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# Wear Resistance of Coating Materials under the Frictional Heating Conditions

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**Abstract**—The problem of the wear of an elastic coating due to a rigid body sliding over the coating surface and heating due to contact friction has been considered. The solution of the quasi-static problem has been constructed in the form of a series over eigenvalues. The area of unstable solutions of the problem, where the thermoelastic instability of a sliding contact takes place, has been determined in the dimensionless parameter space. The wear resistance of a coating has been studied for different kinds of materials depending on the following parameters: the relative sliding velocity of contact surfaces, the mode of the contact interaction of the friction surfaces, the coating thickness, etc. taking into account the temperature and stresses developing at the contact interface.

**Keywords:** sliding, quasi-static, friction, wear, thermoelastic instability, wear resistance, coating

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## INTRODUCTION

In mechanical engineering and other fields of industry and transport, coatings of different applications (antifriction, anticorrosive, etc.) are widely used to protect the working surfaces of mechanisms and machines during operation. The key characteristic for selecting protective coatings for working surfaces on a sliding friction contact interface is the wear resistance. Thus, the study of the coating wear resistance becomes increasingly important.

In friction assemblies, at the sliding contact interface, upon an increase in the relative velocity of friction surfaces, the contact stresses increase, which is accompanied by frictional heating [1, 2] and, under certain conditions, by a significant temperature increase [3–9], which is often referred to as the *thermoelastic instability* (TEI) of the sliding contact. Under these conditions, the coating wear increases and acquires a catastrophic nature. The determination of the conditions under which TEI occurs on sliding friction contact interface assemblies is an important problem for preventing wear in mechanisms and machines. Thus, the reasons for the accelerated wear process have been thoroughly studied, and they are associated with the properties of a coating and a contact interface, such as the hardness and the thermomechanics of a coating material, the friction properties of its surface, the relative velocity of contact surfaces, the coating thickness and the indentation resistance, the mode and nature of the interaction of friction surfaces, and the intensity of the supply of friction heat from a contact interface into a coating.

**The aim of this work** is the theoretical study of the dependence of the coating wear resistance on the mechanical and thermomechanical properties of a coating under the conditions of frictional heating. A study of the wear resistance of the coating has been performed using the quasi-static thermoelasticity model [6, 9].

## FORMULATION OF THE PROBLEM

The contact flat deformation problem of the sliding of a rigid, thermally insulated body in the form of the half-plane $I\,(h \le x < \infty)$ moving at constant velocity $V$ over surface $x = h$ of an elastic heat-conducting coating of thickness $h\,(0 \le x \le h)$ is considered. Coulomb friction develops during the sliding of a contact interface, which is accompanied by heating and the abrasive wear of the coating. The lower surface of the coating is rigidly bonded to an undeformed nonconducting substrate in the form of half-plane $II\,(-\infty < x < 0)$ (Fig. 1) [6, 8, 9]. The heat flow formed on a contact interface due to friction is directed to the elastic coating. Since the initial moment of time, the rigid body moving along the $y$ axis deforms the surface $x = h$ of the elastic coating, causing displacement along the

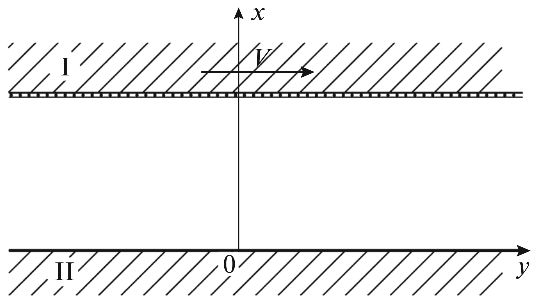


**Fig. 1.** Formulation of the problem on the wear resistance of the coating. Points denote sliding contact interface with friction and coating wear.

direction opposite to the $x$ axis in accordance with the law $\Delta(t)$. Before the initial moment, the coating was at rest and its temperature was equal to zero.

