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Calculation of Friction Characteristics of Disc Brakes Used in Repetitive Short-Term Braking Mode

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Abstract—The numerical solution for the system of equations of heat friction dynamics (HFD) is obtained for the pad-disc tribosystem for use in intermittent braking mode (RST). The system of HFD equations is formulated, considering the temperature dependence of mechanical and thermal physical properties of materials, as well as the friction coefficient. The pattern of the perfect thermal contact of two sliding layers with frictional heat generation taken into account is chosen as a calculation model. The numerical analysis of the metal ceramic pad-cast iron disc friction pair is made. The RST mode consisting of three full (braking-acceleration) cycles and one incomplete (braking) cycles is studied. The change at each braking in the interconnected flashpoint, average temperature of the friction region, volumetric and maximal temperatures of the tribosystem, sliding speed, specific power, and friction coefficient is investigated.

Keywords: repeated short-term braking, heat dynamics of friction, temperature, speed, disc brake

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INTRODUCTION

An intermittent brake operation mode is characterized by recurrent braking-acceleration (heating-cooling) cycles. The characteristics of a friction pair used in intermittent mode are designed by solving the system of heat friction dynamics (HFD) equations [1]. In this case, the initial motion problem and the thermal friction problem as the system's main components are solved either in sequence (divided pattern) or synchronously (complementary pattern). In the former case, the first step is to determine the time profiles of friction velocity and specific friction power, which is followed by using them in thermal brake design. If there are available data on the frictional resistance tests of the target pair that allow recording the functional dependence of the friction coefficient on temperature, the calculations will be made according to the complementary pattern, taking into account the velocity-temperature correlation [2].

The complementary pattern of solving the system of HFD equations, considering the relation among the average, the volumetric temperature, and the flashpoint of the brake used in intermittent mode was first implemented in the works written by professor A.V. Chicinadze and his disciples [2]. In our proposed design technique, the frictional heat generation in the actual disk-pad system was replaced with the linear problem of the separate heating of two different layers by heat flows at an intensity pro rata with the specific friction power. The changes in the mechanical and thermal physical properties of the pad's and the disk's material at each braking were considered by using the values of those properties calculated for the average integral volume temperature of the tribosystem from the respective test curves. To calculate the disk's maximal temperature in intermittent braking mode by the FEM, we considered the current, temperature-dependent coefficient of friction and heat conduction, and the Brinell hardness of the rubbing pair's materials [14]. We exposed the pad and the disk to separate frictional heatings, in which the thermal fields of those elements were interconnected through the heat flows partition coefficient.

This article is aimed at modifying the unidimensional nonlinear complementary system of HDF equations for one-time braking [15] in intermittent disk brake operation mode, considering the sliding speed-temperature ratio at each braking and using the derived numerical solution of this system to study the influence of the braking number on the change in such characteristics such as temperature, speed, specific capacity, and friction coefficient.

FORMULATION OF THE PROBLEM

Braking Pattern

Let us consider a disk brake's intermittent operation mode consisting in the gradual completion of $(n-1)$ full cycles and an nth interrupted cycle. Each of the full cycles consists from two phases. In the first phase (braking), linear sliding velocity V decreases from initial value V_0 to zero for a period of $t = t_{s_k}, k = 1,2,\ldots,n-1$. Then, when the brake is switched off, the second phase starts and continues for $t = t_c$ when the velocity increases to initial value V_0. Thus, the time needed for completing the full cycles is $t_n = \sum_{k=1}^{n-1} t_k$, where $t_k = t_{s_k} + t_c$ and $k = 1,2,\ldots,n-1$ is the cycle's duration. In the last interrupted cycle performed with the brake switched on, velocity V again drops from initial value V_0 to zero at stopping instant $t = t_{s_n}$. The overall period of this operation mode is $t_b = t_n + t_{s_n}$.

Approximating Test Data

In the disk brake's single and, even more so, intermittent, heavy thermal operation mode, not only the surface temperature but also the volume temperature exceeds 450°C [16]. Thus, the thermal physical properties of the disk and pad materials before the braking and at stopping may significantly differ. Elevated temperatures also affect the friction coefficient value. Thus, we assume that heat conduction coefficient K_l, specific heat capacity c_l, and density ρ_l of the friction pair materials, as well as friction coefficient f, are functions of temperature T and recorded as

$$K_l(T) = K_{l,0}K_l^*(T), \quad c_l(T) = c_{l,0}c_l^*(T),$$
$$\rho_l(T) = \rho_{l,0}\rho_l^*(T), \quad f(T) = f_0 f^*(T), \tag{1}$$

$$K_{l,0} = K_l(T_0), \quad c_{l,0} = c_l(T_0),$$
$$\rho_{l,0} = \rho_l(T_0), \quad f_0 = f(T_0), \tag{2}$$

