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Selection of Optimal Parameters for the Jiles–Atherton Magnetic Hysteresis Model

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Abstract—A method is proposed for implementation of the most popular hysteresis model, the Jiles–Atherton model, which has a number of advantages over other models. A technique for optimization of the parameters of the hysteresis model based on a real coded genetic algorithm is presented. The method is implemented in two stages. The first stage involves preliminary estimation of the model parameters and the range of their variation. The second stage is the direct implementation of the genetic algorithm. The criterion of convergence is based on the achievement of a preset value of the standard deviation and the maximum permissible number of generations. The genetic algorithm was implemented with 50 individuals. Each individual is associated with four variables that correspond to the hysteresis model parameters. The maximum number of generations was set to 50 and 100. The initial probabilities of the crossover and mutations were set to 90 and 5%, respectively. A specific feature of the proposed implementation of the genetic algorithm consists in internal optimization of the fifth parameter for each individual of the population. The computer code was developed using the Delphi environment. Comparison of the experimental and simulated curves showed good agreement. A method that involves preliminary estimation of the parameters and further application of the genetic algorithm yields rather accurate results, is easy to implement, and provides a high data-processing speed.

Keywords: modeling of magnetic hysteresis, magnetic materials, identification parameters, genetic algorithm

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Of the hysteresis models used in recent years to describe nonlinear characteristics of magnetic materials, the Jiles–Atherton model (JA-model) is still one of the most popular. This is due to a number of advantages of the model. First, the model is formulated in terms of a differential equation. Second, only five parameters are used that are identified by one measured hysteresis loop [1–3]. In addition, this model can be applied to isotropic and anisotropic media and allows simulation of quasi-static and dynamic loops. This model can underlie a vector hysteresis model.

To obtain reliable calculated parameters of magnetic fields of electrical equipment, one should accurately simulate the characteristics of the materials. For this purpose, correct data have to be selected for the models used in computations. The basic stage of the implementation of the JA-model is the computation of the hysteresis parameters (the model setting) by experimental data. We should note that the computation of the above parameters is a rather laborious process and presents the most serious problem of this model. The problem of estimating and identifying the parameters in question can be solved in the most efficient way by adaptive optimal search techniques such as the simulated annealing algorithm, genetic algorithm, neural network method, fuzzy logic method, particle swam algorithm, and direct search algorithm.

We article propose a method for identification of the JA-model parameters based on solution of the optimization problem. The standard deviation of the hysteresis loop coordinates obtained from the experimentally measured hysteresis loop using the JA-model serves as the optimization function. The JA-model parameters play the role of independent variables in this case. A hybrid genetic algorithm that supposes the presetting of the ranges of variation of the JA-model parameters when estimating the latter is proposed as an optimization method. The genetic algorithm allows us to achieve rather quickly good agreement between the simulated and measured curves. An advantage of the algorithm is that it works with continuous or discrete parameters. It does not require any information on the gradients and potential discontinuities present in the function that evaluates the solution validity. The algorithm is resistant to hitting local optima and can process numerical experimental data and analytical functions. The random nature of the genetic algorithm does not allow finding the absolutely best solution; however, it can help find a good solution of the problem of selecting the JA-model parameters.

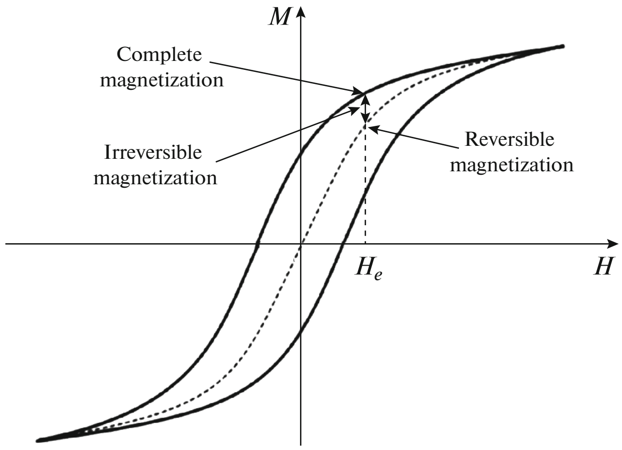

Fig. 1. Hysteresis loop according to the Jiles–Atherton model.

The theory of ferromagnetic hysteresis devised by Jiles and Atherton distinguishes between the reversible and irreversible magnetization in the saturation function [1]. The total magnetization according to the Jiles–Atherton model is shown in Fig. 1.

