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Modeling of Three-Dimensional Fields of Eddy Currents during Induction Heating of Process Equipment

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Abstract—The problem of heat generation during induction heating of process equipment has been analyzed using the example of ferromagnetic plates used for assembling hydraulic-frame presses. We present a mathematical model of induction heating that includes the equations of the electromagnetic field and heat transfer in a three-dimensional formulation. The calculation of three-dimensional fields of eddy currents in ferromagnetic bodies is associated with the large computer time consumption for the solution of Maxwell's equations. In addition, the engineering methods used do not provide the required accuracy since they do not take into account the features of the geometry of the object. A technique has been proposed for calculating three-dimensional fields of eddy currents in ferromagnetic bodies using linear differential equations, which makes it possible to reduce the computation time by more than an order of magnitude. This simplification of the mathematical model of induction heating is based on the assumption that the magnetic permeability of the plate material is constant during the process of heating. Solving the nonlinear equations of the electromagnetic field in the two-dimensional formulation, we have determined the magnetic permeability corresponding to the magnetization curve (in terms of active power) as a function of the characteristics of the inductor and its location. The finite-element method is implemented with the ANSYS software is used. The results obtained can be used in the design of induction heating plants that should satisfy special requirements for the temperature-field configuration.

Keywords: induction heating, ferromagnetic materials, magnetic permeability, Maxwell's equations, heating plate, temperature field model

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For implementation of interrelated technological processes of pressure treatment and heating of products, press equipment with metal plates of induction heating is used. Such electrothermal plants should provide heating of processed articles to 150–250°C for polymers and elastomers, and up to 350–550°C for metals and special compositions. As a rule, in a single operation cycle of the plant, several articles are treated, placed in the molds between the working surfaces of the plates. Therefore, such presses are characterized by a relatively high energy consumption (from 4 to 20 kW per plate) and material consumption (the plates can have a length and a width of more than 1 m and a thickness of 50–80 mm).

To reduce the costs of material in the design, manufacture, and operation of equipment, improve its technological characteristics, it is necessary to optimize the design of the heating plate to provide a temperature field of the required configuration on the working surface. Mathematical description of the processes of induction heating is associated with the formalization of associated non-stationary electromagnetic and thermal processes in the volume of the plate. As a rule, for heating the plates of presses, low-frequency electromagnetic fields are used, being efficient only when heating ferromagnetic materials. The nonlinear dependence of the magnetic field induction on the field strength in ferromagnetic bodies greatly complicates the calculation of heat generation. Current methods for calculating induction electrotechnical plants have the following drawbacks:

—methods of calculation based on solving linear Maxwell's equations in a one-dimensional formulation with the use of empirical data do not allow to obtain and analyze the distribution of energy release in ferromagnetic bodies with complex shape;

—simplified mathematical models of the heating process, which do not take into account the nonlinearity of characteristics of ferromagnetic materials, can lead to significant errors in calculating the heat generation due to eddy currents; and

—the approximation of electric heaters by the bodies of canonical form leads to large errors in most practical problems [1].

Thus, it is necessary to develop effective approaches for the mathematical description of electromagnetic processes and solve the equations of mathematical models. These approaches must take into account all the features of processes under consideration, but they should not lead to an excessive volume of calculations.

MATHEMATICAL MODEL OF INDUCTION HEATING OF A SINGLE PLATE

The mathematical model of induction heating of a single plate of the hydraulic press contains the relationships for calculating heat generation from eddy currents in the volume of the plate and the heat-transfer equation taking into account the heat loss in the environment. Mathematical simulation of eddy currents is based on the allocation of two regions in the volume of the plate and in the surrounding airspace:

—regions of existence of eddy currents with nonzero electric conductivity (Ω_1); and

—regions without eddy currents that can contain currents from an external source (Ω_2).

When considering the problem of induction heating of the plates, region Ω_1 is represented by the material of the plates and region Ω_2 contains inductors and the surrounding airspace.

We make the following assumptions:

—Maxwell displacement current is absent (low-frequency fields in a conducting medium are considered);

—magnetic hysteresis is absent (the processes in strong electromagnetic fields are considered); and

—the properties of materials are isotropic.