$$K_l^*(T) = K_{l,1} + K_{l,2}\big/\{[K_{l,3}(T - K_{l,4})]^2 + 1\}$$
$$+ K_{l,5}\big/\{[K_{l,6}(T - K_{l,7})]^2 + 1\}, \tag{3}$$

$$c_l^*(T) = c_{l,1} + c_{l,2}\big/\{[c_{l,3}(T - c_{l,4})]^2 + 1\}$$
$$+ c_{l,5}\big/\{[c_{l,6}(T - c_{l,7})]^2 + 1\}, \tag{4}$$

$$\rho_l^*(T) = \rho_{l,1} + \rho_{l,2}\big/\{[\rho_{l,3}(T - \rho_{l,4})]^2 + 1\}$$
$$+ \rho_{l,5}\big/\{[\rho_{l,6}(T - \rho_{l,7})]^2 + 1\}, \tag{5}$$

$$f^*(T) = f_1 + f_2\big/\{[f_3(T - f_4)]^2 + 1\}$$
$$+ f_5\big/\{[f_6(T - f_7)]^2 + 1\}, \tag{6}$$

where $K_{l,j}$, $c_{l,j}$, $\rho_{l,j}$, f_j, $l = 1,2$; $j = 1,2,\ldots,7$ are the approximation coefficients of the respective test data resulting from the frictional heat resistance tests of the materials [2]. All of the quantities applied to the disk and the pad hereafter have subscripts $l = 1$ and $l = 2$, respectively.

Maximal Temperature

We shall use the temperature summation hypothesis to record the change in maximal braking temperature $T_{\max}$ [1] as

$$T_{\max}(t) = T_m(t) + T_f(t), \quad 0 \le t \le t_{s_k}, \quad k = 1,2,\ldots,n. \tag{7}$$

Average Temperature of the Contact's Nominal Region

The system of the frictional contact of two heterogeneous layers $0 \le z \le d_1$ (disk) and $-d_2 \le z \le 0$ (pad) that slide at relative speed $V(t) = V_0V^*(t)$, as well as are compressed at a normal pressure, are used as the design geometric pattern for determining T_m:

$$p(t) = p_0p^*(t), \quad p^*(t) = 1 - e^{-t/t_i},$$
$$0 \le t \le t_{s_k}, \quad k = 1,2,\ldots,n. \tag{8}$$

Nonstationary temperature field $T(z,t)$ of this tribosystem is initiated by the frictional heat generation at braking and found by solving the following thermal friction problem:

$$\frac{\partial}{\partial z}\left[K_1(T)\frac{\partial T}{\partial z}\right] = \rho_1(T)c_1(T)\frac{\partial T}{\partial t},$$
$$0 < z < d_1, \quad 0 < t \le t_{s_k}, \tag{9}$$

$$\frac{\partial}{\partial z}\left[K_2(T)\frac{\partial T}{\partial z}\right] = \rho_2(T)c_2(T)\frac{\partial T}{\partial t},$$
$$-d_2 < z < 0, \quad 0 < t \le t_{s_k}, \tag{10}$$

$$K_2(T)\frac{\partial T}{\partial z}\bigg|_{z=0^-} - K_1(T)\frac{\partial T}{\partial z}\bigg|_{z=0^+} = q(t), \quad 0 < t \le t_{s_k}, \tag{11}$$

$$T(0^+,t) = T(0^-,t) \equiv T_m(t), \quad 0 < t \le t_{s_k}, \tag{12}$$

$$K_1(T)\frac{\partial T}{\partial z}\bigg|_{z=d_1} = 0, \quad 0 < t \le t_{s_k}, \tag{13}$$

$$K_2(T)\frac{\partial T}{\partial z}\bigg|_{z=-d_2} = h[T(-d_2,t) - T_0], \quad 0 < t \le t_{s_k}, \tag{14}$$

$$T(z,0) = T_0^{(k)}, \quad -d_2 \le z \le d_1, \quad k = 1,2,\ldots,n, \tag{15}$$

$$q(t) = q_0q^*(t), \quad q_0 = f_0p_0V_0, \quad q^*(t) = F^*(t)V^*(t),$$
$$F^*(t) = f^*[T_{\max}(t)]p^*(t), \quad 0 \le t \le t_{s_k}. \tag{16}$$

Temperature of the Contact's Actual Region

The evolution of flashpoint T_f was calculated by the empirical formulas for the contact pattern of the microasperities of rubbing surfaces [17] as

$$T_f(t) = B(t)e^{-C(t)T_m(t)}, \quad B(t) = \sqrt[3]{p(t)V(t)/\kappa_1},$$
$$C(t) = \kappa_2/V(t) + \kappa_3, \quad 0 \le t \le t_{s_k}, \tag{17}$$

where κ_i, $i = 1,2,3$ are the coefficients determined by experiment.

