n^5 + 106n^4 - 925n^3 - 809n^2 - 66n + 90\right)}{15(2n-5)(2n-3)(2n-1)(2n+1)(2n+3)(2n+1)};$$

$$k_4 = \frac{(n+1)(n+2)(n+3)\left(136n^4 + 936n^3 + 1606n^2 + 36n - 365\right)}{15(2n+7)(2n+5)(2n+3)(2n+1)(2n-1)(2n+1)};$$

$$k_5 = \frac{(n+1)(n+2)(n+3)(n+4)(n+5)}{(2n+7)(2n+5)(2n+3)(2n+1)}.$$

APPENDIX 3

Analytic expressions for the coefficients $G_1^{\pm}(j)$, $G_2^{\pm}(j)$, and $G_3^{\pm}(j)$ in (28):

$$G_1^{+}(j) = \frac{2}{35}\delta_{j,3}F_0 + p_1F_{j-3} + p_2F_{j-1} + p_3F_{j+1} + p_4F_{j+3};$$

$$p_1 = \frac{3(j-1)(j-2)^2\left(j^3 - 7j^2 + 4j + 4\right)}{4(2j-3)(2j-1)(2j+1)(3j-4)};$$

$$p_2 = \frac{j\left(180j^7 - 1212j^6 + 251j^5 + 2985j^4 + 25j^3 - 3453j^2 + 324j + 540\right)}{60(2j-3)(2j-1)(2j+1)^2(2j+3)(3j-4)};$$

$$p_3 = -\frac{(j+1)(j+2)\left(180j^6 + 1572j^5 + 4771j^4 + 6319j^3 + 2816j^2 - 988j - 720\right)}{60(2j-1)(2j+1)^2(2j+3)(2j+5)(3j+2)};$$

$$p_4 = \frac{3(j+1)^2(j+2)(j+3)(j+4)^2}{4(2j+1)(2j+3)(2j+5)(3j+2)};$$

$$G_2^{+}(j) = \frac{(j-1)\left(3j^3 + 9j^2 - 28j + 12\right)}{4(2j-1)(2j+1)(3j-4)(2j-3)} \times \left(j(j-1)F_{j-1} + (j-2)^2F_{j-3}\right);$$

$$G_3^{+}(j) = \frac{(j+1)(j+2)\left(3j^2 + 9j + 8\right)}{4(2j+1)(2j+3)(3j+2)(2j+5)} \times \left((j+4)(j+3)F_{j+3} + (j+2)^2F_{j+1}\right);$$

$$G_1^{-}(j) = -\frac{2}{35}\delta_{j,3}F_1 + q_1F_{j-4} + q_2F_{j-2} + q_3F_j + q_4F_{j+2} + q_5F_{j+4};$$

$$q_1 = -\frac{3(j-1)^2(j-2)(j-3)\left(3j^3-27j^2+20j+12\right)}{4(2j-5)(2j-3)(2j-1)(2j+1)(3j-4)};$$

$$q_2 = -\frac{(j-1)}{60(2j-5)(2j-3)(2j-1)^2(2j+1)^2(2j+3)(3j-4)}$$

$$\times\left(720j^9-8664j^8+20468j^7+9762j^6-56530j^5+17109j^4+25712j^3-837j^2-4320j-540\right);$$

$$q_3 = \frac{(j+1)}{30(2j-3)(2j-1)^2(2j+1)^2(2j+3)^2(3j-4)(3j+2)}$$

$$\times\left(19584j^{10}+84144j^9+17104j^8-327136j^7-329116j^6+317277j^5\right.$$
$$\left.+517866j^4+9421j^3-176508j^2-19116j+17280\right);$$

$$q_4 = \frac{(j+1)(j+2)(j+3)}{60(2j-1)(2j+1)^2(2j+3)^2(2j+5)(2j+7)(3j+2)}$$

$$\times\left(720j^7+11544j^6+65836j^5+166470j^4\right.$$
$$\left.+180340j^3+47181j^2-38771j-17520\right);$$

$$q_5 = \frac{(j+1)(j+2)(j+3)(j+4)(j+5)\left(3j^2+21j+16\right)}{4(2j+1)(2j+3)(2j+5)(2j+7)(3j+2)};$$

$$G_2^-(j) = \frac{(j-1)\left(3j^3+9j^2-28j+12\right)}{4(2j-1)(2j+1)(3j-4)(2j-3)}\times\left(\frac{j(j-1)(j+1)}{(2j-1)}F_j\right.$$

$$+\frac{(j-1)\left(4j^3-14j^2+10j+1\right)}{(2j-1)(2j-5)}F_{j-2}$$
$$\left.+\frac{(j-1)(j-2)(j-3)}{(2j-5)}F_{j-4}\right);$$

$$G_3^-(j) = \frac{(j+1)(j+2)\left(3j^2+9j+8\right)}{4(2j+1)(2j+3)(3j+2)(2j+5)}\times\left(\frac{(j+3)(j+4)(j+5)}{(2j+7)}F_{j+4}\right.$$

$$+\frac{(j+3)\left(4j^3+34j^2+90j+73\right)}{(2j+7)}F_{j+2}$$
$$\left.+\frac{(j+1)(j+2)(j+3)}{(2j+3)}F_j\right).$$

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