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Multi-Objective Optimization of Spur Gearbox with Inclusion of Tribological Aspects[1]

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Abstract—In this paper, a novel multi-objective optimization of a two-stage spur gearbox is carried out with a comprehensive range of constraints. The first objective function aims to reduce the weight/volume and second aims to minimize the power losses in the gearbox. Various design constraints and tribological constraints such as scuffing and wear are included. By using a specially formulated discrete version of NSGA-II optimization code, these objective functions are minimized for three different gear profiles (unmodified profile, smooth meshing, and high load) and for different SAE oil grades. Optimization is first carried out based on standard single objective minimization using regular constraints based on existing literature and then based on multi-objective optimization with comprehensive constraints which include tribological aspects. Finally, these two cases are compared for different gear profiles and oils. The results indicate that there is a high probability of wear failure, for solutions obtained from single objective minimization. The total power loss is reduced by half when using multi-objective compared to single objective optimization.

Keywords: spur gearbox, multi-objective optimization, tribology, scuffing, wear

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INTRODUCTION

Light weight and high-efficiency gearboxes with the maximum service life is the prime necessity of today's high-performance power transmission systems such as automotive and aerospace. Although the gearboxes used currently in many applications have very high efficiency there is still scope for further improvement. For instance, if the power loss of the gearbox could be reduced by 1%, it is equivalent to savings of 4% of the fuel in the case of automotive applications. Similarly, according to [1], a modern wind turbine of the 5 MW class consists of more than eight gear meshes and 12 bearing meshes. A reduction of overall gearbox losses by 50% could save about 200 kW power. Also in wind turbines, gearbox failure causes the longest downtime. Where 25% faults in the gearboxes cause 95% of total downtime and 75% other faults causes only 5% downtime and gearboxes are costliest to repair [2]. Hence, the real challenge is to increase the service life of the gearbox and reduce the power loss with minor impact on the load carrying capacity, and also reduce the volume of the gearbox.

Hungling Wang et al. [3] considered four objective functions, minimum size, weight, tooth deflection and maximum life of spur gear pair using Modified Iterative Weighted Tchebycheff (MIWT) method. The limitation of this method is the difficulty of convergence depending on the initial sample vector, and the time for convergence of the solution is often excessive. Yokota et al. [4] used the improved Genetic Algorithm (GA), for optimizing the weight of a gear pair by considering the gear bending strength and torsional strength of shafts as constraints for the optimization problem. Thompson et al. [5] considered two-stage and three-stage spur gear reduction to minimize the volume in trade-off analysis with surface fatigue life. Chong et al. [6] investigated the multi-objective optimal design of cylindrical gear pairs to reduce the size of gear and meshing vibration force. By using GA, Mendi et al. [7] carried out the optimization of the module, shaft diameter and rolling bearing for single stage spur gearbox by minimizing the volume of the gearbox. Marjanovic et al. [8] selected an optimal gearbox arrangement for minimum volume by selecting optimal materials, gear ratio and optimal position of shaft axes. However, all the design variables were considered as continuous for the optimization. Golabi et al. [9] studied gear train optimization based on minimum volume design with standard regular gear design constraints. The study also showed how the graphs of the results can be used to design minimum volume gearbox.

From the literature survey presented above, it is seen that volume minimization of the gearbox is the prime consideration in many studies. Also, many

[1] The article is published in the original.

researchers have discussed the performance of gears in terms of friction and wear, but while designing gears, the tribological (scuffing and wear) and hunting frequency aspects are not considered. Hence there is the significant scope of additionally minimizing the power loss considering these critical factors.

In this paper, it is proposed to formulate two generalized objective functions for a two-stage spur-gearbox, one for minimum volume, and the other for minimum power losses in the gearbox. These are conflicting objectives and the solution is a Pareto front which gives the locus of trade-off solutions. The novelty here is that in addition to general mechanical design constraints, the tribological constraints (scuffing and wear), as well as the constraints to avoid the hunting tooth frequency of the gears, are also considered for a range of SAE mineral oils. Most of the design constraints are formulated as per American Gear Manufacturers Association (AGMA) standards. These objective functions are then optimized to obtain the Pareto fronts of the two objective functions for the two-stage gearbox for different viscosity grades of mineral oils. Non-dominated Sorting Genetic Algorithm version 2 (NSGA-II) [10] which can use discrete design variables such as gear module is used in this study.