Then, in domain Ω_1, the differential equations of the electromagnetic field (Maxwell's equations) have the form

$$\mathrm{rot}\mathbf{H} - \gamma\mathbf{E} = 0, \tag{1}$$

$$\mathrm{rot}\mathbf{E} + \frac{\partial\mathbf{E}}{\partial t} = 0, \tag{2}$$

$$\mathbf{divB} = 0, \tag{3}$$

where $\mathbf{H}$ is the magnetic-field strength, A/m; $\mathbf{E}$ is the electric-field strength, V/m; γ is the specific electric conductivity, Ω^{-1} m^{-1}; $\mathbf{B}$ is the magnetic-field induction, T; and t is the time in seconds.

Correspondingly, in domain Ω_2, we have

$$\mathrm{rot}\mathbf{H} = \mathbf{J}_{\mathrm{ext}}, \tag{4}$$

$$\mathrm{div}\mathbf{B} = 0, \tag{5}$$

where $\mathbf{J}_{\mathrm{ext}}$ is the density of current from an external source, A/m^2.

For quantities $\mathbf{H}$, $\mathbf{E}$, and $\mathbf{B}$ to be single-valued, the boundary conditions for contact region Γ_{12} between domains Ω_1 and Ω_2 are used, as well as on a far-distant boundary [2].

To solve Maxwell's equations, supplementary functions are introduced: vector magnetic potential $\mathbf{A}$ (Wb/m) and scalar electric potential V (V) related to the vectors $\mathbf{B}$ and $\mathbf{E}$ by the relations

$$\mathbf{B} = \mathrm{rot}\mathbf{A}, \tag{6}$$

$$\mathbf{E} = -\frac{\partial\mathbf{A}}{\partial t} - \mathrm{grad}V. \tag{7}$$

Then, system of equations (1)–(3) can be written in the form of a single equation:

$$\mathrm{rot}(\mu^{-1}\mathrm{rot}\mathbf{A}) + \gamma\frac{\partial\mathbf{A}}{\partial t} + \gamma\mathrm{grad}V = 0, \tag{8}$$

where μ is the absolute magnetic permeability, H/m.

Similarly, we can write Eqs. (4), (5):

$$\mathrm{rot}(\mu^{-1}\mathrm{rot}\mathbf{A}) = \mathbf{J}_{\mathrm{ext}}. \tag{9}$$

It follows from (7) that scalar electric potential V is determined up to a constant. To eliminate this uncertainty, it is sufficient to set the value of the potential V at one point in the computational domain. According to (6), vector magnetic potential $\mathbf{A}$ is determined to within a gradient of an arbitrary scalar function. The uniqueness of the solution is achieved by means of a gauge condition. When calculating the fields of eddy currents, the Coulomb gauge is used most frequently [3]

$$\mathrm{div}\mathbf{A} = 0. \tag{10}$$

From the computational point of view, it is extremely difficult to satisfy condition (10) with zero divergence when the magnetic permeability is nonconstant. The Coulomb-gauge condition in the three-dimensional formulation was first successfully implemented in [2], in which "penalty function" $-\mathrm{grad}(\mu^{-1}\mathrm{div}\mathbf{A})$ was added in the left parts of Eqs. (8), (9) and the conditions of uniqueness of the solution were formulated.

In this case, the modified equations within domain Ω_1 take the form

$$\begin{gathered}\mathrm{rot}(\mu^{-1}\mathrm{rot}\mathbf{A}) - \mathrm{grad}(\mu^{-1}\mathrm{div}\mathbf{A}) \\ + \gamma\frac{\partial\mathbf{A}}{\partial t} + \gamma\mathrm{grad}V = 0,\end{gathered} \tag{11}$$

$$\mathrm{div}\left(-\gamma\frac{\partial\mathbf{A}}{\partial t} + \gamma\mathrm{grad}V\right) = 0, \tag{12}$$

while, in domain Ω_2, we have

$$\mathrm{rot}(\mu^{-1}\mathrm{rot}\mathbf{A}) - \mathrm{grad}(\mu^{-1}\mathrm{div}\mathbf{A}) = J_{\mathrm{ext}}. \tag{13}$$

The boundary conditions on the remote boundary (the conditions of zero normal component of the magnetic induction):

$$\mathbf{B} \times \mathbf{A} = 0; \tag{14}$$

$$\mu^{-1}\mathrm{div}\mathbf{A} = 0, \qquad (15)$$

where $\mathbf{n}$ is the unit vector of normal to the surface of the boundary.