Average Volume Temperature

The average volume temperature of the disk-pad friction pair before the start of the kth braking was calculated by the formulas from [2] as

$$T_0^{(k)} = \frac{T_{0,0}^{(k)} + T_{0,1}^{(k)}}{2},$$
$$T_{0,i}^{(k)} = T_0 + \frac{\gamma_i W_0}{2G_{1,i}c_{1,i}}\left(\frac{e^{-\alpha_i t_c} - e^{-k\alpha_i t_c}}{1 - e^{-\alpha_i t_c}}\right), \quad (18)$$
$$i = 0,1, \quad k = 1,2,\ldots,n,$$

$$\alpha_i = \frac{hA_{\text{vent}}}{G_{1,i}c_{1,i}}, \quad \gamma_i = \frac{\sqrt{K_{1,i}\rho_{1,i}c_{1,i}}}{\sqrt{K_{1,i}\rho_{1,i}c_{1,i}} + \sqrt{K_{2,i}\rho_{2,i}c_{2,i}}}, \quad (19)$$

$$G_{1,i} = A_1 d_1 \rho_{1,i}, \quad i = 0,1, \quad A_{\text{vent}} = A_1 + A_1', \quad (20)$$
$$A_1 = \pi(R_1^2 - r_1^2), \quad A_1' = 2\pi R_1 d_1,$$

$$K_{l,1} = K_l(T_{0,0}^{(k)}), \quad c_{l,1} = c_l(T_{0,0}^{(k)}), \quad (21)$$
$$\rho_{l,1} = \rho_l(T_{0,0}^{(k)}), \quad l = 1,2,$$

whereas the formulas used after the kth braking were

$$T_s^{(k)} = \frac{T_{s,0}^{(k)} + T_{s,1}^{(k)}}{2},$$
$$T_{s,i}^{(k)} = T_0 + \frac{\gamma_i W_0}{2G_{1,i}c_{1,i}}\left(\frac{1 - e^{-k\alpha_i t_c}}{1 - e^{-\alpha_i t_c}}\right), \quad (22)$$
$$i = 0,1, \quad k = 1,2,\ldots,n.$$

Initial Motion Problem

A drop in speed $V(t) = V_0 V^*(t)$ at the kth braking was found by solving the initial motion problem [15] as

$$t_s^0 dV^*(t)/dt = -F^*(t), \quad 0 < t \le t_{s_k}, \quad (23)$$
$$k = 1,2,\ldots,n, \quad V^*(0) = 1,$$

$$t_s^0 = 2W_0/(q_0 A_2), \quad A_2 = \phi_0(R_2^2 - r_2^2), \quad |\phi_0| \le \pi, \quad (24)$$

where rated specific friction power q_0 and dimensionless time profile of friction force $F^*(t)$ are shown in the form of (16). The duration of each braking was determined from the halt condition as

$$V^*(t_{s_k}) = 0, \quad k = 1,2,\ldots,n. \quad (25)$$

In the acceleration phases, the speed linearly increased with time as

$$V^*(t) = (t - t_{s_k})/t_c, \quad t_{s_k} \le t \le t_k, \quad k = 1,2,\ldots,n. \quad (26)$$

Correlations (1)—(26) are determined as the mathematical recorded system of HFD equations for the considered intermittent braking mode. Such key elements of this system, as the boundary problem of heat conduction, considering the heat generation by friction in (9)—(16), flashpoint calculation formulas (17), volume temperature formulas (18)—(22), and initial motion problem (22)—(26) are interrelated through friction coefficient f (16) dependent from maximal temperature $T_{\max}$ (7).

Solving the System of HFD Equations

We shall switch to the dimensionless variables and parameters recorded below as

$$\zeta = \frac{z}{d_1}, \quad \tau = \frac{k_{1,0}t}{d_1^2}, \quad \tau_i = \frac{k_{1,0}t_i}{d_1^2}, \quad \tau_s^0 = \frac{k_{1,0}t_s^0}{d_1^2}, \quad \tau_c = \frac{k_{1,0}t_c}{d_1^2},$$
$$\tau_{s_k} = \frac{k_{1,0}t_{s_k}}{d_1^2}, \quad d^* = \frac{d_2}{d_1}, \quad K_0^* = \frac{K_{2,0}}{K_{1,0}}, \quad c_0^* = \frac{c_{2,0}}{c_{1,0}},$$
$$\rho_0^* = \frac{\rho_{2,0}}{\rho_{1,0}}, \quad k_0^* = \frac{K_0^*}{\rho_0^* c_0^*}, \quad \text{Bi} = \frac{hd_1}{K_{1,0}}, \quad T_a = \frac{q_0 d_1}{K_{1,0}},$$
$$T^* = \frac{T - T_0}{T_a}, \quad T_{\max}^* = \frac{T_{\max} - T_0}{T_a}, \quad T_m^* = \frac{T_m - T_0}{T_a},$$
$$T_f^* = \frac{T_f}{T_a}, \quad T_0^{*(k)} = \frac{T_0^{(k)} - T_0}{T_a}, \quad T_s^{*(k)} = \frac{T_s^{(k)} - T_0}{T_a}.$$