SUMMARY OF NSGA-II

Among the multi-objective Genetic Algorithm's, NSGA-II [10] is widely used because of its speed of convergence and diversity (a uniform spread of solutions) of the Pareto front. The Pareto front gives the locus of the trade of solutions between two conflicting objective functions, such as minimum volume and minimum power loss and the goal of multi-objective optimization is to obtain this front. It uses a simple but effective Pareto ranking concept to find the fitness of a population member. In the NSGA-II, initially a random population is created and the objective functions are evaluated for each solution and then the whole population is sorted into different non-domination ranks, which are based on the degree of non-domination. Now, the population is sorted in ascending order based on the fitness level. From these solutions, new offspring population is created by using a tournament selection scheme which gives importance to both Pareto rank as well as least crowded solutions. Crossover, and mutation operators are applied to the best parents to get the offspring for the next generation. The major steps of NSGA-II are shown as a flow chart in Fig. 1.

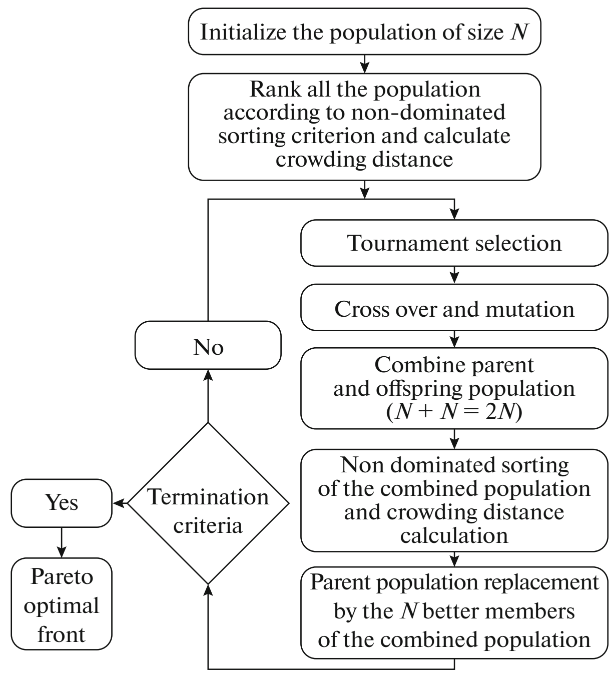

Fig. 1. NSGA-II flowchart.

PROBLEM FORMULATION

In this paper, the multi-objective optimization of a two-stage spur gearbox is carried out using a MATLAB coded NSGA-II algorithm which can take discrete design variables such as gear module. For comparison purpose, two cases are considered here.

(1) A novel multi-objective optimization (simultaneous minimization of volume and power loss) with critical tribology and hunting frequency constraints.

(2) Single objective (volume minimization) optimization without tribology constraints (scuffing and wear) as per current literature [12].

Design Variables

The gearbox parameters which affect the volume and power loss of the gearbox are selected as design variables. The design variable vector X for two-stage gearbox is defined as,

$$X = \{m_1, m_2, z_1, z_2, z_3, b_1, b_2, d_{s1}, d_{s2}, d_{s3}\}.$$

The range of design variables and the type of the design variables used to solve the problem are shown Table 1.

OBJECTIVE FUNCTIONS

Total Volume of the Gearbox

The volume objective function is formulated considering the volume of gears, shafts, and gearbox frame. Thus, the volume function is defined as,

$$\min f_1(X) = V_g + V_s + V_{\text{fr}}. \quad (1)$$

Table 1. Design variables

Design variable	Lower bound	Upper bound	Type
z_1, z_2, z_3	10	125	Integer
m_1, m_2	1, 1.25, 1.5, 2, 2.5, 3, 3.5, 4, 5, 6, 8, 10	–	Discrete
b_1, b_2	10	120	Continuous
d_{s1}, d_{s2}, d_{s3}	15	150	Integer

Volume of gears,

$$V_g = \sum_{i=1}^{2S} \frac{\pi}{4} d_i^2 b_i - \sum_{i=1}^{S} \frac{\pi}{4} d_i^2 (b_{2i-2} + b_{2i-1}) - \frac{\pi}{4}[d_1^2 b_1 + d_{S+1}^2 b_{2S}]. \tag{2}$$

Volume of shafts,

$$V_s = \sum_{i=1}^{S+1} \frac{\pi}{4} d_{si}^2 + \frac{\pi}{4} d_{s1}^2 L_{\text{in}} + \frac{\pi}{4} d_{s(S+1)}^2 L_{\text{out}}. \tag{3}$$

The volume of the frame (V_{fr}) is neglected since the frame does not have a regular shape. Hence the total volume is constituted by the volume of the gears and the shafts. Refer Fig. 2 for gearbox specifications. In Equation (3), the input and output shaft length are taken as,

$$L_{\text{in}} = 2d_{s1}, \quad L_{\text{out}} = 2d_{s2}. \tag{4}$$

Power Losses

The second objective function for minimizing the power losses includes the power losses in the gears, bearings, and seals.