It follows from the formulation of the considered problem that the temperature, stress, and displacement of the distributions in a coating do not depend on the choice of the coordinate on the $y$ axis, which is parallel to the direction of the motion of a rigid body (the half-plane $I$), and are functions of only the $x$ coordinate and time $t$ [3–9]. In this case, the differential equations of thermoelasticity are used to describe the quasi-static stress–strain state of the coating have the following form:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1+\nu}{1-\nu}\alpha\frac{\partial T}{\partial x}, \quad \frac{\partial^2 w}{\partial x^2} = 0, \quad 0 \le x \le h, \quad t > 0, \tag{1}$$

where $\nu$, $\alpha$ are the Poisson's ratio and the coefficient of thermal expansion of coating material, $u(x, t)$, $w(x, t)$ are the vertical and the horizontal displacements, and $T(x, t)$ is the temperature in a coating. The temperatures $T(x, t)$ in the coating are described using the differential equation of thermal conductivity

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{\kappa}\frac{\partial T}{\partial t} = 0, \quad 0 \le x \le h, \quad t > 0, \tag{2}$$

where $\kappa$ is the thermal diffusivity of the coating.

The relation between the normal $\sigma_{xx}(x,t)$ and the tangential $\sigma_{xy}(x,t)$ stresses; the displacements $u(x, t)$, $w(x, t)$; and temperature $T(x, t)$ are determined by the Duhamel–Neumann relations

$$\begin{gathered}\sigma_{xx}(x,t) = \frac{2\mu(1-\nu)}{1-2\nu}\frac{\partial u}{\partial x} - \frac{2\mu(1+\nu)}{1-2\nu}\alpha T, \\ \sigma_{xy}(x,t) = \mu\frac{\partial w}{\partial x},\end{gathered} \tag{3}$$

where $\mu$ is the shear modulus.

The boundary conditions of the considered problem of the coating wear resistance are as follows:

for the mechanical properties ($t > 0$),

$$x = h \quad u(h,t) = -\Delta(t) + u_w(t), \tag{4}$$

$$\sigma_{xy}(h,t) = -f\sigma_{xx}(h,t), \tag{5}$$

$$x = 0 \quad u(0,t) = 0, \tag{6}$$

$$w(0,t) = 0; \tag{7}$$

for the temperature properties ($t > 0$),

$$x = h \qquad K\frac{\partial T(h,t)}{\partial x} = -fV\sigma_{xx}(h,t), \tag{8}$$

$$x = 0 \qquad K\frac{\partial T(0,t)}{\partial x} = kT(0,t), \tag{9}$$

where $f$ is the coefficient of friction, $K$ is the thermal conductivity, $k$ is the heat transfer coefficient, and $u_w(t)$ is the deepening of the half-plane $I$ due to the coating wear. The abrasive wear is described using the model that determines the deepening $u_w(t)$ of a rigid body (the half-plane $I$) due to the coating wear by the following formula (Archard's law):

$$u_w(t) = -fVK^*\int_0^t \sigma_{xx}(h,\tau)d\tau, \quad t > 0, \tag{10}$$

where $\sigma_{xx}(h,t)$ is the normal compression stress on a contact interface and $K^*$ is the coefficient of proportionality between the work of friction forces and the amount of material.

The initial conditions of the problem are zero so that the following equalities are kept:

$$\begin{gathered}u(x,0) = \frac{\partial u(x,0)}{\partial t} = w(x,0) = \frac{\partial w(x,0)}{\partial t} = 0, \\ T(x,0) = 0, \quad \Delta(0) = 0, \quad 0 \le x \le h.\end{gathered} \tag{11}$$

As a result, the considered problem of the coating wear is reduced to solving the system of differential equations (1), (2) under the boundary (5)–(10) and initial conditions (11). The vertical displacements $u(x, t)$, the normal stress $\sigma_{xx}(x,t)$, and the temperature $T(x,t)$ in the coating are determined regardless of the horizontal displacements $w(x, t)$. After the determination of the normal stress $\sigma_{xx}(x,t)$, the horizontal displacements $w(x, t)$ are determined from the second differential equation (4) and the boundary (5), (7), (8) and initial conditions (11).

## SOLUTION OF THE PROBLEM

The solution of the formulated problem of the coating wear is determined using the integral Laplace transform [10]. As a result of the inversion of the Laplace transform, the solution of the problem (the temperature $T(x, t)$, displacements $u(x, t)$, and stresses

