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Model for Calculating the Mean Temperature on the Friction Area of a Disc Brake

A. Yevtushenko[a], * and K. Topczewska[a]

[a] *Bialystok University of Technology, Bialystok, Poland*

**e-mail: a.yevtushenko@pb.edu.pl*

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Abstract—The calculation scheme for determining the temperature mode of the disc brake taking into account the change of the contour contact area during braking was proposed. For this purpose, the exact solutions of the initial problem of motion and the corresponding heat problem of friction were obtained. On their basis, formulas were obtained in the analytical forms for calculating the evolution of the sliding speed, specific power and work of friction, and the mean temperature on the contact area. The thermal sensitivity of the friction pair materials was taken into account by introducing into the calculation the values of the thermal conductivity and the specific heat capacity at the volumetric temperature. Numerical analysis was performed for a brake with discs made from Termar-ADF carbon composite friction material. The values of such characteristics as speed, braking duration, and maximum and volumetric temperatures, found with and without taking into account the change in the contour contact area, were compared. The performed comparison analysis allowed us to determine the influence the contour friction area has on the evolution of mean temperature during braking. It was found that the level of the temperature calculated with consideration of the friction contour surface increases during braking is higher. Despite the fact that the average contact area is equal and the compared braking processes take place at practically the same time. During analysis, the influence of the change of the apparent contact surface area on the temperature time profile was also examined. It was noticed that a larger apparent area of contact causes a decrease in the value of the maximum temperature achieved and the time of reaching it.

Keywords: braking, friction heating, temperature, contour contact area, disc brake

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INTRODUCTION

The friction surfaces of the working elements of the disc brake are not perfectly smooth. Due to waviness and roughness, their contact interaction is discrete and occurs in three areas: nominal, due to the size of the rubbing surface of the lining; contour, formed at the points of contact of waves and macrodeviations; actual, consisting of a set of contacts of microroughnesses within the contour area of contact [1]. According to the hypothesis of summation of temperatures of A.V. Chichinadze, the maximum temperature of the friction surface is equal to the sum of the average temperature of the nominal contact area and the temperature of the actual contact area (temperature flash) [2]. The calculation of the average temperature of the friction surface is usually performed with a constant value of the area of the nominal contact area [3–6]. When a temperature flash is found, the areas of the contour and actual contact areas changing during deceleration are used [7, 8].

Objective—development of an analytical model for calculating the average temperature of the friction surface of a disc brake, taking into account the relationship between the contour and nominal areas. Study on this basis of the evolution of speed, specific friction power, and temperature of a multi-disc brake with rubbing elements made of a carbon friction composite material (CFCM).

CALCULATION MODEL

For a given initial value of kinetic energy W_0, a decrease in velocity V with time t from initial value V_0 at $t = 0$ to zero at the moment of stop $t = t_s$ describes the relationship [3]:

$$V(t) = V_0\left[1 - \frac{V_0}{W_0}\int_0^t F(s)ds\right], \quad 0 \le t \le t_s, \tag{1}$$

where friction force F acting in the contour area with area A_c has form

$$F(t) = fp(t)A_c(t), \quad 0 \le t \le t_s. \tag{2}$$

Taking into account the monotonic increase in pressure p from zero to nominal value p_0:

$$p(t) = p_0 p^*(t), \quad p^*(t) = 1 - e^{t/t_i}, \quad 0 \le t \le t_s, \tag{3}$$

and relations [9, 10]:

$$A_c(t) = A_a A^*(t), \quad A^*(t) = 0.4 + \frac{0.5t}{t_s}, \quad 0 \le t \le t_s, \tag{4}$$

after integration from formulas (1), (2) we obtain:

$$V(t) = V_0 V^*(t),$$

$$V^*(t) = 1 - \frac{tA^*(t) - t_i p^*(t) A^*(t) - \dfrac{0.25t}{t_s}(t - 2t_i e^{-t/t_i})}{t_s^0}, \tag{5}$$
$$0 \le t \le t_s,$$

$$t_s^0 = \frac{W_0}{q_0 A_a}, \quad q_0 = f p_0 V_0. \tag{6}$$

The duration of braking was determined numerically from condition of stop $V^*(t_s) = 0$.