The boundary conditions at the boundary Γ_{12} of the contact of domains Ω_1 and Ω_2:

$$\mathbf{A}_1 = \mathbf{A}_2, \qquad (16)$$

$$\mu_1^{-1}\mathrm{rot}\mathbf{A}_1 \times \mathbf{n}_1 + \mu_2^{-1}\mathrm{div}\mathbf{A}_2 \times \mathbf{n}_2 = 0, \qquad (17)$$

$$\mu_1^{-1}\mathrm{rot}\mathbf{A}_1 - \mu_2^{-1}\mathrm{div}\mathbf{A}_2 = 0, \qquad (18)$$

$$\mathbf{n}\left(-\gamma\frac{\partial\mathbf{A}}{\partial t} + \gamma\mathrm{grad}V\right) = 0, \qquad (19)$$

where subscripts 1 and 2 indicate that the characteristics correspond to the appropriate regions.

In [2], it was proved that system (11)–(19) of equations and boundary conditions is equivalent to Maxwell's system of differential equations and provides an automatic fulfillment of Coulomb gauge (10).

To calculate the generation of heat in the volume of the heating plate, the required total current density is

$$\mathbf{J} = \mathbf{J}^{\mathrm{ind}} + \mathbf{J}^{\mathrm{ext}}, \qquad (20)$$

where $\mathbf{J}^{\mathrm{ind}}$ is the density of eddy (induced) current (A/m^2) determined by the expression

$$\mathbf{J}^{\mathrm{ind}} = -\gamma\frac{\partial\mathbf{A}}{\partial t} + \gamma\mathrm{grad}V. \qquad (21)$$

The vector of current density of an external source is determined, in general, by the current of the inductors and their design characteristics:

$$\mathbf{J}^{\mathrm{ext}} = f(I_i, \mathbf{G}^{\langle i\rangle}),\ i = 1,\dots,n_{\mathrm{ind}}, \qquad (22)$$

where I_i is the current through the ith inductor, A; $\mathbf{G}^{\langle i\rangle}$ is the vector of construction parameters of the ith inductor; and n_{ind} is the number of inductors in a heating plate.

The modulus of the average current density of the inductor can be calculated by the formula

$$J_i^{\mathrm{ext}} = \frac{\omega_i I_i}{b_i h_i},$$

where ω_i is the number of turns of the ith inductor and b_i and h_i are the width and depth of the transverse slot behind the ith inductor (in meters).

The direction of current is expressed via the basis vectors of the coordinate system, depending on the shape of the inductor. The design characteristics of the most common rectangular inductors can be represented as the vector

$$\mathbf{G}^{\langle i\rangle} = \{d_i, \omega_i, x_i^c, y_i^c, l_i, s_i, b_i, h_i\},$$

where d_i is the wire diameter, m); x_i^c and y_i^c are the coordinates of the center of the ith inductor, m; and l_i and s_i are the length and width of the inductor (m).

It should be noted that the currents of the external source and eddy currents do not intersect, and equation (20) describing the merger of regions Ω_1 and Ω_2 is necessary to correctly write the relations characterizing the heat propagation in the bulk of the plate.

To describe the process of heat transfer in the plate, the heat-conduction equation is used with allowance for the internal heat sources:

$$\frac{\partial T}{\partial\tau} = a\nabla^2 T + \frac{q}{c\rho}, \qquad (23)$$

where $T = T(x, y, z, \tau)$ is the temperature (°C) in the point of the volume of the plate with coordinates x, y, z at moment τ; $q = q(x, y, z)$ is the specific heat production (W/m^3); $a = \lambda/(c\rho)$ is the thermal-diffusivity coefficient (m^2/s) of the material of the plate; and c, ρ, and λ are the specific heat capacity (J/(kg K)), density (kg/m^3), and thermal conductivity (W/(m K)).

The steady-state average specific heat generation in the volume of the plate is determined by the Joule–Lenz law,

$$q = f\int\limits_{\tau_{si}}^{\tau_{si}+\frac{1}{f}} \frac{J^2}{\gamma}\,d\tau, \qquad (24)$$

where f is the frequency of current, Hz); J is the total current-density modulus (see (20)); and τ_{si} is the conditional stabilization time of electromagnetic processes after which the changes in the amplitude values of the current density can be neglected (c).

The initial condition for Eq. (23) is

$$T(x, y, z, 0) = T_0, \qquad (25)$$

where T is the temperature of a surrounding air, °C.