$\sigma_{xx}(x,t)$ in a coating) is written in the form of the Laplace convolutions

$$T(x,t) = \frac{1-\nu}{1+\nu}\frac{\hat{V}}{\alpha h}\int_0^t \Delta(\tau) f_T(x,t-\tau)d\tau, \quad 0 \le x \le h, \quad t > 0, \tag{12}$$

$$f_T(x,t) = \frac{1}{2\pi i}\int_\Gamma \frac{N_T(x,z)}{t_\kappa R(z)} e^{z\tilde{t}} dz, \quad \tilde{t} = \frac{t}{t_\kappa}, \quad t_\kappa = \frac{h^2}{\kappa}, \tag{13}$$

$$N_T(x,z) = \sqrt{z}\left(\mathrm{Bi}\sinh\sqrt{z}\frac{x}{h} + \sqrt{z}\cosh\sqrt{z}\frac{x}{h}\right), \tag{14}$$

$$R(z) = zr(z) - \hat{V}\left[(1-k_w)r(z) - \mathrm{Bi}\right], \quad r(z) = \mathrm{Bi}\cosh\sqrt{z} + \sqrt{z}\sinh\sqrt{z}, \tag{15}$$

$$\hat{V} = \frac{fV\alpha}{K}\frac{2\mu(1+\nu)h}{1-2\nu}, \quad \mathrm{Bi} = \frac{kh}{K}, \quad k_w = \frac{1-\nu}{1+\nu}\frac{KK^*}{\alpha\kappa},$$

$$u(x,t) = -\Delta(t)\frac{x}{h} + \int_0^t \Delta(\tau) f_u(x,t-\tau)d\tau, \quad 0 \le x \le h, \quad t > 0, \tag{16}$$

$$f_u(x,t) = \frac{1}{2\pi i}\int_\Gamma \frac{N_u(x,z)}{t_\kappa R(z)} e^{z\tilde{t}} dz, \tag{17}$$

$$N_u(x,z) = \hat{V}\left[\mathrm{Bi}\cosh\sqrt{z}\frac{x}{h} + \sqrt{z}\sinh\sqrt{z}\frac{x}{h} - \frac{x}{h}(1-k_w)r(z) - \left(1-\frac{x}{h}\right)\mathrm{Bi}\right], \tag{18}$$

$$\sigma_{xx}(x,t) = -\frac{2\mu(1-\nu)}{(1-2\nu)h}\left(\Delta(t) - \int_0^t \Delta(\tau) f_\sigma(x,t-\tau)d\tau\right), \quad 0 \le x \le h, \quad t > 0, \tag{19}$$

$$f_\sigma(x,t) = \frac{1}{2\pi i}\int_\Gamma \frac{N_\sigma(x,z)}{t_\kappa R(z)} e^{z\tilde{t}} dz, \tag{20}$$

$$N_\sigma(x,z) = \hat{V}\left[\mathrm{Bi} - (1-k_w)r(z)\right], \tag{21}$$

where $k_w$ in (21) is given after (15). In the quadratures (13), (17), (20), the integrands are regular on infinity ($|z| \to \infty$), while the extraintegral terms in the formulas for $u(x, t)$ (16) and $\sigma_{xx}(x,t)$ (19) are regular components of the obtained generalized solutions. To calculate the quadratures (13), (17), (20), where the integrands are meromorphic in the complex plane, the methods of the complex analysis are used. Under the assumption that the poles of integrands in (13), (17), (20) are known in the complex plane and single, these quadratures are calculated using the residue theorem. Substituting the resulting formulas for $f_T$, $f_u$, $f_\sigma$ in (12), (16), (19), then calculating the convolution integrals yields the new formulas for $T(x,t)$, $u(x,t)$, $\sigma_{xx}(x,t)$ in the form of series for poles $\zeta_k$ of the integrands from (13), (17), (20)

$$T(x,t) = \frac{1-\nu}{1+\nu}\frac{\hat{V}}{\alpha h}\sum_{k=0}^{\infty} B_T(x,\zeta_k)\, D(\zeta_k,t), \quad 0 \le x \le h, \quad t > 0, \tag{22}$$

$$u(x,t) = -\frac{x}{h}\Delta(t) + \sum_{k=0}^{\infty} B_u(x,\zeta_k)\, D(\zeta_k,t), \quad 0 \le x \le h, \quad t > 0, \tag{23}$$

$$\sigma_{xx}(x,t) = -\frac{2\mu(1-\nu)}{(1-2\nu)h} \times \left[\Delta(t) - \sum_{k=0}^{\infty} B_\sigma(x,\zeta_k)\, D(\zeta_k,t)\right], \quad 0 \le x \le h, \quad t > 0, \tag{24}$$

where

$$B_a(x,z) = \frac{N_a(x,z)}{t_\kappa R'(z)}, \quad a = T, u, \sigma, \tag{25}$$

$$D(z,t) = \int_0^t \Delta(\tau)\exp\frac{z(t-\tau)}{t_\kappa}d\tau, \quad t > 0. \tag{26}$$

In (25), the formula for calculating $B_a(x,z)$ is given, where index $a$ is substituted for $T$, $u$, $\sigma$; functions $N_T(x,z)$, $N_u(x,z)$, $N_\sigma(x,z)$ are given in (14), (18), (21); and the derivative $R(z)$ is denoted by $R'(z)$.