We present the change in the specific power of friction in form:

$$q(t) = q_0 q^*(t), \quad q^*(t) = p^*(t) V^*(t), \quad 0 \le t \le t_s, \tag{7}$$

where the time profiles of pressure $p^*(t)$ and velocity $V^*(t)$ were determined by formulas (3)–(6), respectively. We look for the specific friction work in form:

$$w(t) = w_0 w^*(t), \quad w_0 = q_0 t_s, \quad w^*(t) = t_s^{-1}\int_0^t q^*(t)dt, \tag{8}$$
$$0 \le t \le t_s.$$

Substituting function $q^*(t)$ (7) under the integral sign in formula (8), we find

$$w^*(t) = \frac{1}{t_s^0 t_s} \times \left\{ \begin{array}{l} t_s^0 t - \dfrac{1}{5}t^2 + \dfrac{1}{4}\left(t_i^2 - t_i t - \dfrac{1}{3}t^2\right)\dfrac{t}{t_s} \\ + \left[\begin{array}{l} \dfrac{1}{4}t^2 - t_i^2 - t_s^0 t_s \\ + \dfrac{2}{5}t_s(t - t_i) \end{array} \right] \dfrac{t_i}{t_s} p^*(t) \\ + \dfrac{1}{4}\left(\dfrac{4}{5}t_s + \dfrac{3}{2}t_i + t\right)\dfrac{t_i^2}{t_s} p^*(2t) \end{array} \right\}, \quad 0 \le t \le t_s. \tag{9}$$

To determine the temperature of the contour region, we will use the design scheme of the thermal contact of two semi-bounded bodies, taking into account frictional heat generation. The evolution of the average temperature of the contact surface of such a tribosystem is determined by formulas [11, 12]:

$$T(t) = T_a + \gamma T_0 T^*(\tau), \quad 0 \le t \le t_s, \tag{10}$$

$$T^*(\tau) = \frac{1}{\sqrt{\pi}}\int_0^\tau \frac{q^*(s)}{\sqrt{\tau - s}}ds, \quad 0 \le \tau \le \tau_s, \tag{11}$$
$$\gamma = \frac{\sqrt{k^*}}{\sqrt{k^*} + \sqrt{K^*}}, \quad T_0 = \frac{q_0 a}{K_1}, \quad \tau = \frac{k_1 t}{a^2}, \quad \tau_s = \frac{k_1 t_s}{a^2},$$

$$\tau_s^0 = \frac{k_1 t_s}{a^2}, \quad \tau_i = \frac{k_1 t_i}{a^2}, \quad K^* = \frac{K_2}{K_1}, \quad k^* = \frac{k_2}{k_1}, \tag{12}$$

$a = \max(a_l)$, $a_l = \sqrt{3k_l t_s^0}$ are the effective depths of heat penetration. Hereinafter, all values related to the disc and the pad are provided with subscripts $l = 1$ and $l = 2$, respectively.

Substituting under the integral sign in formula (11) function $q^*(\tau)$ (7), taking into account relations (3) and (5), we obtain:

$$T^*(\tau) = \frac{1}{\sqrt{\pi}}\int_0^\tau \left\{ 1 - \frac{1}{\tau_s^0}\left[\begin{array}{l} \dfrac{2}{5}s + \dfrac{s^2}{4\tau_s} - (1 - e^{-s/\tau_i}) \\ \times\left(\dfrac{2}{5} + \dfrac{\tau_i}{2\tau_s}\right)\tau_i + \dfrac{\tau_i s}{2\tau_s}e^{-s/\tau_i} \end{array} \right] \right\} \times \frac{(1 - e^{-s/\tau_i})}{\sqrt{\tau - s}}ds, \quad 0 \le \tau \le \tau_s. \tag{13}$$