In the absence of thermal insulation, heat transfer from the working surface of the cover and the ends of the heating plate is described by the boundary conditions of the third kind,

$$-\lambda\left.\frac{\partial T}{\partial n}\right|_{\Omega_{\mathrm{pl},F}} = \alpha_r(T_r - T_0),\ r = 1,\dots,6, \qquad (26)$$

where $\Omega_{\mathrm{pl},F}$ is the rth surface of the heating plate, m^2; α_r is the heat conductivity of the surface of this plate, W/(m^2 K), determined from the criteria equation according to [4]; and T_r is the average temperature of the rth surface of the plate, °C.

To calculate the temperature field of heating plates in the mode of thermal stabilization with the help of a two-position controller, the following expression is proposed:

$$I_i(\tau) = \frac{I_i}{2}(1 + (-1)^{I_{\mathrm{sw}}(\tau)+1},\ i = 1,\dots,n_{\mathrm{ind}}, \qquad (27)$$

where $I_{sw}(\tau) = \begin{cases} 1, \text{ if } T_s(\tau) < T_d \\ 0, \text{ if } T_s(\tau) > T_u \end{cases}$

is the state of the heaters (1 is "on," 2 is "off"); $T_s(\tau)$ is the temperature of the plate at the location of a reference thermocouple, °C; and T_d and T_u are the lower and upper operation thresholds of the regulator, °C.

Thus, the mathematical model of the process of induction heating and automatic stabilization of the working surface temperature of a single plate of hydraulic press is described by the system of equations (11)–(27).

THE METHOD OF SOLVING EQUATIONS OF THE MATHEMATICAL MODEL

To solve Eqs. (11)–(27) of the model, the ANSYS system of finite-element analysis was used. It was shown in [5] that electromagnetic and thermal analysis should be carried out consecutively using the SOLID97 and SOLID70 finite-element software, respectively. The specific electric conductivity of the plate material declines in the process of heating. Therefore, the procedure of electromagnetic analysis should be repeated to clarify the heat generation.

Solving three-dimensional nonlinear equations (11)–(13) with the use of the finite-element method (FEM) requires a significant expenditure of computer time due to the slow convergence of iterations and the nonstationarity of the electromagnetic processes.

Assuming that the external source of current is sinusoidal and the magnetic permeability of the plate material is constant, relations (11)–(13) are simplified to linear quasi-stationary equations in complex representation

$$-\mu^{-1}\Delta\dot{\mathbf{A}} + j2\pi f\gamma\dot{\mathbf{A}} + \gamma\operatorname{grad}\dot{V} = 0; \tag{11'}$$

$$\operatorname{div}(j2\pi j\gamma\dot{\mathbf{A}} + \gamma\operatorname{grad}\dot{V}) = 0, \tag{12'}$$

$$-\mu^{-1}\Delta\dot{\mathbf{A}} = \dot{\mathbf{J}}_{\text{ext}}, \tag{13'}$$

where j is the imaginary unit ($j^2 = -1$).

Boundary conditions (14)–(19) are simplified similarly, and the calculation of heat generation in this case is carried out by the formula

$$q = \frac{|J|^2}{2\gamma}, \tag{24'}$$

where $|J| = \sqrt{\operatorname{Re}\dot{J}^2 + \operatorname{Im}\dot{J}^2}$ is the modulus of amplitude of the complex density of current.

The time spent on solving Eqs. (11')–(13') can be reduced by more than an order of magnitude compared with the solution of original equations (11)–(13). However, the necessary condition of the magnetic-permeability constancy being equivalent to the magnetization curve for the mean value of the active power per period hinders the direct use of these equations when calculating eddy-current fields in ferromagnetic materials.

To solve this problem, a technique was developed based on solving two-dimensional nonlinear electromagnetic-field equations, since in this case the mathematical model becomes substantially simpler. If vector $\mathbf{J}_{\text{ext}}$ of the external current is perpendicular to the XY plane, then the vector magnetic potential has only one component A_z. Therefore, the Coulomb gauge condition is automatically satisfied and it is not necessary to introduce the scalar electric potential [6]. Taking into account the characteristics of ferromagnets in the two-dimensional formulation is not a serious problem. Therefore, two-dimensional models can be used very effectively in cases in which this does not lead to a distortion of the description of physical processes.