To implement the model, five parameters need to be set, i.e., saturation magnetization M_s (A/m); α, a parameter that considers the effective magnetic field strength in the core; k (A/m), the constant of irreversible deformation of the domain walls; c, the constant of the elastic displacement of the domain boundaries; and A, the anhysteretic curve shape parameter.

In [2], an alternative solution is proposed that allows simplification of the modeling procedure by replacing the equation

$$dM = \left(\frac{1}{k\delta}dH_e\right)(M_{an} - M) + cdM_{an}$$

by the equation

$$dM = \frac{\chi_f}{|\chi_f|}(\chi_f dH_e) + cdM_{an},$$

where $\chi_f = \frac{1}{k}[M_{an} - M]$.

Here, M is the magnetization of the substance, H_e is the effective field, M_{an} is the anhysteretic magnetization curve, c is the constant of the elastic displacement of the domain boundaries, k is the factor of adhesion or loss factor, and $\delta = \begin{cases} \text{if } \Delta H \geq 0 \Rightarrow \delta = 1 \\ \text{else} \Rightarrow \delta = -1 \end{cases}$ is the sign of change in the magnetic field strength; for the rest of the variables, conventional notation is adopted.

Consequently, the algorithm for implementation of the scalar JA-model (dependence $B = f(H)$) can be written in the following form:

count = 1,

$He_0 = H(t_0) = 0,$

$B_1 = B(t_0) = 0,$

While count $\leq N4$

$t_1 = [t_0 + \Delta t(\text{count} - 1)],$

$H_1 = H_m \sin(\omega t_1),$

$H_2 = H_m \sin(\omega(t_1 + \Delta t)),$

$\Delta H = H_2 - H_1,$

$M_1 = \dfrac{B_1}{\mu_0} - H_1,$

$He_1 = H_1 + \alpha M_1,$

$\Delta H_e = He_1 - He_0,$

$$\text{if } \frac{|He_1|}{A} > 0.1 \Rightarrow \begin{cases} M_{an} = M_s\left[\coth\left(\dfrac{He_1}{A}\right) - \dfrac{A}{He_1}\right] \\ \dfrac{dM_{an}}{dHe} = \dfrac{M_s}{A}\left[1 - \coth^2\dfrac{He_1}{A} + \left(\dfrac{A}{He_1}\right)^2\right] \end{cases},$$

$$\text{else} \Rightarrow \begin{cases} M_{an}(t) = M_s\dfrac{He_1}{3A} \\ \dfrac{dM_{an}}{dHe} = \dfrac{M_s}{3A}, \end{cases}$$

$$\chi_f = \frac{1}{k}[M_{an} - M_1],$$

$$\text{if } (\chi_f \Delta He) > 0 \Rightarrow \frac{dM}{dH} = \frac{\dfrac{\chi_f}{|\chi_f|}\chi_f + c\dfrac{dM_{an}}{dHe}}{1 - \alpha\left(\dfrac{\chi_f}{|\chi_f|}\chi_f - c\dfrac{dM_{an}}{dHe}\right)},$$

$$\text{else} \Rightarrow \frac{dM}{dH} = \frac{c\dfrac{dM_{an}}{dHe}}{1 - c\alpha\dfrac{dM_{an}}{dHe}},$$

$$M_2 = M_1 + \frac{dM}{dH}\Delta H,$$

$B_2 = \mu_0[H_2 + M_2],$

$B_1 = B_2,$

$He_0 = He_1,$

$M_1 = M_2.$

To apply the genetic algorithm, the initial values of the independent variables and the range of their variation have to be determined. We adopt M_s, c, A, and k as independent variables and calculate then a number of auxiliary parameters, i.e.,

(i) permeability of saturation $\mu_s = \dfrac{B_s}{\mu_0 H_s}$;

(ii) permeability of demagnetization $\mu_r = \dfrac{B_r}{\mu_0 H_C}$; and

(iii) mean magnetic susceptibility. To determine this variable, we use the hysteresis model of [4] as an auxiliary model. This model uses three parameter that can be found in the manufacturer's specifications, i.e., saturation induction B_s, coercive force H_C, and residual induction B_r. The model is based on the representation of the magnetic properties of the material in the form of a hysteresis loop of the major cycling hysteresis formed by three curves, namely, the upward and downward branches of the hysteresis cycle and the initial magnetization curve. To provide "interlocking" of the downward and upward branches of the major cycling hysteresis for the model of [4], the saturation induction is corrected by calculating auxiliary coefficients as follows:

$$a = \frac{2B_s H_C^2}{B_r^2} - \frac{2\mu_0 H_s H_C^2}{B_r^2} - \frac{2H_s H_C}{B_r},$$

$$b = \frac{4B_s H_s H_C}{B_r} - \frac{4\mu_0 H_s^2 H_C}{B_r} - \frac{4B_s H_C^2}{B_r} + \frac{4\mu_0 H_s H_C^2}{B_r} - 2H_s^2 + 2H_s H_C + 2H_C^2,$$

$$c = 2B_s H_s^2 - 2\mu_0 H_s^3 - 4B_s H_s H_C + 4\mu_0 H_s^2 H_C,$$

$$B_{s1} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad B_{s2} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