Therefore, in general form, the second objective function is written as,

$$\min f_2(X) = P_{L\text{gear}} + P_{L\text{bearing}} + P_{L\text{seal}}. \tag{5}$$

Power losses in the gears $P_{L\text{gear}}$ are divided as load dependent and load independent losses [1]. Load dependent losses are due to the friction between the meshing gears and largely influenced by the coefficient of the friction in the mesh and the type of gear lubrication oil used.

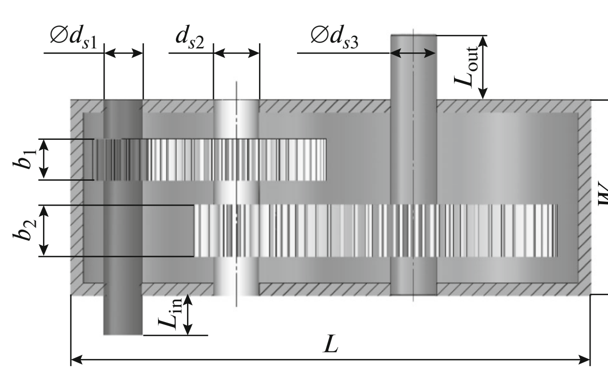

Fig. 2. Typical two stage spur-gearbox.

$$P_{L\text{gear}} = P_{VZP} + P_{VZ0}. \tag{6}$$

The power loss due to meshing of gears,

$$P_{VZP} = P\mu_{mz} H_V. \tag{7}$$

Since the no-load gear power losses, P_{VZ0} are greatly depends on the viscosity, and the volume of the oil used for lubrication, these losses are neglected.

The power losses in the bearings are calculated as per the following equation [11].

$$P_{L\text{bearing}} = P_{VL} = \mu F v. \tag{8}$$

And the power loss in the seals is calculated according to Simrit equation [12].

$$P_{L\text{seal}} = P_{VD} = 7.69 \times 10^{-6} d_s^2 n. \tag{9}$$

Constraints Formulation

According to [13], constraints are formulated for the bending and pitting resistance of the gear.

Bending strength of the gears,

$$F_t K_o K_v K_s \frac{1}{bm_t} \frac{K_H K_B}{Y_J} - \frac{\sigma_{FP} Y_N}{S_F Y_\theta Y_Z} \le 0. \tag{10}$$

Pitting resistance of the gears,

$$Z_E \sqrt{F_t K_o K_v K_s \frac{K_H}{d_{w1} b} \frac{Z_R}{Z_I}} - \frac{\sigma_{HP} Z_N Z_W}{S_H Y_\theta Y_Z} \le 0. \tag{11}$$

Shaft diameter constraint by considering both torque and maximum bending moment on shaft is given by,

$$\left[\frac{32 S_{FS}}{\pi} \sqrt{\left(\frac{T}{S_y}\right)^2 + \left(\frac{M}{S_e}\right)^2} \right]^{1/3} - d_s \le 0. \tag{12}$$

To avoid interference between the gears the constraint for minimum number of teeth on pinion is considered as,

$$\frac{2}{\left(1 + 2\frac{z_g}{z_p}\right)\sin^2(\alpha_t)} \times \left(\frac{z_g}{z_p} + \sqrt{\left(\frac{z_g}{z_p}\right)^2 + \left(1 + 2\frac{z_g}{z_p}\right)\sin^2(\alpha_t)} \right) - z_p \le 0. \tag{13}$$

Since the gearbox is a reducer, the diameter of pinion should always be less than that of the mating gear.

$$d_p < d_g. \quad (14)$$

The gears face widths are limited to,

$$3\pi m \le b \le 5\pi m. \quad (15)$$

Following constraint is considered to avoid the interference between a gear and the next shaft. Here i varies from 1 to the number of stages S.