The horizontal displacements $w(x, t)$ in the coating are determined by the following formula:

$$w(x,t) = -f\mu^{-1}x\sigma_{xx}(h,t), \quad 0 \le x \le h, \quad t > 0,$$

where $\sigma_{xx}(h,t)$ is from (24).

## POLES OF INTEGRANDS $\zeta_k$

The poles of the integrands $\zeta_k$ $k = 0, 1, 2, \ldots$, according to which the solutions of the problem (22)–(24) are constructed, are the roots of the equation $R(z) = 0$ in the complex plane, where $R(z)$ is from (15). The poles $\zeta_k$ depend on three dimensionless parameters $\hat{V}$, $k_w$, and Bi contained in $R(z)$, which influence their location in the complex plane. In Fig. 2, for $\hat{V} \in [0,\infty)$, Bi = 100, and different $k_w$, the trajectories $\zeta_k(\hat{V})$ $k = 0,1$ are given as follows: $\zeta_0(\hat{V})$ is the solid line and $\zeta_1(\hat{V})$ is the dashed line. The solid square denotes the location of the poles for $\hat{V} = 0$ and the punctured square is for $\hat{V} \to \infty$. The arrows show the direction of the location of the poles $\zeta_k(\hat{V})$ $k = 0,1$ upon an increase in $\hat{V}$ from 0 to $\infty$. If

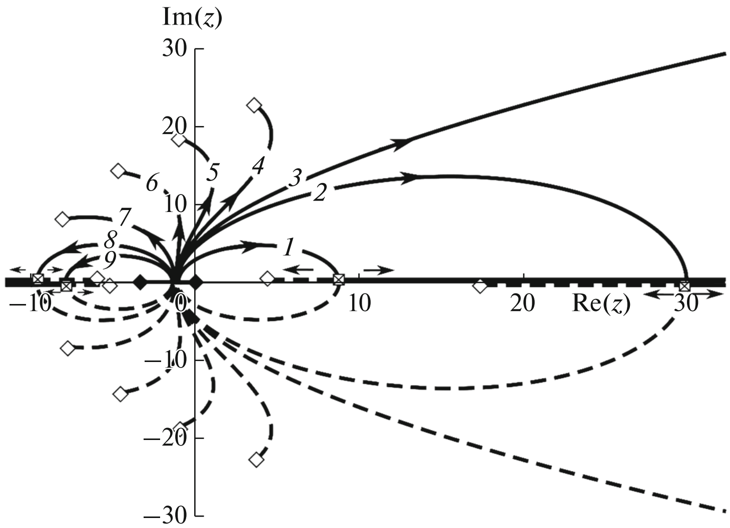


**Fig. 2.** Trajectories of poles $\zeta_0$ (solid line) and $\zeta_1$ (dashed line) of integrands (12), (16), (19) for Bi = 100, $k_w$ = (*1*) 0.75, (*2*) 0.97, (*3*) 1.0, (*4*) 1.05, (*5*) 1.1, (*6*) 1.2, (*7*) 1.5, (*8*) 2, (*9*) 2.5 upon change in $\hat{V}$ from 0 to ∞.

$\mathrm{Re}(\zeta_k) > 0$, then the solutions (22)–(24) are unstable because, in this case, $\lim_{t\to\infty}\{T(x,t), \sigma_{xx}(x,t)\} = \infty$ (if $\mathrm{Im}(\zeta_k) = 0$) or do not exist $(\text{if } \mathrm{Im}(\zeta_k) \neq 0)$. If $\mathrm{Re}(\zeta_k) < 0$, then solutions (22)–(24) are stable. In the space of the dimensionless parameters of the problem $\hat{V}$, $k_w$, Bi, the regions of stable solutions and unstable solutions of the problem have been developed. In Fig. 3, the regions of stable and unstable solutions of the problem in the plane $(k_w, \hat{V})$ are shown for different values Bi. As mentioned above, the unstable solutions of the problem are characterized by the unlimited increase in the amplitudes $T(h,t)$, $\sigma_{xx}(h,t)$ at $t \to \infty$. This property of the unstable solutions on a contact interface is often called the TEI of the sliding contact interface. Note that, upon an increase in $k_w$, the region of stable solutions is widened while the unstable one narrows (Fig. 3).