and thus record formulas and equations (7)—(17), (23)—(25) as

$$T_{\max}^*(\tau) = T_m^*(\tau) + T_f^*(\tau), \quad (27)$$
$$0 \le \tau \le \tau_{s_k}, \quad k = 1,2,\ldots,n,$$

$$\frac{\partial}{\partial\zeta}\left[K_1^*(T^*)\frac{\partial T^*}{\partial\zeta}\right] = \rho_1^*(T^*)c_1^*(T^*)\frac{\partial T^*}{\partial\tau}, \quad (28)$$
$$0 < \zeta < 1, \quad 0 < \tau \le \tau_{s_k},$$

$$\frac{\partial}{\partial\zeta}\left[K_2^*(T^*)\frac{\partial T^*}{\partial\zeta}\right] = \frac{1}{k_0^*}\rho_2^*(T^*)c_2^*(T^*)\frac{\partial T^*}{\partial\tau}, \quad (29)$$
$$-d^* < \zeta < 0, \quad 0 < \tau \le \tau_{s_k},$$

$$K_0^* K_2^*(T^*)\frac{\partial T^*}{\partial\zeta}\bigg|_{\zeta=0^-} - K_1^*(T^*)\frac{\partial T^*}{\partial\zeta}\bigg|_{\zeta=0^+} = q^*(\tau), \quad (30)$$
$$0 < \tau \le \tau_{s_k},$$

$$T^*(0^+,\tau) = T^*(0^-,\tau) \equiv T_m^*(\tau), \quad 0 < \tau \le \tau_{s_k}, \quad (31)$$

$$K_1^*(T^*)\frac{\partial T^*}{\partial\zeta}\bigg|_{\zeta=1} = 0, \quad 0 < \tau \le \tau_{s_k}, \quad (32)$$

$$K_0^* K_2^*(T^*)\frac{\partial T^*}{\partial\zeta}\bigg|_{\zeta=-d^*} = \text{Bi}\,T^*(-d^*,\tau), \quad 0 < \tau \le \tau_{s_k}, \quad (33)$$

$$T^*(\zeta,0) = T_0^{*(k)}, \quad -d^* \le \zeta \le 1, \quad (34)$$

$$q^*(\tau) = F^*(\tau)V^*(\tau), \quad F^*(\tau) = f^*[T_{\max}^*(\tau)]p^*(\tau), \quad (35)$$
$$p^*(\tau) = 1 - e^{-\tau/\tau_i}, \quad 0 \le \tau \le \tau_{s_k},$$

$$T_f^*(\tau) = B^*(\tau)e^{-C^*(\tau)T_m^*(\tau)}, \quad B^*(\tau) = B(t)e^{-C(t)T_0}\Big/T_a, \quad C^*(\tau) = T_aC(t), \quad 0 \le \tau \le \tau_{s_k}, \tag{36}$$

$$\tau_s^0 dV^*(\tau)\big/d\tau = -F^*(\tau), \quad 0 \le \tau \le \tau_{s_k}, \quad V^*(0) = 1, \quad V^*(\tau_{s_k}) = 0. \tag{37}$$

The boundary problem of heat conduction (27)—(37) is essentially nonlinear since it contains both internal (in differential operators (28) and (29)) and external (in boundary conditions (30), (32), and (33)) nonlinearities. We shall solve this problem by integral interpolative discretization from [18] and introduce the following lattices in tribocoupling elements:

$$\zeta_{l,j} = (-1)^{l-1} j\Delta\zeta_l, \quad l = 1,2, \quad j = 0,1,\ldots,n_l, \quad \Delta\zeta_1 = 1/n_1, \quad \Delta\zeta_2 = d^*/n_2 \tag{38}$$

and record

$$a_{l,j}(\tau) = 0.5[K_{l,j}^*(\tau) - K_{l,j-1}^*(\tau)], \quad b_{l,j} = \rho_{l,j}^*(\tau)c_{l,j}^*(\tau)\Delta\zeta_l, \quad b_0(\tau) = b_{1,0}(\tau) - \rho_0^* c_0^* b_{2,0}(\tau), \tag{39}$$

$$K_{l,j}^*(\tau) = K_l^*[T_{l,j}^*(\tau)], \quad c_{l,j}^*(\tau) = c_l^*[T_{l,j}^*(\tau)], \quad \rho_{l,j}^*(\tau) = \rho_l^*[T_{l,j}^*(\tau)], \quad T_{l,j}^*(\tau) = T^*(\zeta_{l,j},\tau). \tag{40}$$