Designating:

$$I_{m,n}(\tau) = \frac{1}{\sqrt{\pi}}\int_0^\tau \frac{s^m e^{-ns/\tau_i}}{\sqrt{\tau - s}}ds, \quad m, n = 0, 1, 2. \tag{14}$$

integral (13) can be written in form

$$\begin{aligned} T^*(\tau) &= \left(1 + \frac{2\tau_i}{5\tau_s^0} + \frac{\tau_i^2}{2\tau_s^0\tau_s}\right) I_{0,0}(\tau) \\ &- \left(1 + \frac{4\tau_i}{5\tau_s^0} + \frac{\tau_i^2}{\tau_s^0\tau_s}\right) I_{0,1}(\tau) + \frac{\tau_i}{\tau_s^0}\left(\frac{2}{5} + \frac{\tau_i}{2\tau_s}\right) I_{0,2}(\tau) \\ &- \frac{2}{5\tau_s^0} I_{1,0}(\tau) - \frac{1}{\tau_s^0}\left(\frac{2}{5} + \frac{\tau_i}{2\tau_s}\right) I_{1,1}(\tau) \\ &+ \frac{\tau_i}{2\tau_s^0\tau_s} I_{1,2}(\tau) - \frac{1}{4\tau_s^0\tau_s}[I_{2,0}(\tau) - I_{2,1}(\tau)]. \end{aligned} \tag{15}$$

Using substitution $x = \sqrt{\tau - s}$, integrals (14) are presented as follows:

$$I_{m,n}(\tau) = \frac{1}{\sqrt{\pi}}\int_0^{\sqrt{\tau}} (\tau - x^2)^m e^{-n(\tau - x^2)/\tau_i} dx, \quad m, n = 0, 1, 2. \tag{16}$$

First, let us consider the particular case when $n = 0$. Using result [13]

$$\int_0^{\sqrt{\tau}} (\tau - x^2)^m dx = \tau^m \sqrt{\tau} B(m + 1; 0.5), \quad m = 0, 1, 2,$$

taking into account the values of beta function $B(1;\,0.5)=2$, $B(2;\,0.5)=4/3$, and $B(3;\,0.5)=16/15$ [14], from formula (16) we obtain

$$I_{0,0}(\tau)=2\sqrt{\frac{\tau}{\pi}},\quad I_{1,0}(\tau)=\frac{4}{3}\tau\sqrt{\frac{\tau}{\pi}},\quad I_{2,0}(\tau)=\frac{16}{15}\tau^2\sqrt{\frac{\tau}{\pi}}. \tag{17}$$

For $n=1,2$ the integrals in formula (16) can be represented in form:

$$I_{0,n}(\tau)=J_{0,n}(\tau),\quad I_{1,n}(\tau)=\tau J_{0,n}(\tau)-J_{2,n}(\tau),\quad I_{2,n}(\tau)=\tau^2 J_{0,n}(\tau)-2\tau J_{2,n}(\tau)+J_{4,n}(\tau), \tag{18}$$

$$J_{l,n}(\tau)=\frac{2}{\sqrt{\pi}}e^{-n\tau/\tau_i}\int_0^{\sqrt{\tau}}x^l e^{nx^2/\tau_i}dx,\quad l=0,2,4. \tag{19}$$

After substitution $y=nx^2/\tau_i$, integrals (19) can be written as follows:

$$J_{l,n}(\tau)=\frac{1}{\sqrt{\pi}}\left(\frac{\tau_i}{n}\right)^{(l+1)/2}e^{-n\tau/\tau_i}\int_0^{n\tau/\tau_i}y^{(l-1)/2}e^y dy,\quad l=0,2,4,\quad n=1,2. \tag{20}$$