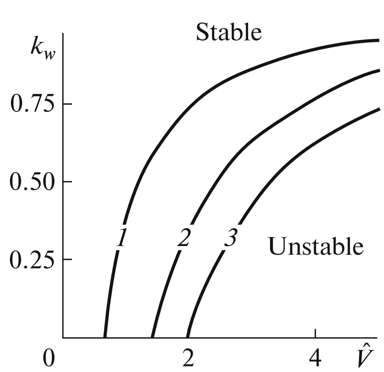


**Fig. 3.** Boundaries and regions of stable and unstable solutions of the problem for different Bi values: (*1*) 1, (*2*) 5, (*3*) 100. Numbers of the curves are given in parentheses.

## NUMERICAL ANALYSIS OF OBTAINED SOLUTIONS

The obtained solutions are used to study the wear resistance of the coating materials during the sliding of a rigid body (half-plane *I*) over the coating surfaces of different materials often used in practice. Let the law of the indentation $\Delta(t)$ of a rigid body into a coating consists of the active phase in $0 < t < t_\varepsilon$ and the passive phase in $t_\varepsilon < t < \infty$ of indentation, e.g.,

$$\Delta(t) = \Delta_0 \begin{cases} -1 + e^{\varepsilon t}, & 0 < t < t_\varepsilon \\ 1, & t_\varepsilon < t < \infty \end{cases}, \tag{27}$$

where $t_\varepsilon = \varepsilon^{-1}\ln 2$ is the time of the end (or the duration) of the active phase of the indentation and $\Delta_0$ is the maximum level of the deepening (indentation) of the rigid body (half-plane *I*) into a coating. The study of the wear resistance of the coating considers three coating materials, i.e., nickel alloy, nodular cast iron, and aluminum alloy, whose parameters are shown in Table 1.

In Fig. 4, the graphs of the change in the temperature $T(h, t)$ calculated according to the formula (22); the wear $u_w(t)$ according to the formulas (23), (4); and the contact stress $p(t) = -\sigma_{xx}(h,t)$ according to formula (24) are shown, which occur and develop over time on a sliding thermal friction contact interface between a rigid body (the half-plane *I*) and a coating made of materials shown in Table 1. The values of thermomechanical and geometric characteristics of a

**Table 1.** Parameters of coating materials

| Material | $\mu$, GPa | $\nu$ | $K$, W/(m K) | $\kappa$, $10^{-6}$ m$^2$/s | $\alpha$, $10^{-6}$ K$^{-1}$ | $f$ | $K^*$, $10^{-12}$ m$^2$/N |
|---|---|---|---|---|---|---|---|
| Nickel alloy | 76.34 | 0.31 | 90.9 | 22.2 | 13.4 | 0.64 | 5 |
| Nodular cast iron | 62.8 | 0.25 | 62.8 | 17.94 | 10.4 | 0.16 | 4.5 |
| Aluminum alloy | 24.81 | 0.34 | 209.3 | 88.09 | 22.09 | 0.47 | 7.5 |

**Table 2.** Values of $k_w$, $\hat{V}$, Bi, and time of wear $t_w$ for calculating the graphs in Fig. 4

| Material | $k_w$ | $\hat{V}$ | Bi | $t_w$, s |
|---|---|---|---|---|
| Nickel alloy (*1*) | 0.8047 | 0.4000 | 5.5006 | 60.4485 |
| Nodular cast iron (*2*) | 0.9088 | 0.0489 | 7.9618 | 254.9745 |
| Aluminum alloy (*3*) | 0.3833 | 0.0861 | 2.3889 | 99.6870 |