Considering formulas (36)—(40) after the respective transformations made to replace the differential operators by dimensionless spatial variable ζ with their finite difference analogs, we shall record the nonlinear boundary problem of heat conduction (28)—(34) as

$$\frac{dT_{1,0}^*(\tau)}{d\tau} = \frac{2}{b_0(\tau)}\left\{q^*(\tau) + a_{1,1}(\tau)\frac{[T_{1,1}^*(\tau) - T_{1,0}^*(\tau)]}{\Delta\zeta_1} - K_0^* a_{2,1}(\tau)\frac{[T_{2,1}^*(\tau) - T_{2,0}^*(\tau)]}{\Delta\zeta_2}\right\}, \tag{41}$$

$$\frac{dT_{1,j}^*(\tau)}{d\tau} = \frac{1}{b_{1,j}(\tau)}\left\{a_{1,j+1}(\tau)\frac{[T_{1,j+1}^*(\tau) - T_{1,j}^*(\tau)]}{\Delta\zeta_1} - a_{1,j}(\tau)\frac{[T_{1,j}^*(\tau) - T_{1,j-1}^*(\tau)]}{\Delta\zeta_1}\right\}, \quad j = 1,2,\ldots,n_1 - 1, \tag{42}$$

$$\frac{dT_{1,n_1}^*(\tau)}{d\tau} = -\frac{2}{b_{1,n_1}(\tau)} a_{1,n_1}(\tau)\frac{[T_{1,n_1}^*(\tau) - T_{1,n_1-1}^*(\tau)]}{\Delta\zeta_1}, \quad 0 < \tau \le \tau_{s_k}, \quad k = 1,2,\ldots,n, \tag{43}$$

$$\frac{dT_{2,j}^*(\tau)}{d\tau} = \frac{k_0^*}{b_{2,j}(\tau)}\left\{a_{2,j+1}(\tau)\frac{[T_{2,j+1}^*(\tau) - T_{2,j}^*(\tau)]}{\Delta\zeta_2} - a_{2,j}(\tau)\frac{[T_{2,j}^*(\tau) - T_{2,j-1}^*(\tau)]}{\Delta\zeta_2}\right\}, \quad j = 1,2,\ldots,n_2 - 1, \tag{44}$$

$$\frac{dT_{2,n_2}^*(\tau)}{d\tau} = \frac{2k_0^*}{K_0^* b_{2,n_2}(\tau)} \times \left\{\mathrm{Bi}\, T_{2,n_2}^*(\tau) - K_0^* a_{2,n_2}(\tau)\frac{[T_{2,n_2}^*(\tau) - T_{2,n_2-1}^*(\tau)]}{\Delta\zeta_2}\right\}, \tag{45}$$

$$T_{l,j}^*(0) = T_0^{*(k)}, \quad l = 1,2, \quad j = 0,1,\ldots,n_l. \tag{46}$$

According to boundary condition (31), we have $T_{1,0}^*(\tau) = T_{2,0}^*(\tau) \equiv T_m^*(\tau)$, $0 < \tau \le \tau_{s_k}$; therefore, system $n_1 + n_2 + 1$ of common differential equations (41)—(46) contains the same number of sought-for functions $T_{l,j}^*(\tau)$. The tribosystem's dimensionless volume temperature $T_0^{*(k)}$ included in initial condition (46) was calculated by formulas (18)—(21). It is important to note that initial motion problem (37) and heat conduction problems (41)—(46) are conjugate to each other through the friction coefficient dependent from maximal temperature (27). Thus, both problems were solved synchronously in the DIFSUB suite [19]. The maximal number of units for having a relative calculation accuracy of 0.1% was $n_1 = 20$ and $n_2 = 40$ for the disk and pad, respectively.

We shall note that the speed and temperature at constant friction coefficient $f = f_0$ are no longer interdependent. In this case, the divided pattern of solving the system of HFD equations consists in solving the initial motion problem (37) and then, at the known time profile of speed $V^*(\tau) = 1 - \tau/\tau_s^0 + p^*(\tau)\tau_m/\tau_s^0$, $0 \le \tau \le \tau_{s_k}$, $\tau_{s_k} \cong \tau_s^0 + 0.99\tau_m$, $k = 1,2,\ldots,n$ [20], in determining the temperature by solving initial problem (41)—(46). As a result, the time profile of speed and the stop time are equal at each braking.

Numerical Analysis

We studied the intermittent thermal operational mode of the disk-pad tribosystem for three full cycles and an interrupted cycle ($n = 4$) with such input parameters, as $f_0 = 0.448$, $p_0 = 1.47$ mPa, $V_0 = 28$ mps, $W_0 = 392.1$ kJ, $t_i = 0.5$ s, $t_c = 5$ s, $d_1 = 5.5$ mm, $d_2 = 10$ mm, $r_1 = r_2 = 76.5$ mm, $R_1 = R_2 = 113.5$ mm, $\phi_0 = \pi$, $h = 100$ W/(m^2 K), $T_0 = 20$°C. The disk was made from gray cast iron CHNMH ($K_{1,0} = 52.17$ W/(m K), $c_{1,0} = 444.6$ J/(kg K), $\rho_{1,0} = 7100$ kg/m^3), whereas the pad was made from ceramic metal material FMC-11 ($K_{2,0} = 35$ W/(m K), $c_{2,0} = 478.9$ J/(kg K), $\rho_{1,0} = 4700$ kg/m^3) [2]. The values of coefficients $K_{l,j}$, $c_{l,j}$, $\rho_{l,j}$, f_j, $j = 1,2,\ldots,7$; $l = 1,2$ in formulas (1)—(6) and κ_i, $i = 1,2,3$, in correlations (17) for the CHNMH/ FMC-11 friction pair are provided in study