By integrating by parts, taking into account substitution $s=\sqrt{y}$, from formulas (20) we obtain:

$$J_{0,n}(\tau)=\sqrt{\tau}D\left(\sqrt{n\frac{\tau}{\tau_i}}\right),\quad J_{2,n}(\tau)=\frac{\tau_i}{n}\left[\sqrt{\frac{\tau}{\pi}}-\frac{1}{2}J_{0,n}(\tau)\right],\quad J_{4,n}(\tau)=\frac{\tau_i}{n}\left[\tau\sqrt{\frac{\tau}{\pi}}-\frac{3}{2}J_{2,n}(\tau)\right], \tag{21}$$

$$D(x)=\frac{2}{\sqrt{\pi}}\frac{e^{-x^2}}{x}\int_0^x e^{s^2}ds. \tag{22}$$

Taking into account relations (21), (22), it follows from formulas (18):

$$I_{0,n}(\tau)=\sqrt{\tau}D\left(\sqrt{\frac{n\tau}{\tau_i}}\right),\quad I_{1,n}(\tau)=\left(\frac{\tau_i}{2n}+\tau\right)I_{0,n}(\tau)-\frac{\tau_i}{n}\sqrt{\frac{\tau}{\pi}}, \tag{23}$$

$$I_{2,n}(\tau)=\left[\frac{\tau_i}{n}\left(\frac{3\tau_i}{4n}+\tau\right)+\tau^2\right]I_{0,n}(\tau)-\frac{\tau_i}{n}\left(\frac{3\tau_i}{2n}+\tau\right)\sqrt{\frac{\tau}{\pi}},\quad n=1,2. \tag{24}$$

Substituting functions $I_{m,n}(\tau)$ (17), (23), (24) to the right side of solution (15), we get:

$$\begin{aligned}T^*(\tau)={}&\left[2+\frac{1}{5\tau_s^0}\left(2\tau_i-\frac{8}{3}\tau\right)+\frac{1}{\tau_s^0\tau_s}\times\left(\frac{7}{8}\tau_i^2-\frac{1}{4}\tau\tau_i-\frac{4}{15}\tau^2\right)\right]\sqrt{\frac{\tau}{\pi}}\\&+\frac{\tau_i}{\tau_s^0}\left[\frac{2}{5}-\frac{1}{2\tau_s}\left(\frac{3}{4}\tau_i-\tau\right)\right]\sqrt{\tau}D\left(\sqrt{2\frac{\tau}{\tau_i}}\right)\\&-\frac{1}{\tau_s^0}\left[\frac{1}{5}(3\tau_i-2\tau_i)+\frac{1}{4\tau_s}\times\left(\frac{1}{4}\tau_i^2+\tau_i\tau-\tau^2\right)+\tau_s^0\right]\sqrt{\tau}D\left(\sqrt{\frac{\tau}{\tau_i}}\right),\\&0\le\tau\le\tau_s.\end{aligned} \tag{25}$$

When calculating function $D(x)$ (22) in solution (25) we used expansions [15]:

$$D(x)=\frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}(-1)^n\frac{(2x^2)^n}{(2n+1)!!},\quad 0\le x\le 3,$$

$$D(x)=\frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(2n-1)!!}{(2x^2)^{n+1}},\quad x>3.$$

Knowing the time profile of dimensionless temperature (25), using formulas (10), (12), we find the temperature change during deceleration.

The volumetric temperature, averaged over the braking time, of the brake elements is determined by formulas [3]:

$$\theta_l=T_a+\gamma_l\,\theta_{l,0}\,\theta^*,\quad \theta_{l,0}=\frac{\psi_l W_0}{c_l G_l},\quad G_l=A_a a_l \rho_l,\quad l=1,2, \tag{26}$$

$$\theta^*=\frac{2}{t_s}\int_0^{t_s}w^*(t)dt, \tag{27}$$

where $\gamma_1=1-\gamma$, $\gamma_2=\gamma$ are coefficients of distribution of heat flows, ψ_l are correction factors taking into account the temperature decrease due to heat propagation to the sides of the friction track. Taking into account relation (9) from formula (27) we obtain:

$$\theta^*=1+\frac{t_i}{t_s t_s^0}\times\left\{\left[\frac{2}{5}(t_s+t_i-5t_s^0)-\frac{t_i^2}{t_s}\left(2\frac{t_i}{t_s}+\frac{31}{20}\right)-\frac{7}{40}\frac{t_s^2}{t_i}\right]+\frac{t_i^2}{t_s}\left[\left(\frac{11}{20}-\frac{3}{10}\frac{t_s}{t_i}+2\frac{t_s^0}{t_i}+3\frac{t_i}{t_s}\right)p^*(t_s)-\frac{1}{2}\left(\frac{9}{10}+\frac{t_i}{t_s}\right)p^*(2t_s)\right]\right\}. \tag{28}$$

Let us consider two particular cases of the model under consideration that are frequently encountered in applications. The first of these is constant contact pressure at the contour site. Passing in formulas (3), (5), (7), (9), (25), and (28) to limit $t_i \to 0$, we obtain

$$p^*(t) = 1, \quad V^*(t) = 1 - \left(0.4 + \frac{0.25t}{t_s}\right)\frac{t}{t_s^0},$$

$$q^*(t) = 1 - \frac{0.4t + \dfrac{0.25t^2}{t_s}}{t_s^0},$$

$$w^*(t) = \left[1 - \left(0.2 + \frac{t}{12t_s}\right)\frac{t}{t_s^0}\right]\frac{t}{t_s}, \quad 0 \le t \le t_s,$$

$$T^*(\tau) = 2\sqrt{\frac{\tau}{\pi}}\left[1 - \left(1 - \frac{2\tau}{\tau_s}\right)\frac{4\tau}{15\tau_s^0}\right], \quad 0 \le \tau \le \tau_s,$$

$$\theta^* = 1 - \frac{7t_s}{40t_s^0},$$

and from the stop condition we find braking time $t_s = 20/(13t_s^0) \cong 1.54t_s^0$.

The second case concerns the determination of the characteristics of the braking process at a constant value of contour area ($A^*(t) = 1$). With monotonically increasing pressure $p^*(t)$ (3) time profiles of speed, specific power and work of friction, as well as temperature and volumetric temperature have form [16]:

$$V^*(t) = 1 - \frac{t - t_i p^*(t)}{t_s^0},$$
$$q^*(t) = \left(1 - \frac{t - t_i p^*(t)}{t_s^0}\right)p^*(t), \tag{29}$$

$$w^*(t) = \left(1 - \frac{0.5t}{t_s^0}\right)\frac{t}{t_s^0}$$
$$- p^*(t)\left(1 - \frac{t}{t_s^0}\right)\frac{t_i}{t_s^0} - 0.5\left(\frac{p^*(t)t_i}{t_s^0}\right)^2, \quad 0 \le t \le t_s, \tag{30}$$

$$T^*(\tau) = \sqrt{\tau}\left[\begin{array}{c} 2\dfrac{1 + \dfrac{0.5\tau_i}{\tau_s^0}}{\sqrt{\pi}} - \left(1 - \dfrac{\tau}{\tau_s^0} + \dfrac{1.5\tau_i}{\tau_s^0}\right) \\ \times D\sqrt{\dfrac{\tau}{\tau_i}} + \dfrac{\tau_i}{\tau_s^0} D\sqrt{\dfrac{2\tau}{\tau_i}} \end{array}\right], \quad 0 \le \tau \le \tau_s, \tag{31}$$

$$\theta^* = \frac{\left\{\begin{array}{c} t_s - 2t_i + \dfrac{t_i^2 + t_i t_s - \dfrac{t_s^2}{3}}{t_s^0} \\ + 2\left(\dfrac{1}{t_s} - \dfrac{1}{t_s^0}\right)t_i^2 p^*(t_s) - [0.5 + p^*(2t_s)]\dfrac{t_i^3}{3t_s t_s^0} \end{array}\right\}}{t_s^0}, \tag{32}$$

for $t_s \cong t_s^0 + 0.99t_i$.