contact interface are as follows: $k = 10^5$ W/(m$^2$ K), $h$ = 5 mm, $V$ = 1.6 mm/s, $\Delta_0 = 0.1h$ = 0.5 mm, $\varepsilon$ = 0.0154 s$^{-1}$, $t_\varepsilon$ = 45 s. The values of the dimensionless parameters $k_w$, $\hat{V}$, and Bi from (15) are shown in Table 2. It is assumed that the wear of the coating surface of the value $\Delta_0 = 0.1h$ ends at $t = t_w$ when the stresses turn to zero ($p(t_w) = -\sigma_{xx}(h,t_w) = 0$). The time of coating wear $t_w$ to the preset value $\Delta_0$ characterizes the wear resistance of the coating at fixed $t_\varepsilon$ and other parameters of the friction contact interface. The values of $t_w$ according to the calculation results for the materials from Table 1 are shown in Table 2. The theoretical wear resistance of the coating $t_w$ made of nodular cast iron is several times higher than the coating wear resistance made of other materials, although it is achieved at the maximum values of temperature and stresses on a contact interface.

In Fig. 5, the graphs of the change in the temperature $T(h,t)$ (22); the wear $u_w(t)$ (23), (4); and the contact stress $p(t) = -\sigma_{xx}(h,t)$ (24) for a coating made of aluminum alloy at three different values of the sliding velocity $V$ are shown, i.e., 0.8, 1.6, and 2.4 mm/s. The dimensionless parameter $\hat{V}$ acquires values of 0.0430, 0.0861, and 0.1291, respectively, while the time of wear $t_w$ = 159.2, 99.7, and 79.8 s, respectively. Note that, upon an increase in the sliding velocity $V$, the temperature on the contact interface increases insignificantly, while the stresses decrease.

The dependence of the thickness $h$ of aluminum alloy coating on the process of its wear to the fixed depth $\Delta_0$ = 0.5 mm is demonstrated in Fig. 6, which shows the graphs of the change in $T(h,t)$ (22), $u_w(t)$ (23), (4), $p(t) = -\sigma_{xx}(h,t)$ (24) on the contact interface for three values of $h$, namely 5, 10, and 20 mm. The time of wear $t_w$ of a coating to the same value $\Delta_0$ = 0.5 mm for different thicknesses of the coating $h$ differs significantly. For $h$ = 5 mm, the time of wear is 159.3 s; for $h$ = 10 mm, it is 268.2 s; and, for $h$ = 20 mm, it is 469.9 s. It is noteworthy that the wear of the thin coating ($h$ = 5 mm) occurs at a maximum contact pressure $p(t)$, which indicates a higher resistance to the indentation of a rigid body.

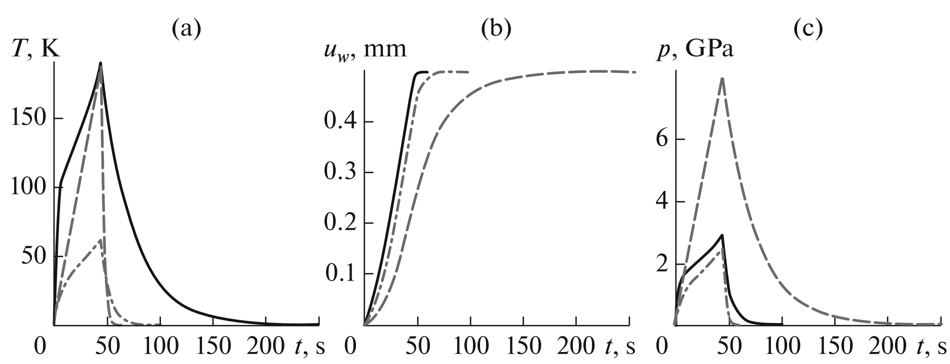


**Fig. 4.** Graphs of temperature $T(h,t)$, wear $u_w(t)$, and contact stress $p(t) = -\sigma_{xx}(h,t)$ for different coating materials: nickel alloy (solid line), high-strength cast iron (dashed line), and aluminum alloy (dash-and-dot line).