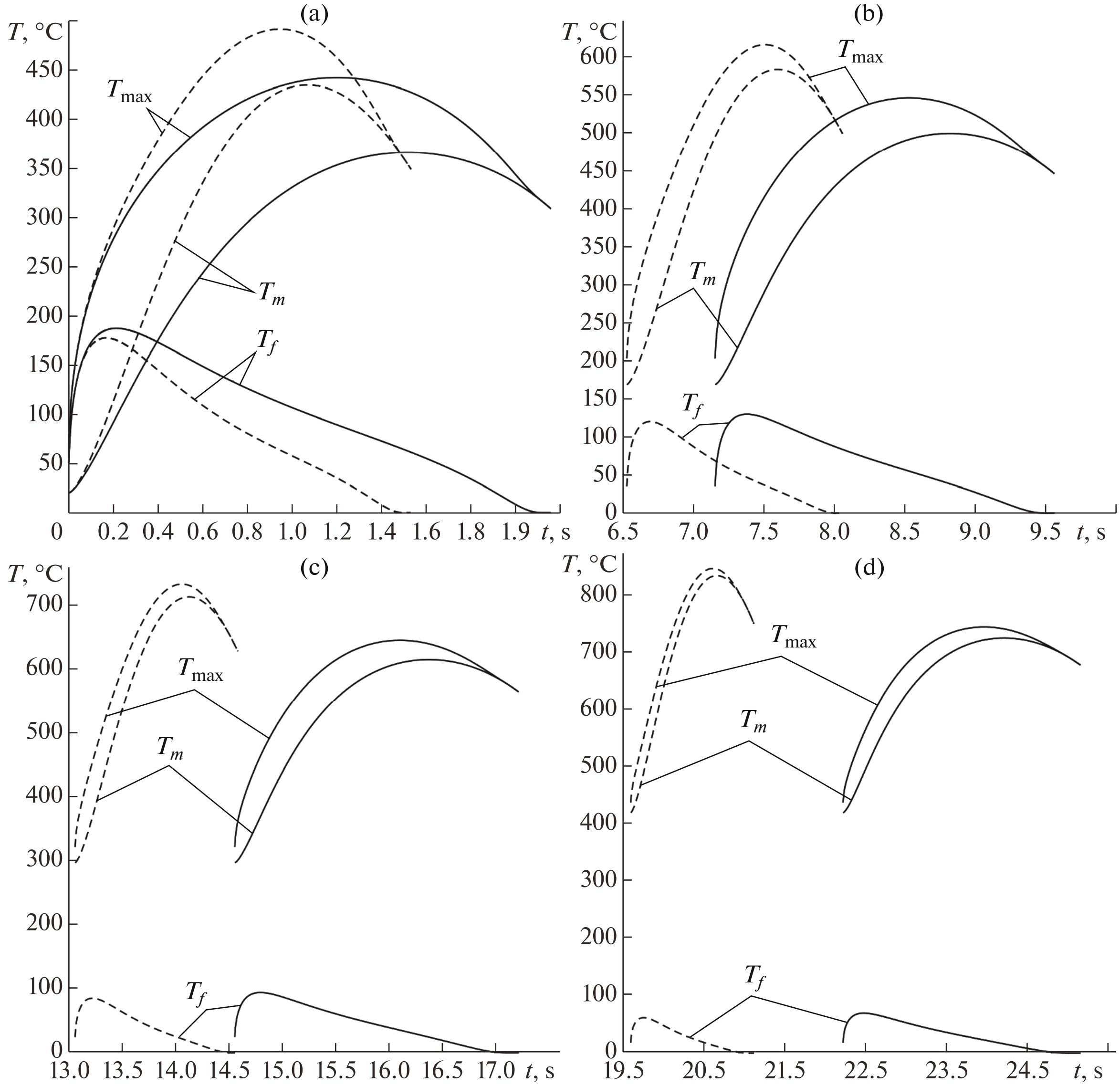

Fig. 1. Change in time of maximal temperature T_{max}, average friction surface temperature T_m, and flashpoint T_f, considering (solid curves) and not considering (dashed curves) the temperature dependence of the friction coefficient at braking one (a), two (b), three (c), and four (d).

[15]. The calculations were made taking into account the change in the friction coefficient at braking $f(t) = f_0 f^*[T_{max}(t)]$ (variant 1, solid curves) and at constant friction coefficient $f = f_0$ (variant 2, dashed curves).