NUMERICAL ANALYSIS

Investigated the characteristics of the braking system, consisting of three identical discs made of CFCM Termar-ADF. Due to the geometric and force symmetry, the temperature regime of such a system can be determined using the analytical two-element model proposed above. The values of the input parameters are as follows [16]: $p_0 = 0.602$ MPa, $V_0 = 23.8$ m/s, $W_0 = 103.54$ kJ, $A_a = 22.1$ cm^2, $f = 0.27$, $t_i = 0.5$ s with, $\rho_l = 7100$ kg/m^3, and $\psi_l = 0.92$, $l = 1, 2$. Then, from formulas (6) and (8) we find $q_0 = 3.87$ MW/m^2, $w_0 = 46.87$ MJ/m^2. Calculations are made taking into account the change in contour area A_c (4) (variant 1) and at a constant, averaged over deceleration time $\bar{A}_c = 0.65A_a$ (variant 2). The thermal sensitivity of the disc material was taken into account by using in the calculations instead of the values of the thermophysical properties $K_l = 21$ W/(m K), $c_l = 728.5$ J/(kg K) at initial temperature $T_a = 20$°C, their values at bulk temperature θ_l, $l = 1, 2$ (26), (28), and (32) equal to 585°C (variant 1) and 566°C (variant 2). Using experimental data in the form of curves of the dependence of thermal conductivity coefficients and the specific heat capacity of Termar-ADF on temperature [9], it was found that $K_l = 16.5$ W/(m K), $c_l = 1737$ J/(kg K), and $l = 1, 2$.

Velocity change over time V, specific power of friction q, and specific friction work w as well as temperature T for variants 1 (solid curves) and 2 (dashed curves) are shown in Fig. 1. When calculating according to variant 2, we used formulas (29)–(32). With the constant value of the contour area equal to $0.65A_a$, braking occurs with constant deceleration and lasts 19.13 s (Fig. 1a). Linear increase of A_c in the process of braking leads to the appearance of nonlinearity in the time profile of the velocity and insignificant reduction of the stop time up to the 18.95 s. The time profile of the specific power of friction with a maximum after the start of the process (Fig. 1b) is characteristic of rational braking modes [2]. The specific work of friction monotonically increases with the deceleration time, reaching maximum values 40.11 MJ/m^2 (variant 1) and 36.04 MJ/m^2 (variant 2) at the moment of stop (Fig. 1c). The evolution of the temperature of the contour area depends on the time profile of the specific power of friction. Its characteristic feature in a rational braking mode is the presence of a temperature maximum, which is reached, depending on the pressure rise time, approximately in the middle of the braking time (Fig. 1d). In the case under consideration, maximum temperature values 714°C (variant 1) and 645°C (variant 2) were achieved respectively after 10.7 s and 9.8 s from the start of braking. At the moment of stop, the temperature was 525°C (variant 1) and 462°C (variant 2).