The dependence of the equivalent magnetic permeability on the characteristics of the inductor and its location was studied by solving systems of equations (11)–(13) and (11')–(13') for an axisymmetric system including a ferromagnetic disk and an inductor (Fig. 1). The radius of location of the inductor, the number of its coil turns, the current, the filling factor (the fraction of "pure" wire in the volume of the slot), and the position of the inductor relative to the outer surfaces of the disk altered alternately. For each combination of these parameters, we carried out a "reference" nonlinear calculation and a series of linear calculations in which the magnetic permeability was determined. In the course of the conducted numerical experiments, we found that:

—the magnetic permeability is most affected by the dimensions of the cross section of the inductor and its magnetomotive force (MDS);

—if the radius of the inductor location exceeds the inductor width by more than two times, its influence can be neglected;

—the distance from the inductor to the outer surfaces of the disk significantly affects eddy currents if it is less than twice the depth of the electromagnetic-wave penetration into the body of the disk;

—the filling factor of the slot of the inductor with the wire affects only the heat dissipation of the inductor (ohmic heating); and

—the inductor location inside the slot and the gaps between the inductor and the walls of the slot do not affect the induced eddy currents due to screening.

These conclusions made it possible to propose a method to determine the equivalent magnetic permeability, a block diagram of which is shown in Fig. 2. The solution is continued until the active powers of the

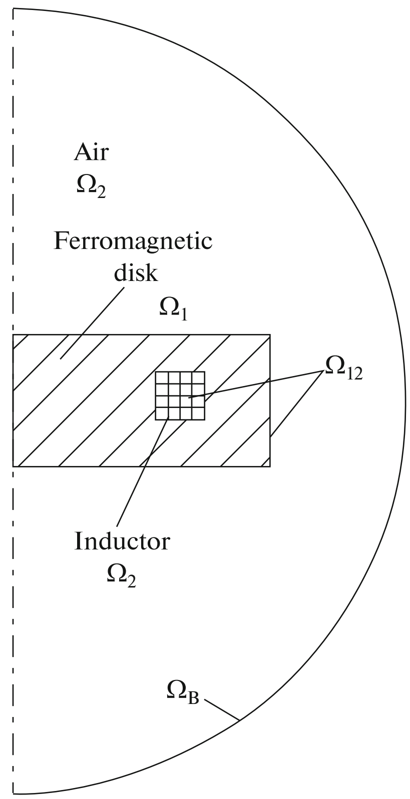

Fig. 1. Axisymmetrical model of a disk with an inductor.

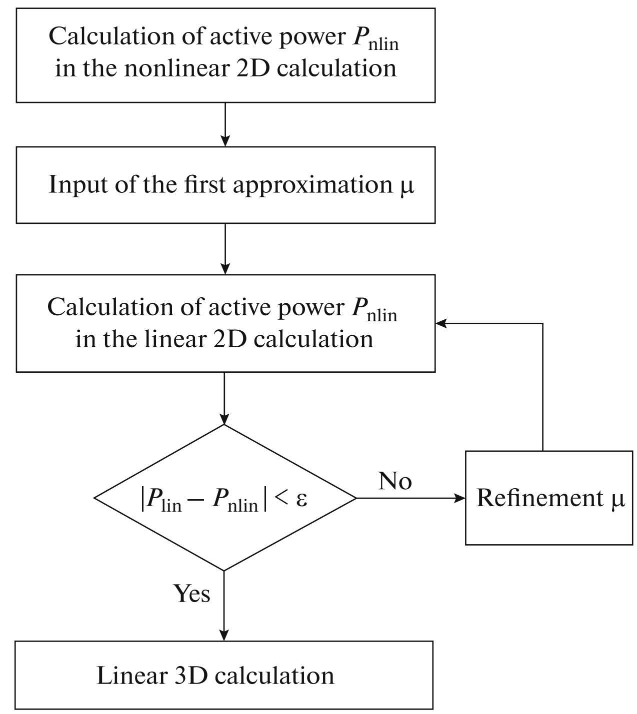

Fig. 2. Block diagram of the algorithm for determining the equivalent magnetic permeability.

inductor in the linear and nonlinear calculations (P_{lin} and P_{nlin}) differ by more than allowable inaccuracy ε.

Using this technique, we calculated and approximated by power functions the dependence of the equivalent magnetic permeability of structural ferromagnetic steel of the MDS inductor with cross-sectional dimensions of 25 × 25 mm when connected to an ideal current generator (Fig. 3, curve *1*) and to an ideal voltage generator (Fig. 3, curve *2*).

The differences are caused by the use of a linear description of electromagnetic processes. In fact, harmonic curves with nonlinear distortions, characteristic of ferromagnetic materials, are replaced by equivalent sinusoidal curves. In this case, the active power released in the form of heat is determined in different ways: the values of the power coefficient do not coincide. We note that for the vast majority of industrial presses, voltage generators are applied, that is, curve *2* is used in the calculations.