The change with time of maximal temperature T_{max}, average friction surface temperature T_m, and flashpoint T_f for four brakings is shown in Fig. 1. The highest values of these temperatures, as well as the values of average volume temperatures (18)—(22) before start $T_0^{(k)}$ and after completion $T_s^{(k)}$, $k = 1, 2, 3, 4$ of each braking, are shown in Table 1, where braking durations t_{s_k}, $k = 1, 2, 3, 4$, are shown as well. The tribosystem's volume, average, and maximal temperature rises with each braking. The maximal flashpoint is observed in the initial phase of each braking and rapidly decreases with time. Thus, the maximal values of T_f decrease with an increase in the number of brakings. This evolution of the flashpoint stems from the plastic straining of microasperities of the rubbing pad and disk surfaces, which is implied in formulas (17). The maximal temperature at each braking, determined by the temperature relation of the friction coefficient (variant 1), is always lower than this coefficient's constant (initial) value (variant 2). The difference in the highest maximal temperatures determined for these two variants increases with each subsequent braking.

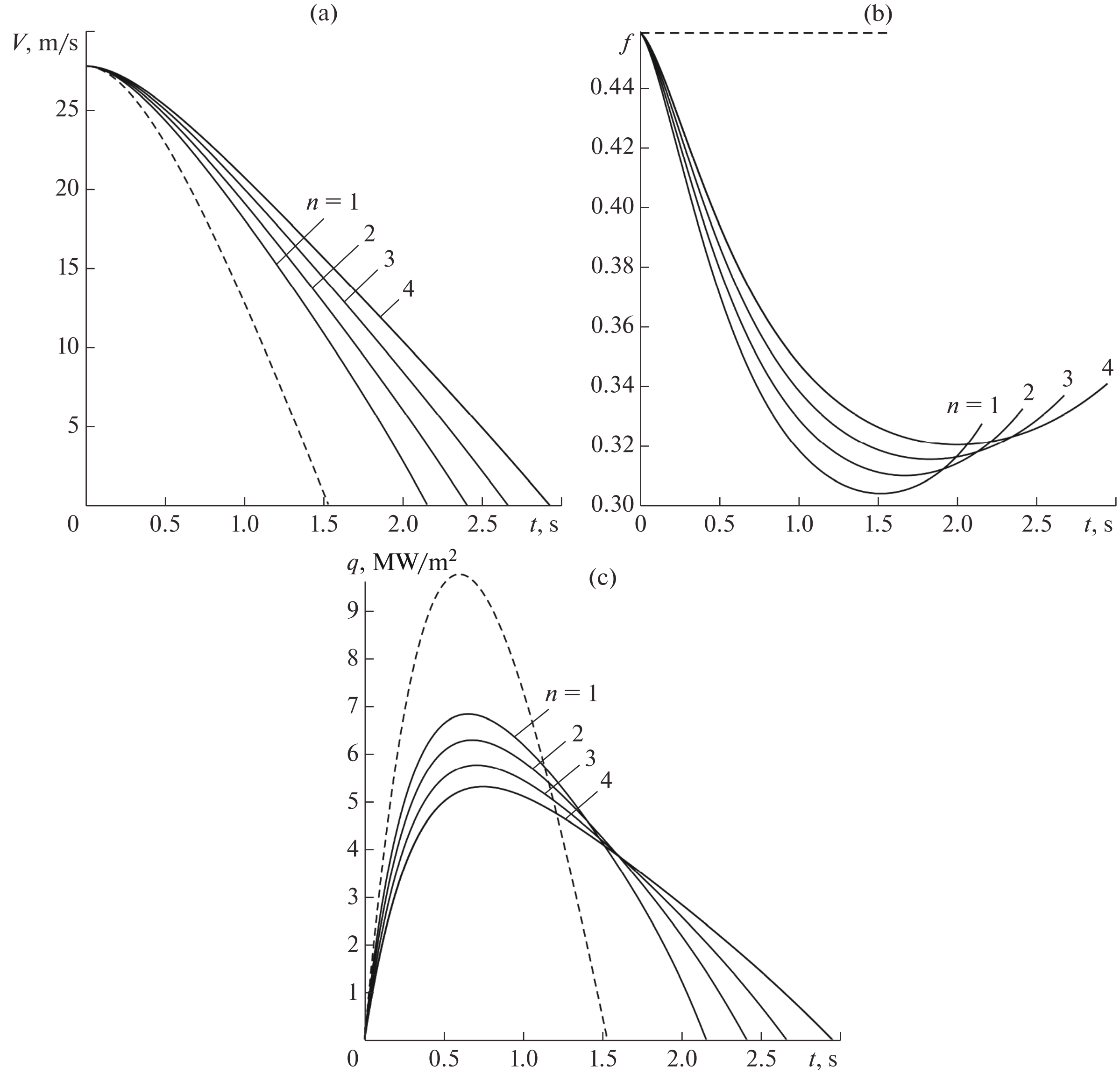

Fig. 2. Time profiles of speed V (a), friction coefficient f (b), and specific friction power q (c), considering (solid curves) and not considering (dashed curves) the temperature dependence of the friction coefficient, at each of the four brakings.

