

# Numerical Investigations of Electromagnetic Processes in a Solid Cylindrical Shield

A. G. Shcherbinin and A. S. Mansurov

Perm National Research Polytechnic University, Perm, 614990 Russia

e-mail: journal-elektrotehnika@mail.ru

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**Abstract**—A mathematical model is suggested to describe the processes in a solid cylindrical shield in protection against an alternating magnetic field. The model is constructed with respect to the complex amplitude of magnetic vector potential. Since magnetic field lines are in a plane perpendicular to the axis of a cylindrical shield, the problem becomes two-dimensional. The electromagnetic parameters of the considered media are constant and isotropic. The plates at which the magnetic potential is set are the source of the magnetic field. A distribution of real and imaginary components of the complex amplitude of magnetic potential is described by four differential equations in the conducting medium and by two equations in the dielectric one. An equality of magnetic potential at both sides of the interface is predetermined at the interfaces. The Robin boundary condition provides equality of the magnetic vector potential to zero at an infinite distance from the shield. The obtained differential equation system supplemented with the boundary conditions can be numerically solved by the finite elements method using the Galerkin method. As a result, distributions of magnetic potential and magnetic field intensity in the absence and presence of a shield are determined; shielding attenuation is then calculated. It is found that, with increasing shield thickness and noise frequency, the efficiency of electromagnetic shielding is increased. The adequacy of the suggested model and technique of determination of the shielding efficiency is corroborated by comparison with the results of an analytical model for a copper cylindrical shield.

**Keywords:** electromagnetic shielding, shielding efficiency, finite element method, numerical investigations

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Shielding is the most effective way to protect electrical equipment from electromagnetic fields [1–3]. Sources of electromagnetic fields can be both natural and artificial in origin. Atmospheric phenomena and solar activity can be designated as natural sources (noise), while broadcasting, telecommunications, TV, navigation, and power supply sources are regarded as artificial sources. Shielding is implemented by means of mounting barriers between an object and noise source; metallic screens intended for attenuation of electromagnetic fields are used as barriers. Since electric fields are shielded much better than magnetic fields, let us determine the protection properties of a cylindrical electromagnetic shield subjected to the action of an alternating magnetic field. To calculate magnetic fields, a vector potential of magnetic field is widely used [4, 5]:

$$\bar{B} = \text{rot} \bar{A}, \quad (1)$$

where  $\bar{B}$  is the magnetic induction vector.

This is a 2D problem since, magnetic field lines are in the plane perpendicular to axis of a cylindrical shield. Let us assume that the electromagnetic parameters of the considered media are constant and isotropic.

Using the approach suggested in [6], a differential equation for the complex amplitude of magnetic potential in 2D formulation can be expressed in the following form:

for a conducting medium,

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} - i\omega\sigma\mu_a A + i\omega\sigma\mu_a G = 0; \quad (2)$$

$$\int_{S_C} (-i\omega\sigma\mu_a A + i\omega\sigma\mu_a G) ds = \mu_a I, \quad (3)$$

for a dielectric medium,

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = 0. \quad (4)$$

Here,  $A$  is the magnetic potential (directed along coordinate  $z$  in 2D formulation);  $\omega$  is the circular frequency;  $\mu_a$  is the absolute permeability;  $\sigma$  is the specific conductivity,  $G = \frac{J_s}{i\omega\sigma}$ ;  $J_s$  is the current density determined by Ohm's law in differential form;  $S_C$  is the cross-section area of a cylindrical shield; and  $I$  is the given current in a shield ( $I = 0$ ).

In system of equations (2)–(4), values  $A$  and  $G$  are unknown. The complex amplitude of magnetic potential and  $G$  are determined using real ( $A^R$ ,  $G^R$ ) and imaginary ( $A^I$ ,  $G^I$ ) parts:

$$A = A^R + iA^I, \quad (5)$$

$$G = G^R + iG^I. \quad (6)$$

When substituting Eqs. (5) and (6) in Eqs. (2)–(4) and separating real and imaginary parts, we can obtain for a conducting medium,

$$\frac{\partial^2 A^R}{\partial x^2} + \frac{\partial^2 A^R}{\partial y^2} + \omega\sigma\mu_a A^I - \omega\sigma\mu_a G^I = 0; \quad (7)$$

$$-i\omega\sigma\mu_a A^R + \frac{\partial^2 A^I}{\partial x^2} + \frac{\partial^2 A^I}{\partial y^2} + \omega\sigma\mu_a G^R = 0; \quad (8)$$

$$\int_{S_C} (\omega\sigma\mu_a A^I - \omega\sigma\mu_a G^I) ds = \mu_a I^R; \quad (9)$$

$$\int_{S_C} (-\omega\sigma\mu_a A^R + \omega\sigma\mu_a G^R) ds = \mu_a I^I; \quad (10)$$

for a dielectric medium

$$\frac{\partial^2 A^R}{\partial x^2} + \frac{\partial^2 A^R}{\partial y^2} = 0; \quad (11)$$

$$\frac{\partial^2 A^I}{\partial x^2} + \frac{\partial^2 A^I}{\partial y^2} = 0. \quad (12)$$