$$d_{2i} < (d_{2i+1} + d_{2i+2}). \quad (16)$$

Teeth on the gear should be less than the maximum number of teeth allowed.

$$z_g - z_{g\,\max} \le 0. \quad (17)$$

The total transmission ratio should not exceed more than 3% from the given value, regardless of the number of steps [11].

$$u - 0.03u_t \le 0. \quad (18)$$

To reduce the frequency of mesh of identical teeth and thus to reduce the possibility of vibrational excitation, it is desirable that gear teeth are not a multiple of pinion teeth.

$$z_{i+1} \ne \text{multiple}(z_i). \quad (19)$$

Further, the total volume should be less than the maximum allowed volume of the gearbox.

$$f_1 - V_{\max} \le 0. \quad (20)$$

The constraints for the probability of failure of gears due to the scuffing and wear are considered as tribological constraints. The probability of scuffing and wear failures are calculated as in [14] and are kept less than 10%.

Scuffing Constraint

"Scuffing is defined as localized damage caused by solid phase welding between surfaces in relative motion. It is accompanied by a transfer of metal from one surface to another due to welding and subsequent tearing and may occur in any highly-loaded contact where the oil film is too thin to adequately separate the surfaces" [14].

$$P_{\text{scuf}i} - 0.1 \le 0. \quad (21)$$

Wear Constraint

"Wear is a term describing the change to a gear tooth surface involving removal or displacement of material, due to mechanical, chemical or electrical action." [14]

$$P_{\text{wear}i} - 0.1 \le 0. \quad (22)$$

Table 2. Design parameters of two-stage gearbox

Parameter	Symbol	Value
Power to be transmitted [kW]	P	50
Total gear ratio	u_{tot}	10
Input pinion speed [rpm]	n_1	1500
Pressure angle [deg]	α_t	20
Gear material	–	18CrNiMo7-6
Gears surface roughness [µm]	R_a	0.3
Shafts material	–	SAE 1060
Safety factor-shaft design	S_{FS}	1.5
Safety factor-bending	S_F	1.15
Safety factor-pitting	S_H	1.2
Oil temperature [deg C]	θ_{oil}	70

RESULTS AND DISCUSSION

To find out the effect of power loss and the tribological constraints, i.e., scuffing and wear on the design of the gearbox, four types of mineral oils ISO VG 680, ISO VG 150, ISO VG 320, and ISO VG 680 are considered with three cases of gear tooth profiles viz., without tooth modification, smooth meshing profile, and high load capacity gear profile. The parameters considered for the design of gearbox are as shown in Table 2.

Figure 3 shows the Pareto optimal curve between total power loss and volume of gearbox material. ISO VG 680 gives the minimum value combinations for an unmodified tooth profile. VG 320 and VG 150 give higher combinations; however, ISO VG 150 gives an incomplete (narrow range) Pareto front perhaps due to the violation of constraints. The Figs. 4 and 5 respec-

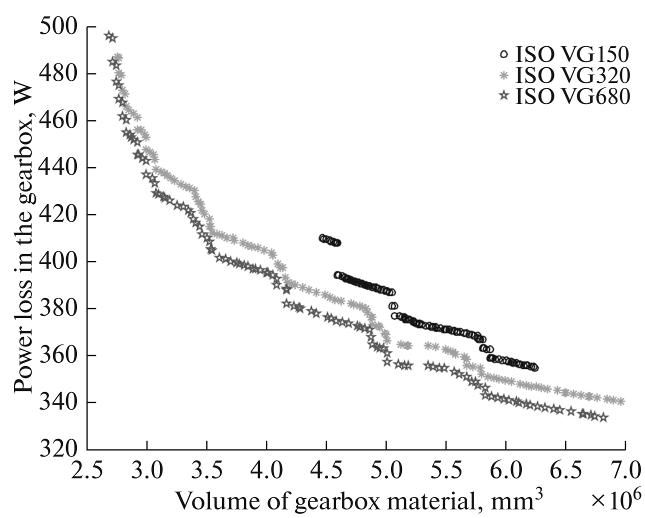

Fig. 3. Pareto fronts for different oils with unmodified gear tooth profile.