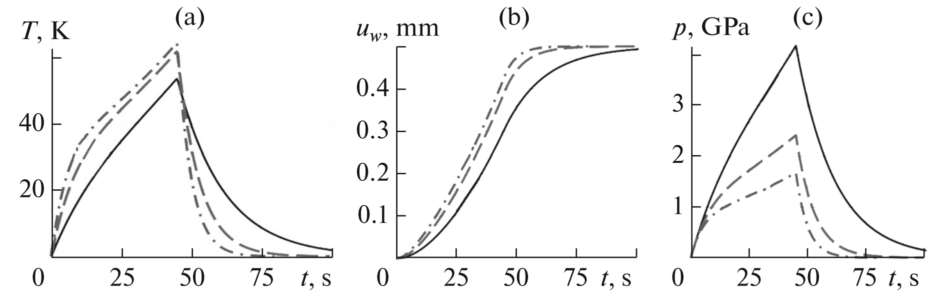


**Fig. 5.** Graphs of temperature $T(h,t)$, wear $u_w(t)$, and contact stress $p(t) = -\sigma_{xx}(h,t)$ for coating made of aluminum alloy for $V = 0.8$ (solid line), 1.6 (dashed line), and 2.4 mm/s (dash-and-dot line).

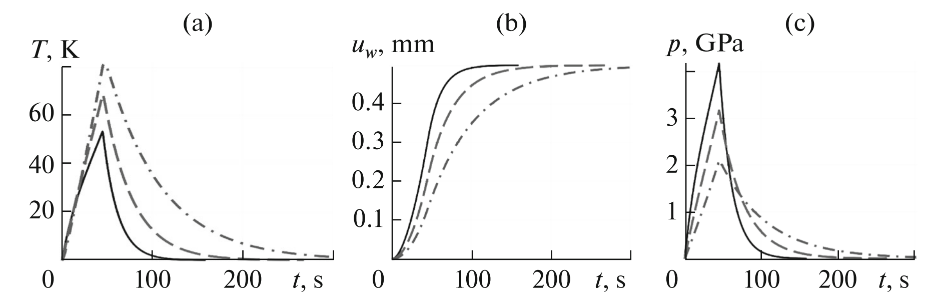


**Fig. 6.** Graphs of temperature $T(h,t)$, wear $u_w(t)$, and contact stress $p(t) = -\sigma_{xx}(h,t)$ for coating made of aluminum alloy for $h = 5$ (solid line), 10 (dashed line), and 20 mm (dash-and-dot line).

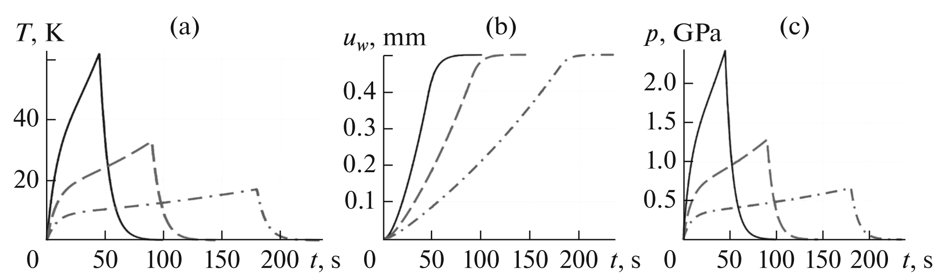


**Fig. 7.** Graphs of temperature $T(h,t)$, wear $u_w(t)$, and contact stress $p(t) = -\sigma_{xx}(h,t)$ for a coating made of aluminum alloy for $t_\varepsilon = 45$ (solid line), 90 (dashed line), and 180 s (dash-and-dot line).

Figure 7 shows the dependence of the temperature $T(h,t)$ (22), the wear $u_w(t)$ (23), (4), and the contact stress $p(t) = -\sigma_{xx}(h,t)$ (24) on a sliding friction contact interface on the value of the parameter $t_\varepsilon$, which characterized the duration of the active phase of indentation of a rigid body (the half-plane $I$) into a coating $0 < t < t_\varepsilon$. The sliding velocity $V = 1.6$ mm/s, the coating thickness $h = 5$ mm, and the indentation depth $\Delta_0 = 0.5$ mm are fixed, while the duration of the active phase of indentation $t_\varepsilon$ acquires values of 45, 90, and 180 s. The corresponding time of wear $t_w$ for the above $t_\varepsilon$ is 99.6, 144.7, and 234.9 s. It can be seen from Fig. 7 that regulating the time of indentation $t_\varepsilon$ of the rigid half-plane $I$ can provide lower values of the temperature and the stresses on a contact interface at the same amount of wear.

## CONCLUSIONS

The considered model for the wear of an elastic coating during the sliding of a rigid body on it taking into consideration the friction and frictional heating of the coating was used for a theoretical study of the wear resistance of the coating depending on the thermomechanical properties of the coating material, the relative sliding velocity of the friction surfaces at the contact interface, the thickness of the coating material, the time interval of the active phase of indentation of a rigid body into an elastic coating, and other wear characteristics. The resulting formulas can be used to find the optimal modes of the coating wear in terms of temperature and stresses on the contact interface.