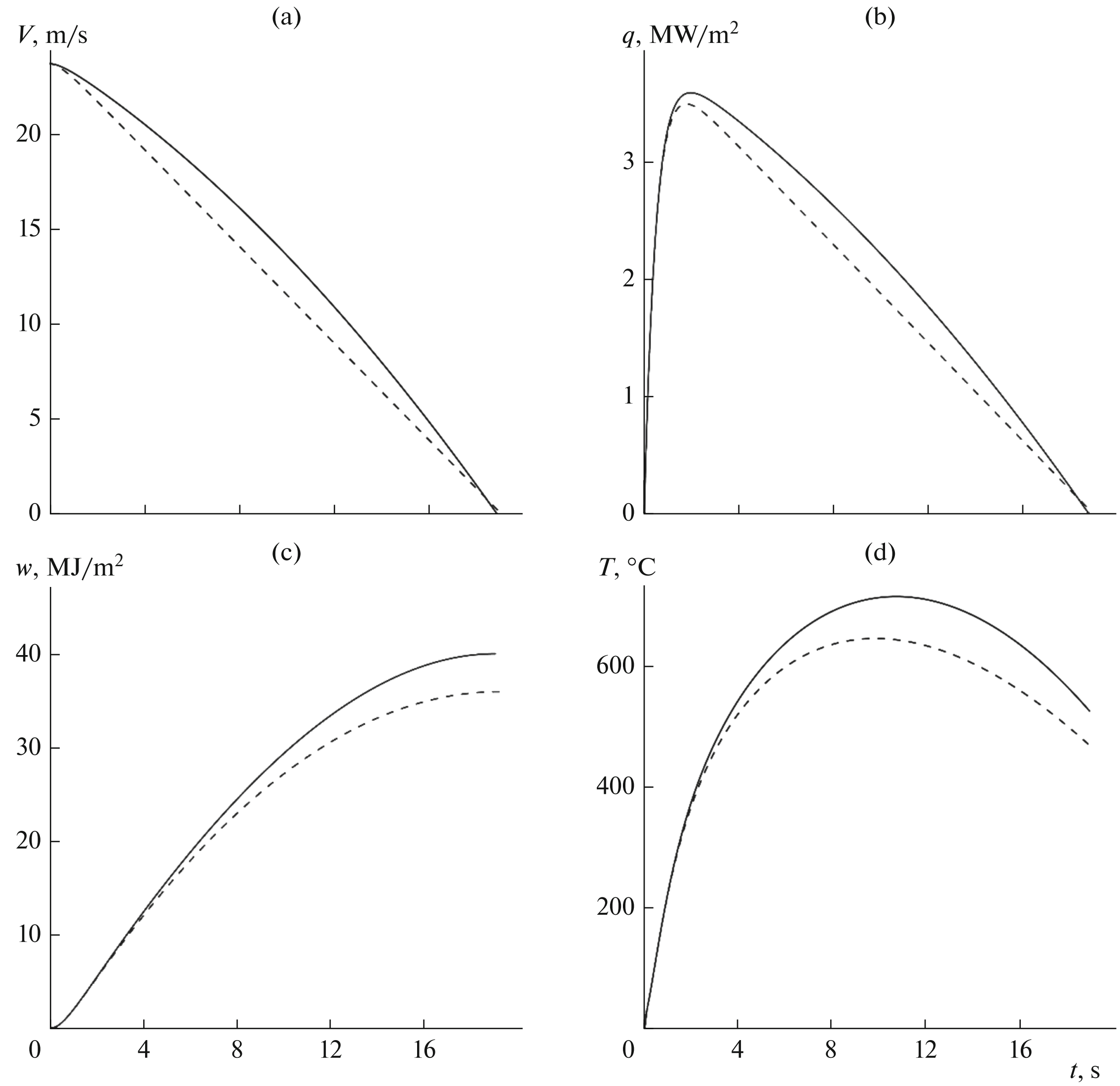

Fig. 1. Changes during braking of the: (a) sliding velocity V; (b) specific power of friction q; (c) specific work of friction w; and (d) temperature T calculated with (solid lines) and without (dashed lines) consideration of the contour contact area variations.

The results presented in Fig. 1 were obtained at a fixed value of the nominal contact area with a variable (variant 1) or constant (variant 2) contour area. Taking previously used value $A_a = 2212$ mm^2 for the base, Fig. 2 shows the change in maximum temperature $T_{\max}$, the time to reach it $t_{\max}$, and the duration of braking t_s for values λA_a, $0.5 \le \lambda \le 2$. When the nominal contact area is halved compared to the baseline, the maximum temperature rises to 1007 and 733°C, the time to reach it increases to 21 and 19 s, and the braking process continues for 37.6 and 37.8 s when calculating according to variants 1 and 2, respectively. With an increase in the nominal contact area, all of the above characteristics monotonically decrease and for value $2A_a$ equal: $T_{\max} = 507$ and 376°C, $t_{\max} = 5.6$ and 5.19 s, $t_s = 9.6$ and 9.8 s.