A similar approach to solving three-dimensional nonlinear electromagnetic-field equations for a ferromagnetic parallelepiped with a regular mesh of finite elements is presented in [1]. In contrast to the method used, the magnetic permeability is here determined for the regions of the body sensing the electromagnetic radiation. These domains are divided onto border blocks of finite elements taking into account the geometry of the design scheme for which one-dimensional calculations are performed. In this case, a three-dimensional model is used in the iteration process. On one hand, such an approach should provide a high accuracy since the magnetic permeability is not averaged over the volume of the ferromagnetic and, in fact, the field of magnetic permeability is determined. On the other hand, a large amount of computation is unavoidable when solving real tasks.

The proposed methodology seems to be a compromise: it ensures an acceptable accuracy of the solution, it does not require the large amount of calculations, and it is simple in implementation.

To verify the adequacy of model (11')—(13'), (14)—(23), (24'), (25)—(27) and the applied methodology in determining the magnetic permeability, we used the results of measuring the temperature field of the heated plate with dimensions of 500 × 410 mm having four rectangular inductors obtained as a result of experiment [7] (in these temperature measurements, we used five thermocouples mounted in the center and in the corners of the working surface, as well as a control thermocouple). Figure 4 compares the experimental and calculated data for the point of location of the reference thermocouple. The average absolute deviation was 2.6°C, and the relative deviation was

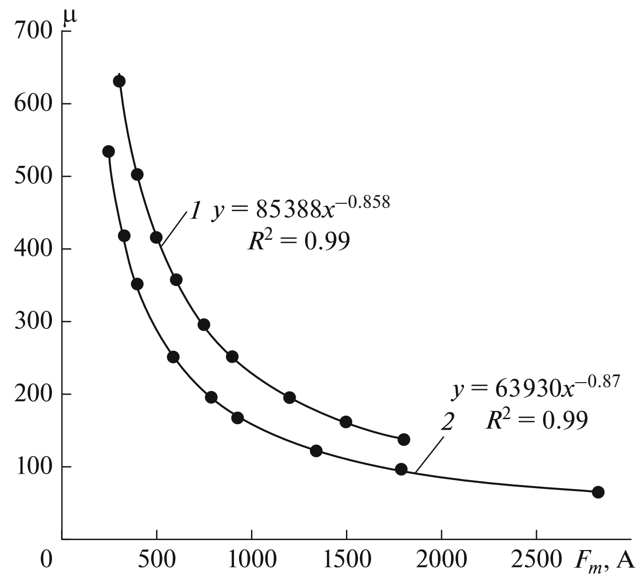

Fig. 3. Dependence of the magnetic permeability on the magnetomotive force of the inductor: (*1*) when connected to a current generator; (*2*) when connected to a voltage generator.

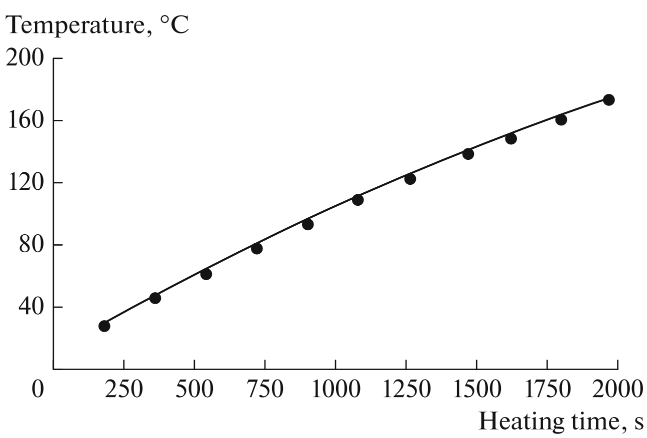

Fig. 4. Comparison of calculation results with experiment.

0.7%. For this plate, a finite-element model containing more than 800 thousand SOLID97 elements was developed. The calculation time on a computer with a performance of 18 GFlops was 25 h.

Earlier, in [8], a method for optimization of constructional characteristics of a press heating plate was proposed on the basis of an empirical method for calculating heat generation. The use of Eqs. (11')—(13'), (24') and a method for determining the equivalent magnetic permeability allows us to describe the electrothermal processes in the plates on a qualitatively new level (with allowance for the parameters of the electrical network, the magnetization curve of the heated material, and the relative dislocation of inductors) and to increase the efficiency of the optimization technique.

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