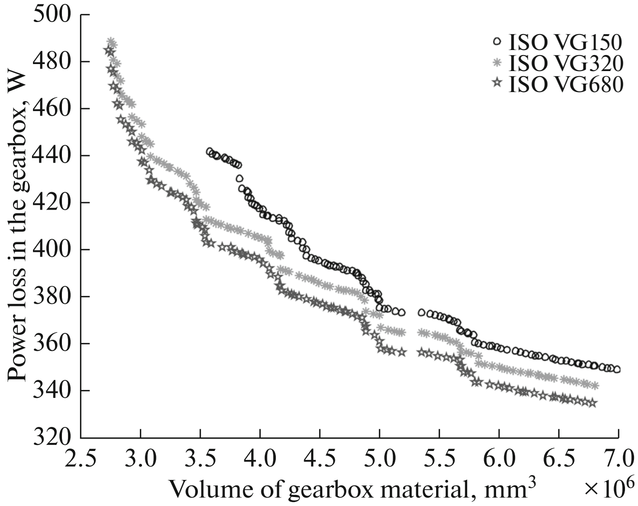

Fig. 4. Pareto fronts for different oils with smooth meshing gear tooth profile.

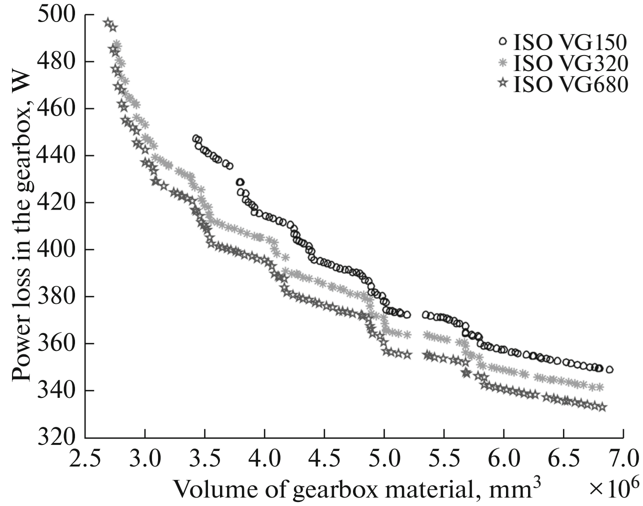

Fig. 5. Pareto fronts for different oils with high load gear tooth profile.

tively represent Pareto fronts for smooth meshing and high load teeth profiles. They follow the same trend as Fig. 3.

Table 3 shows the comparison of volumes and efficiencies of the gearbox for different oils and profiles. It is observed that the volume of the gearbox corresponding to ISO VG 320 and ISO VG 680 oils is almost the same. Whereas, for ISO VG 150, the possible minimum volume obtained is 66.1, 31.37, and 27.3% more than the volume values of ISO VG 680 oil for unmodified, smooth meshing and high load capacity teeth profiles respectively. For further comparison of these results from multi-objective optimization, the same gearbox is studied as a single objective optimization problem as per [12]. i.e., minimize the volume only using constraints Equations 10–20. The solution values of design variables from single objective optimization are used to calculate the corresponding power loss for the multi-objective optimization problem and to check the gearbox safety under scuffing and wear by checking constraint violations. Table 4 shows the results obtained based on values of design variable obtained from single objective optimization.

Since there are no solutions for ISO VG 68 and for VG 150 the gearbox volume obtained is very large for all three cases shown in Table 3. Hence, only the volumes obtained from ISO VG 320 and 680 oils are compared with the volume obtained from single objective optimization. The single objective optimization based volume is about 5.5 and 3.5% less when compared to the average volume obtained by multi-objective optimization for oils ISO VG 320 and 680

Table 3. Minimum volume for different oils and gear tooth profiles from Figs. 3–5

Profile	Volume [mm^3]	Power loss [W]	η_{ab} [%]
	ISO VG 150		
Unmodified	4470900	410.3	99.18
Smooth meshing	3586300	441.2	99.12
High load capacity	3426440	447.2	99.11
	ISO VG 320		
Unmodified	2759680	487.4	99.03
Smooth meshing	2752130	488.1	99.02
High load capacity	2759640	487.4	99.03
	ISO VG 680		
Unmodified	2691660	496.1	99.01
Smooth meshing	2731990	484.3	99.03
High load capacity	2691620	496.1	99.01