CONCLUSIONS

An exact solution to the thermal problem of friction for a disc brake is obtained, taking into account the linear increase with time of the contour contact area and the thermal sensitivity of materials. The calculations were performed for a friction pair consisting of two identical discs made of Termar-ADF CFCM. The change in the process of deceleration of the sliding speed, power and work of friction, as well as temperature at a changing (variant 1) and constant, averaged over time (variant 2), the contour contact area is investigated. It is shown that the temperature obtained in the calculations according to variant 1 is always slightly higher than that found according to variant 2. In this case, the relative difference between the corresponding maximum temperatures does not exceed 10%. An increase in the contour contact area leads to

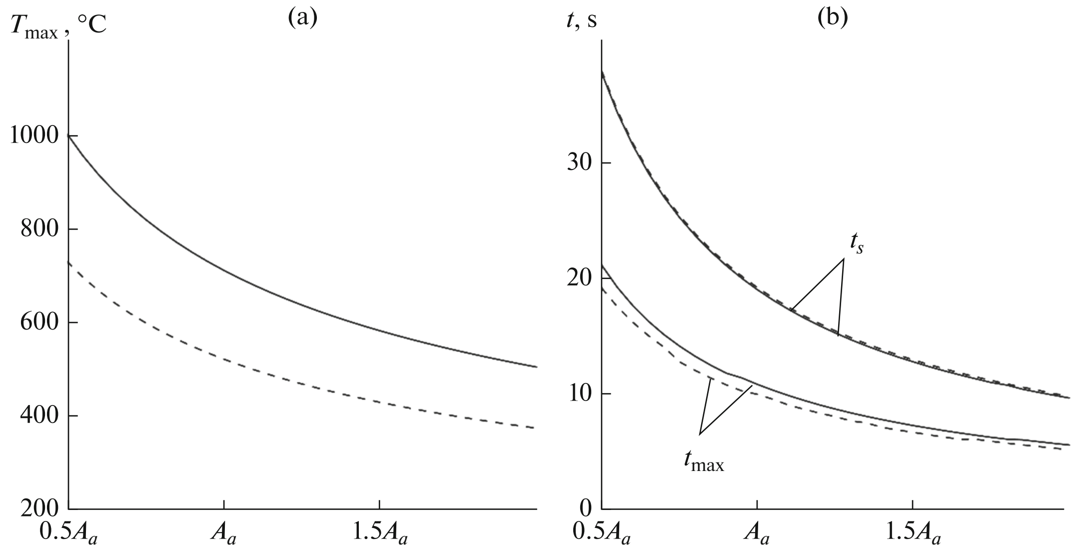

Fig. 2. Dependences on the nominal area of friction contact A_a of the: (a) maximum temperature T_{max}; (b) the time of its reaching t_{max} and stopping time t_s calculated with (solid lines) and without (dashed lines) consideration of the contour contact area variations.

an insignificant, not more than 1%, reduction in the duration of braking. Consequently, the calculation of the temperature regime of the tribosystem under consideration can be performed with sufficient accuracy at a constant value of the contour area.

Additionally, the influence of the dimensions of the nominal friction surface on some characteristics of the braking process has been studied. In both design variants, it was found that an increase in the nominal contact area leads to a reduction in the braking time, a decrease in the maximum temperature, and a decrease in the time to reach it.

NOTATION

A_a	nominal contact area
A_c	contour contact area
$\bar{A}_c$	averaged contour area
c	specific heat
F	friction force
f	coefficient of friction
K	thermal conductivity
p	contact pressure
p_0	nominal contact pressure
q	specific power of friction
q_0	nominal value of specific friction power
T	temperature
T_a	ambient temperature
T_{max}	maximum temperature
t	time
t_i	time of pressure increase
t_s	time of braking
V	velocity
V_0	initial velocity
W_0	initial kinetic energy
w	specific work of friction
w_0	nominal value of specific friction work
θ	volumetric temperature
ρ	density

Subscripts:

$l = 1$	disc
$l = 2$	pad

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