The duration of individual brakings extends with an increase in the number of brake actuations (variant 1) or remains unchanged (variant 2). The duration of four brakings was $t_4 = 10.16$ and 6.22 s for variants 1 and 2, respectively. The overall brake operation duration is $t_b = 25.16$ (variant 1) and 21.22 s (variant 2).

The changes with time in speed of advance V, friction coefficient f, and specific friction power q are shown in Fig. 2. The speed decreases linearly but for the short period after the start of braking (Fig. 2a). The first braking is the shortest one, and the stop duration increases with each subsequent braking phase (variant 1). Since the frictional heat resistance curve for the considered friction pair is an almost linearly decreasing function of temperature, the friction coefficient evolves (Fig. 2b) in the manner contrary to the evolution of the change in the maximal temperature with time (Fig. 1). A reduction in f occurs with the start of each braking, which continues up to the moment coinciding with the moment reaching the highest value of $T_{\max}$. The friction coefficient rises in the subsequent period of a minor drop in $T_{\max}$ until stopping. The most and least significant reduction in f are observed at the first and fourth braking, respectively (variant 1). The results presented in Fig. 2b were used to determine average friction coefficient f_m, its stability $f_s = f_m / f_{\max}$, fluctuation $f_f = f_{\min} / f_{\max}$, and braking efficiency $f_{\mathrm{eff}} = f_s / t_{s_k}^2$, $k = 1, 2, 3, 4$ (Table 2). All of the brakings are highly stable, although the first

Table 1. Duration of t_{s_k} and maximal values of $T_0^{(k)}$, $T_s^{(k)}$, T_f, T_m, $T_{\max}$ at each braking ($k = 1, 2, 3, 4$)

Property	Variant	k			
		1	2	3	4
t_{s_k}, s	1	2.16	2.4	2.66	2.94
	2	1.53	1.53	1.53	1.53
$T_0^{(k)}$, °C	1, 2	20	168	297	420
$T_s^{(k)}$, °C	1, 2	185	320	441	560
T_f, °C	1	187	129	94	70
	2	177	120	85	62
T_m, °C	1	366	498	614	725
	2	434	582	711	834
$T_{\max}$, °C	1	442	545	644	744
	2	491	615	731	847

Table 2. Parameters of brake performance efficiency at four brakings

k	1	2	3	4
f_m	0.359	0.359	0.357	0.355
f_s	0.782	0.783	0.779	0.775
f_f	0.663	0.677	0.688	0.699
f_{eff}, s^{-2}	0.168	0.136	0.11	0.0896

braking was the most efficient one for the chosen friction pair. The braking efficiency decreases with an increase in the number of brake actuations.

The timeline of specific friction power q (16) corresponds to rational braking characterized by the maximum within the braking range (Fig. 2c). The areas under all of the curves are equal, which indicates that the equality conditions in full friction effort at each braking are fulfilled.

CONCLUSIONS

We have formulated and derived the numerical solution of the nonlinear single-dimension system of HFD equations for the disk-pad tribosystem used in intermittent mode. The calculations for the cast iron disk (CHNMKH) and the ceramic metal pad (FMC-11) have been made based on the material's thermal sensitivity and friction coefficient. The influence of the number of brakings on the evolution of temperature, speed, specific power, and coefficient of friction has been studied. The volumetric and the maximal temperature rise with each subsequent braking, as well as at halt (the latter increases from 2.16 s at the first braking to 2.94 at the fourth (quadruple) braking). The time profile of the friction coefficient is pretty much stable (≥0.775), whereas its efficiency decreases from 0.168 to 0.0896 with an increase in the number of brakings.

NOTATION

c_l	is heat capacity ratio
d_l	is layer thickness
f	is friction coefficient
f_0	is rated friction coefficient
F	is friction force
h	is the coefficient of heat exchange between the buildup's free surface and the environment
K_l	is the heat conduction coefficient
n	is the number of brakings
n_l	is the number of nodes in the partitioning mesh
p	is pressure
p_0	is rated pressure
q	is specific friction power
q_0	is rated specific friction power
r_l	is internal radius
R_l	is external radius
T	is temperature
T_0	is ambient temperature
$T_{\max}$	is maximal temperature
T_m	is the average temperature of the nominal region of contact
T_f	is the temperature of the actual region of contact (flashpoint)
$T_0^{(k)}$	is the volume temperature before the start of the kth braking
$T_s^{(k)}$	is the volume temperature after the kth braking
t	is time
t_c	is the duration of acceleration
t_{s_k}	is the duration of the kth braking
t_i	is the pressure increment time
V	is velocity
V_0	is initial velocity
W_0	is the initial kinetic energy of the tribosystem
z	is the spatial coordinate
ρ_l	is density
$2\phi_0$	is the pad expansion angle

The respective subscripts for the disk and the pad are $l = 1$ and $l = 2$

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