Table 4. Results of single objective optimization

Profile	First stage P_{wear1} (%)	Second stage P_{wear2} (%)	Power loss [W]	η_{gb} (%)
	ISO VG 150			
Unmodified	93.69	67.91	839.0	98.32
Smooth meshing	83.33	42.42		
High load capacity	79.52	37.22		
	ISO VG 320			
Unmodified	88.90	37.72	818.2	98.36
Smooth meshing	60.99	<10		
High load capacity	54.37	<10		
	ISO VG 680			
Unmodified	78.50	<10	798.2	98.40
Smooth meshing	25.10	<10		
High load capacity	17.50	<10		
Volume (mm^3)	2613540			

respectively. However, from the results in Table 4, it is observed that for the first stage, the probabilities of wear failure (P_{wear1}) are greater than the permitted value of 10% for all types of oil and profiles. For instance, for oil ISO VG 150, the probabilities of wear failure are 93.69, 83.33, and 79.52% for unmodified, smooth meshing, and high load carrying capacity profiles respectively. Similarly, for ISO 320 and 680, the probabilities of wear failure for the first stage exceeds the permitted value of 10%, which indicates the high risk of wear in the first stage gear pairs. From Table 4 for the second stage, it is also observed that for ISO VG 320 the probability of wear failure corresponding to smooth meshing profile and high load capacity are less than 10% and same things hold good for ISO VG 680 oil for all three profiles. Hence, these pairs are safe under wear, however, since all the first stage gear pairs are at high risk of wear failure, the overall failure of the gearbox is expected.

CONCLUSIONS

The main contributions of this study are summarized as follows:

• Despite considering all the reliability factors and factor of safeties for gear design, it is seen that the gear pairs fail due to wear. Hence it is very important to consider the wear and scuffing failures as tribological constraints in the design stage itself.

• Compared to the method of single objective minimization of volume, with no tribological considerations as followed in [12], the multi-objective approach gave slightly larger gearbox volumes (about 5% more). However, there is a 50% reduction in power loss compared to single objective results, and the multi-objective design is also safer because of violation of wear and hunting constraints for single objective optimization.

NOTATIONS

b_i	Face width of gear pair i, [mm]
d_i	Diameter of gear i, [mm]
d_{si}	Diameter of shaft i, [mm]
z_i	Teeth of gear i, [mm]
m_i	Module of gear pair i, [mm]
P	Input power, [kW]
F_t	Transmitted tangential load, [N]
V_g	Total volume of gears, [mm^3]
V_s	Total volume of shafts, [mm^3]
S	Number of stages
L_{in}	Extended length of input shaft, [mm]
L_{out}	Extended length of output shaft, [mm]
$P_{L\text{gear}}$	Total power loss in gears, [W]
$P_{L\text{bearing}}$	Total power loss in bearings, [W]
$P_{L\text{seal}}$	Total power loss in shaft seals, [W]
P_{VZP}	Meshing gears power loss, [W]
H_V	Gear power loss factor, [–]
v	Peripheral speed, [m/s]
F	Bearing load, [N]
n	Rotational speed, [rpm]
K_o	Overload factor, [–]
K_v	Velocity factor, [–]
K_s	Size factor, [–]
K_H	Load distribution factor, [–]
K_B	Rim thickness factor, [–]

Y_N	Stress cycle life factor for bending strength, [–]
Y_J	Geometry factor for bending strength, [–]
Y_θ	Temperature factor, [–]
Y_Z	Reliability factor, [–]
S_y	Yield strength of shaft material, [N/mm^2]
S_e	Endurance limit of shaft material, [N/mm^2]
Z_E	Elastic coefficient, [N/mm$^{2\ 0.5}$]
Z_R	Surface condition factor for pitting resistance, [–]
Z_I	Geometry factor for pitting resistance, [–]
Z_N	Stress cycle life factor for pitting resistance, [–]
Z_W	Hardness ratio factor for pitting resistance, [–]
T	Torque transmitted by shaft, [N mm]
M	Maximum bending moment on shaft, [N mm]
d_g	Diameter of gear, [mm]
d_p	Diameter of pinion, [mm]
$P_{\mathrm{scuf}i}$	Probability of scuffing failure of gear pair i, [%]
$P_{\mathrm{wear}i}$	Probability of wear failure of gear pair i, [%]
d_{w1}	Operating pitch diameter of pinion, [mm]
η_{oil}	Dynamic viscosity of oil at operating temperature, [mPas]
μ_{mz}	Average coefficient of friction, [–]
μ	Coefficient of friction in bearing, [–]
σ_{FP}	Allowable bending stress number, [N/mm^2]
σ_{HP}	Allowable contact stress number, [N/mm^2]
η_{gb}	Gearbox efficiency, [%]

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