In addition, the magnetic field strength is found that corresponds to the saturation induction for the model of [4] by the condition

$$\text{if } B_{s1} > 0 \Rightarrow \begin{cases} B_{\text{SNEW}} = B_{s1} \\ H_{\text{SNEW}} = H_s + \dfrac{B_{\text{SNEW}} - B_s}{\mu_0 \mu_s}, \end{cases}$$

$$\text{if } B_{s2} > 0 \Rightarrow \begin{cases} B_{\text{SNEW}} = B_{s2} \\ H_{\text{SNEW}} = H_s + \dfrac{B_{\text{SNEW}} - B_s}{\mu_0 \mu_s}. \end{cases}$$

The limit hysteresis loop according to the model of [4] is constructed by implementing the algorithm

count = 1

While count $\leq N + 1$

$t = dt(\text{count} - 1)$,

$H_{\text{count}} = H_{\text{SNEW}} \sin(\omega t)$

$$Bup_{\text{count}} = \frac{B_{\text{SNEW}}(H_{\text{count}} + H_C)}{|H_{\text{count}} + H_C| + H_C\left(\dfrac{B_{\text{SNEW}}}{B_r} - 1\right)} + \mu_0 H_{\text{count}},$$

$$Bdn_{\text{count}} = \frac{B_{\text{SNEW}}(H_{\text{count}} - H_C)}{|H_{\text{count}} - H_C| + H_C\left(\dfrac{B_{\text{SNEW}}}{B_r} - 1\right)} + \mu_0 H_{\text{count}},$$

$$B0_{\text{count}} = \frac{Bdn_{\text{count}} + Bup_{\text{count}}}{2}.$$

Here, Bup_{count} are the points of the upward branch, Bdn_{count} are the points of the downward branch of the limit hysteresis loop, and $B0_{\text{count}}$ are the points of the initial magnetization curve.

The intermediate induction and magnetic field strength value to assess the shape parameter is

$$B_r + \frac{B_{\text{SNEW}} - B_r}{5} \leq B_x < B_{\text{SNEW}} - \frac{B_{\text{SNEW}} - B_r}{5} \Rightarrow B_x H_x.$$

The mean magnetic permeability is $\mu_{sr} = \dfrac{B_s}{\mu_0 H_s}$.

The constant of the elastic displacement of the domain boundaries—for modeling in weak fields—is determined by the relation from [5] as follows:

$$c = \frac{\mu_{sr} - 1}{\mu_r - \mu_{sr}} = \frac{\mu_0(\mu_{sr} - 1)H_C}{B_r - \mu_0 \mu_{sr} H_C}.$$

The constant of the irreversible deformation of the domain walls is

$$k = \frac{H_C \mu_r}{\mu_r - 1}.$$

The saturation magnetization is

$$M_s = \frac{B_s}{\mu_0} - H_s.$$

The intermediate magnetization for determination of the shape parameter is

$$M_x = \frac{B_x}{\mu_0} - H_x.$$

The shape parameter of the anhysteretic magnetization curve is [5]

$$A = \frac{H_x\left[1 - \dfrac{c}{1 - c}\right]}{\dfrac{1}{1 - \dfrac{M_x}{M_s}} - 3\dfrac{M_x}{M_s}}.$$

The coefficient of the magnetic couple of the domains is [6]

$$\alpha = \left|\frac{3A}{M_s} - \frac{H_x - H_C}{M_x}\right|.$$

The ranges of variation of the parameters are set according to Table 1.

Table 1. Preset boundaries of the parameters under optimization

M_s	Left-hand boundary	Right-hand boundary	A	Left-hand boundary	Right-hand boundary
	$0.5M_s$	$1.5M_s$		$0.1A$	$1.2A$
c	Left-hand boundary	Right-hand boundary	k	Left-hand boundary	Right-hand boundary
	0	$1.5c$		$0.5k$	$1.5k$

The work of the algorithm starts with the formation of a set of solutions called populations. The solutions from the current population are used to form a new population. This procedure is iterated until a certain state is reached, i.e., a set number of generations or improvement of the best solution. The formation of a population starts from characterization of individuals, these individuals including five JA-model parameters. The initial values of four of them assigned to the population present random values within a permissible range (Table 1). Each individual of this population is estimated from the agreement between the calculated and experimental data.