At the interface between media, the equality of magnetic potential at both sides of the interface is predetermined. At an infinite distance from a shield, the magnetic potential is equal to zero, which is provided by the Robin condition [7, 8]

$$\frac{\partial A}{\partial \rho} + \frac{1}{\rho} A = 0, \quad (13)$$

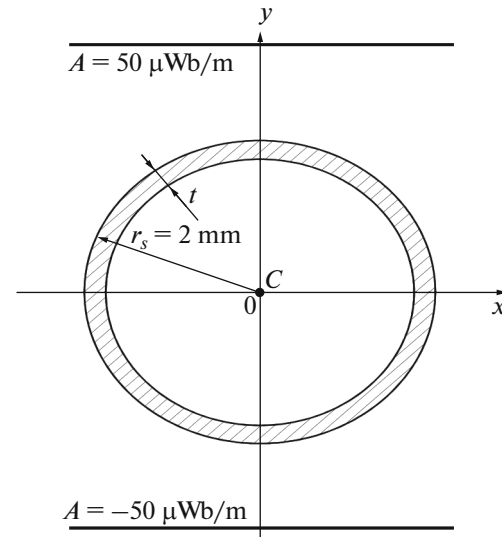
where  $\rho$  is the distance between a conductor center and boundary of the area where the condition of an infinite interface is given.

The presented system of differential equations is solved by the finite elements method [4, 6]. To obtain a system of algebraic equations, the Galerkin method was used [4].

The components of magnetic field intensity in distributions of magnetic potential are calculated by the formulas

$$H_x = \frac{1}{\mu_a} \frac{\partial A}{\partial y}; \quad H_y = \frac{1}{\mu_a} \frac{\partial A}{\partial x}. \quad (14)$$

The calculated scheme is for a copper cylindrical shield placed in the alternating magnetic field between two plates at which magnetic vector potentials are given. The magnetic vector potential is  $A = 50 \mu\text{Wb/m}$  and  $A = -50 \mu\text{Wb/m}$  at upper and lower plates, respectively. The magnetic field changes by the cosine



Schematic of the computational region.

law in the frequency range from 10 to 100 MHz. The shield thickness ranges from 0.10 to 0.14 mm with a step of 0.02 mm.

Attenuation of shielding at point  $C$  (see figure) is determined by the formula [1, 2]

$$\dot{A}_S = 20 \log \left| \frac{H}{H_s} \right|, \quad (15)$$

where  $H$  is the magnetic field intensity at point  $C$  without a shield and  $H_s$  is the magnetic field at point  $C$  with a shield.

To determine shielding attenuation using the proposed model, the distributions of magnetic potential and magnetic intensity in the presence and absence of a shield are calculated. The results of calculations of shielding attenuation are listed in Table 1. As is seen, with increasing frequency of the interfering signal and shield thickness, the efficiency of electromagnetic shielding improves.

The effectiveness of the suggested mathematical model and technique of determination of shielding efficiency is shown by comparison with the results obtained from the corresponding analytical equations [1, 2], by which the shielding effect is determined by the overall action of absorption  $A_{Ab}$  and reflection  $A_R$  attenuation, respectively:

$$A_{S0} = A_{Ab} + A_R, \quad (16)$$

where

$$A_{Ab} = 20 \log |\cosh(\sqrt{ikt})|; \quad (17)$$

$$A_R = 20 \log \left| 1 + \frac{1}{2} \left( \frac{Z_A}{Z_1} + \frac{Z_1}{Z_A} \right) \tanh(\sqrt{ikt}) \right|. \quad (18)$$

**Table 1.** Shielding attenuation

$A_S$ , dB		$f$ , MHz									
		10	20	30	40	50	60	70	80	90	100
Numerical method	$t = 0.10$	72.2	92.4	107.4	119.7	130.5	140.1	149.0	157.1	164.8	172.0
	$t = 0.12$	80.5	104.1	121.7	136.4	149.0	160.5	171.0	180.6	189.7	198.2
	$t = 0.14$	88.8	115.9	136.1	153.0	167.6	180.8	192.9	204.1	214.6	224.5
Analytical method	$t = 0.10$	73.2	101.3	106.5	122.9	135.4	138.9	152.3	157.4	164.6	171.6
	$t = 0.12$	79.3	103.8	131.2	134.9	154.7	157.0	171.4	177.6	179.9	192.4
	$t = 0.14$	95.8	120.5	134.3	149.0	164.0	169.0	178.4	190.4	195.5	203.7

**Table 2.** Relative discrepancies of calculations of shielding attenuation

Shield thickness $s$ , mm	Result discrepancy (%) at frequency $f$ , MHz									
	10	20	30	40	50	60	70	80	90	100
$t = 0.10$	1.4	9.6	0.8	2.7	3.7	0.8	2.2	0.2	0.1	0.2
$t = 0.12$	1.5	0.3	7.8	1.1	3.8	2.2	0.2	1.7	5.1	2.9
$t = 0.14$	7.9	3.9	1.3	2.6	2.14	6.5	7.5	6.7	8.9	9.2

Here,  $k = \sqrt{\omega\mu_a\sigma}$  is the coefficient of the eddy currents,  $Z_M = \sqrt{i\omega\mu_a/\sigma}$  is the wave impedance of the metallic shield,  $Z_A = i\omega\mu_a r_s$  is the wave impedance of the dielectric,  $r_s$  is the shield radius, and  $t$  is the shield thickness.

Results of calculations of shielding attenuation using analytical equations (16)–(18) are also listed in Table 1. The relative discrepancies between shielding attenuation values calculated using the suggested model and the analytical expressions are given in Table 2. For the considered ranges of frequencies and shield thicknesses, they are no higher than 10%, which testifies to the effectiveness of the mathematical model of Eqs. (7)–(12).

Thus, to describe the processes in electromagnetic shields, mathematical models solved by the finite elements method are used. Thanks to the use of such models, shielding efficiency can be determined with a necessary accuracy for both solid cylindrical shields and shields with more complicated geometrical forms and structures.

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