## NOTATION

| | |
|---|---|
| Bi, $k_w$, $\hat{V}$ | dimensionless parameters |
| $D$ | Laplace transform of the indentation law |
| $f$ | coefficient of friction |
| $h$ | thickness of the elastic coating |
| $i$ | imaginary unit |
| $k$ | heat transfer coefficient |
| $K$ | thermal conductivity |
| $K^*$ | coefficient of proportionality between the work of friction forces and the amount of material |
| $N_a$ | numerator of an integrand |
| $R$ | denominator of an integrand |
| Re | real part of a complex number |
| Im | imaginary part of a complex number |
| $t$ | time |
| $t_\varepsilon$ | duration of the active phase of indentation |
| $t_\kappa$ | ratio of squared coating thickness to thermal diffusivity |
| $\tilde{t}$ | dimensionless time (time divided by $t_\kappa$) |
| $T$ | temperature of the coating |
| $u$, $w$ | vertical and horizontal displacements in the coating |
| $u_w$ | deepening of the half-plane $I$ due to the coating wear |
| $V$ | sliding velocity of the half-plane $I$ |
| $x$, $y$ | coordinates |
| $z$ | complex integration variable |
| $\zeta_k$ | set of poles of an integrand |
| $\alpha$ | coefficient of thermal expansion |
| $\Delta$ | law of the half-plane $I$ indentation into a coating |
| $\Delta_0$ | maximum indentation depth |
| $\varepsilon$ | parameter of the indentation law |
| $\kappa$ | thermal diffusivity of the coating |
| $\sigma_{xx}$, $\sigma_{xy}$ | normal and tangential stresses in the coating |
| $\mu$ | shear modulus of the coating material |
| $\nu$ | Poisson's ratio |
| $\rho$ | density |

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## REFERENCES

1. Aleksandrov, V.M. and Kovalenko, E.V., Methods of solving contact thermoelasticity problems with allowance for the wear of interacting surfaces, *J. Appl. Mech. Tech. Phys.*, 1985, vol. 26, no. 3, pp. 418–421.
2. Aleksandrov, V.M. and Gubareva, E.A., Consideration of nonlinear friction, wear and heat generation in nonideal heat contacts for the problem on interacting solid and coating, *J. Frict. Wear*, 2004, vol. 25, no. 5, pp. 455–460.
3. Barber, J.R., Thermoelastic Instabilities in the Sliding of Conforming Solids, *Proc. R. Soc. A*, 1969, vol. 312, no. 1510, 381–394.
4. Dow, T.A. and Burton, R.A., The role of wear in the initiation of thermoelastic instabilities of rubbing contact, *J. Lubr. Technol.*, 1973, vol. 95, no. 1, pp. 71–75.
5. Aleksandrov, V.M. and Annakulova, G.K., Interaction of coatings of bodies taking into account the deformability, wear, and heat release from friction, *Trenie Iznos*, 1992, vol. 13, no. 1, pp. 154–160.
6. Evtushenko, A.A. and Pyr'ev, Yu.A., Influence of wear on the development of thermoelastic instability of a frictional contact, *Izv. Ross. Akad. Nauk, Mekh. Tverd. Tela*, 1997, no. 1, pp. 114–121.
7. Mao, J.-J., Ke, L.-L., and Wang, Y.-S., Thermoelastic instability of a functionally graded layer interacting with a homogeneous layer, *Int. J. Mech. Sci.*, 2015, vol. 99, pp. 218–227.
8. Zelentsov, V.B., Mitrin, B.I., and Aizikovich S.M., Dynamic and quasi-static instability of sliding thermoelastic frictional contact, *J. Frict. Wear*, 2016, vol. 37, no. 3, pp. 213–220.
9. Zelentsov, V.B., Mitrin, B.I., and Lubyagin, I.A., Effect of wear on frictional heating and thermoelastic instability of sliding contact, *Comp. Cont. Mech.*, 2016, vol. 9, no. 4, pp. 430–442.
10. Ditkin, V.A. and Prudnikov, A.P., *Operatsionnoe ischislenie* (Operational Calculations), Moscow: Vysshaya Shkola, 1975.

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