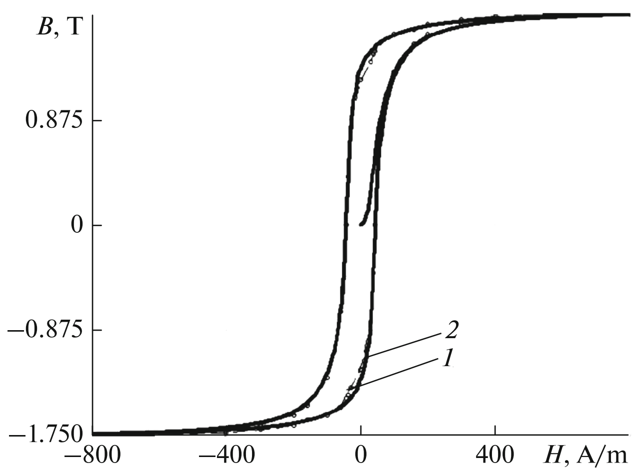

Fig. 2. (*1*) Experimental and (*2*) simulated curves of the magnetic hysteresis for grade E330A steel.

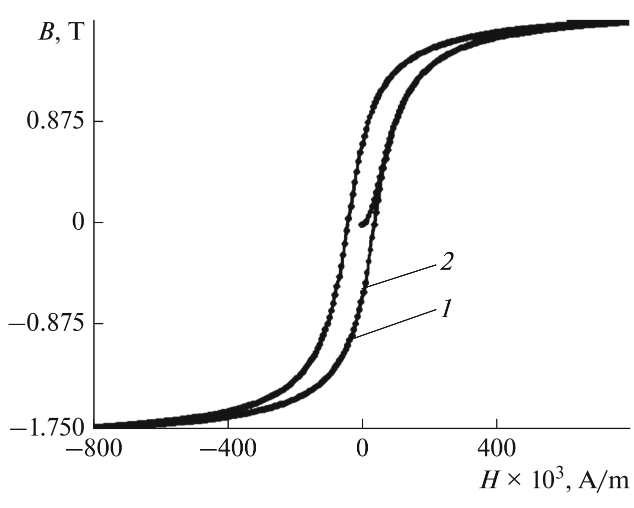

Fig. 3. (*1*) Experimental and (*2*) simulated curves of the magnetic hysteresis for grade 20 steel.

The criterion of convergence is based on reaching the preset value of the standard deviation and the maximum permissible number of generations. If convergence is not achieved, genetic operators are used such as selection, crossover, mutation, and improvement techniques. The selection procedure is responsible for the formation of the pairs to be passed to other genetic operators. Global elitism is used as the improvement technique. It allows avoiding losses of good solutions in the course of the optimization process.

The genetic algorithm was implemented using 50 individuals each with four variables that correspond to the parameters of the JA hysteresis model. The maximum number of generations (iterations) was set equal to 50 and 100. The initial probabilities of the crossover and mutation were set to 90 and 5%, respectively. The permissible ranges of variations for each variable are presented in Table 1. The target function that has to be minimized corresponds to the overall standard deviation between the experimental and simulated magnetic hysteresis curves.

The distinguishing feature of the above variant of implementing the genetic algorithm is the internal optimization of the fifth α parameter for each individual of the population. The initial value of this parameter is selected according to the formula proposed in the section devoted to the preliminary estimation of the parameters. Then, this value is reduced in variable steps until the domain of permissible solutions for the JA-model with preset parameters (independent variables) M_s, k, c, and A has been provided and the α_{init} value is determined. Thereafter, this parameter is reduced again in constant step $\Delta\alpha = 0.01\alpha_{init}$ until the target function minimum—the minimum standard deviation between the experimental and simulated curves—has been achieved. Thus, the best value for each individual in the population is selected.

In Figs. 2 and 3, experimental and simulated hysteresis curves for grade E330A and 20 steels are shown. Curves *1* correspond to the experimental data obtained at a remagnetization frequency of 50 Hz, and curves *2* were simulated using the model with the additional inter-

Table 2. Calculated preliminary and optimized parameters for grade E330A steel (40 experimental points)

Parameter calculated from the preliminary estimate		Optimal parameter			
		without additional internal optimization of α	with additional internal optimization of α	without additional internal optimization of α	with additional internal optimization of α
		50 generations (iterations)		100 generations (iterations)	
M_s	1430130.5292140	1428490.2694216	1434078.4954145	1429887.3259198	1434078.4954144
k	45.0021206	52.2385321	52.1945417	57.6053539	50.6548797
c	0.0866415	0.0303303	0.0335987	0.1013189	0.0019610
A	22.6183058	26.3724118	26.5149653	26.4199296	26.5149659
α	0.0005112	0.0000572	0.0000555	0.0000599	0.0000554
Computation time, s	0.01	5.97	120.41	24.33	499.81

Table 3. Calculated preliminary and optimized parameters for grade 20 steel (1282 experimental points)

Parameter calculated from the preliminary estimate		Optimal parameter			
		without additional internal optimization of α	with additional internal optimization of α	without additional internal optimization of α	with additional internal optimization of α
		50 generations (iterations)		100 generations (iterations)	
M_s	1350507.1183000	1337965.7520671	1355127.6216052	1348526.9025521	1349847.0463627
k	240.1034478	305.4688537	300.0706335	314.1529471	300.0706335
c	0.1247856	0.1643957	0.1244216	0.13740020	0.1195762
A	332.4384217	383.1978366	398.9261060	385.3426013	398.9261067
α	0.0031558	0.0005199	0.0004793	0.0004042	0.0004932
Computation time, s	0.01	5.77	193.56	51.59	1494.31

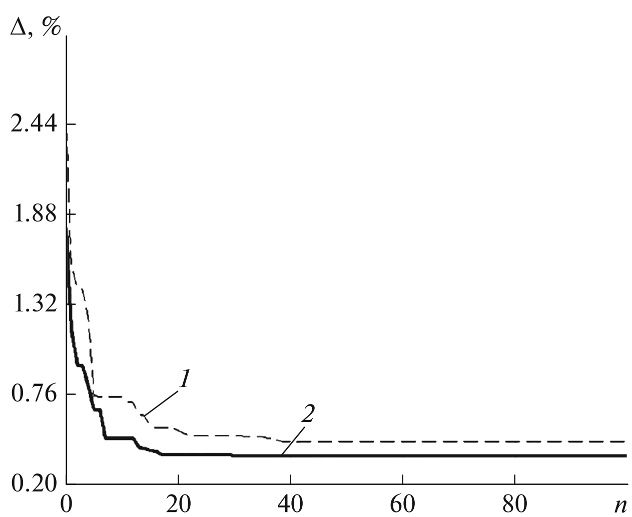

Fig. 4. Total error depending on the number of iterations for grade E330A steel: (*1*) without additional internal optimization and (*2*) with additional internal optimization.

nal optimization with 100 iterations; the latter were constructed based on the data of Tables 2 and 3.

In Figs. 4 and 5, the dependences of the total error (Δ%) on the number of generations for grade E330A and 20 steels are shown. It can be seen that the error quickly decreases and the search algorithm allows us to achieve the optimal set of parameters with minimum computational effort.

The proposed method that involves preliminary estimation of the parameters followed by the use of the genetic algorithm is rather efficient. The method and the corresponding algorithm allow simulation of the change in the magnetic parameters of ferromagnetic materials in the course of remagnetization quickly and with good accuracy. The method is especially promising when applied to designing electromechanical converters the principle of operation of which is based on the magnetic hysteresis effect, i.e., hysteresis-reluctance motors and hysteresis couplings. Comparison of experimental and simulated hysteresis curves shows

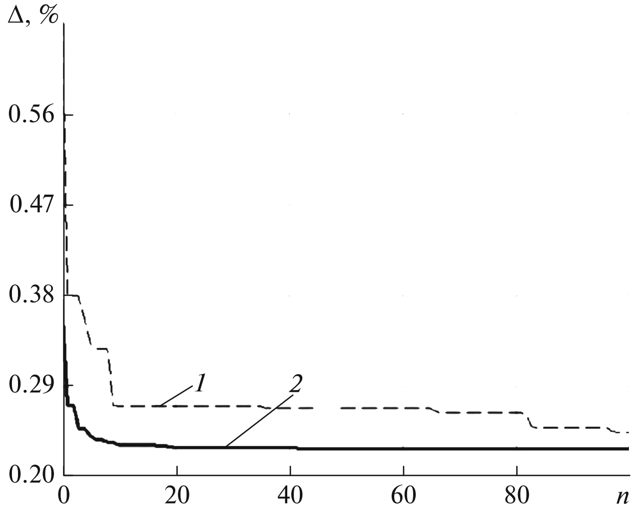

Fig. 5. Total error depending on the number of iterations for grade 20 steel: (*1*) without additional internal optimization and (*2*) with additional internal optimization.

their good agreement. The main requirement for construction of a precise model is sufficient and accurately measured input data uniformly distributed by the hysteresis loop.

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