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Effect of Convective Cooling on Temperature and Thermal Stresses in Disk during Repeated Intermittent Braking

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Abstract—A numerical solution of the heat-friction problem for a brake pad—brake disk system during repeated intermittent braking has been obtained using the finite-element method. The obtained temperature field has been used to study the thermal stress state of the disk. A numerical analysis has been carried out for a metal-ceramic pad—cast iron disk friction pair during ten intermittent brakings. The effect of the coefficient of the coefficient of heat transfer on the temperature, the stress tensor components, and the Huber—Mises—Henky stresses on the working surface of the disk has been investigated.

Keywords: temperature, thermal stresses, braking, frictional heating, convective cooling

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INTRODUCTION

In some cases during the operation of heavy-duty brakes, temperature gradients in the friction members lead to a substantial increase in local thermal stresses on their working surfaces [1]. In particular, high temperature gradients in zones of the real contact of the brake pad with a brake disk induce sharp thermal shocks and the appearance of thermal-damage spots, over the boundaries of which thermal cracks rapidly propagate after even a few brakings [2]. Moreover, thermal stresses result in the warping of disks, which reduces the contour and apparent areas of contact, as well as increases the local thermal loading and the local wear of the working surfaces [3]. Therefore, the study of the thermal stress state based on the known temperature distribution is an important stage of calculating the thermal operating conditions of brake systems [4].

State-of-the-art of works on methods of determining thermal stresses in disk brakes are presented in reviews [5, 6]. We note that the majority of the works are devoted to studying frictional heating during single intermittent or long-term brakings. The thermal stress state of a brake disk during repeated intermittent braking is less well understood. This mode of braking is "a series of successive brakings after each of which the temperature of sliding contact and the temperature of volumes of the brake members increase and reach a steady value" [7]. This mode of braking is typical of car brakes during city and mountain driving. When modeling the thermal state of a disk, which is heated during repeated intermittent brakings, the convection cooling of the working surfaces of the disk should be taken into account [8].

The aim of the work was to study the effect of heat transfer from the free surfaces of the disk during repeated brakings on the distributions of the temperature and thermal stresses in the disk.

FORMULATION OF THE PROBLEM

Let us consider a brake disk with thickness 2δ and an inner radius r_d and outer radius R_d, which rotates at a constant angular velocity ω_0 (Fig. 1a). At the initial moment of time $t = 0$, two pads with angular lengths θ_0, inner radii r_p, and outer radii $R_p = R_d$ are pressed to the end surfaces of the disk by pressure p_0; here and below, all variables and parameters related to the disk and pad are designated by the subscripts d and p, respectively. Because of friction, the angular velocity of the disk linearly decreases to the zero value at the moment it stops, $t = t_s$, and heat is generated in the zones of the contact between the pads and the disk. Upon stopping, the pads immediately depart from the surfaces of the disk, which again accelerates to the initial velocity ω_0 for the time $t = t_c$ after which new braking begins. The total number of breakings and accelerations of the disk is n and the duration of each cycle is $t_{sc} = t_s + t_c$. Upon completing the last nth cycle, the stationary disk undergoes convection heating for the time $t = t_{cn}$. Thus, the total duration of the process of the heating and cooling of the disk end surfaces is $t_{\text{end}} = nt_{sc} - t_c + t_{cn}$. During this time, convection heat transfer to the environment occurs on the free surfaces of the disc with the constant coefficient of heat transfer h. Because of the axial symmetry of the thermal load, when determining the temperature and thermal stresses, it is sufficient to consider the heating and cooling of a disk of the thickness δ, which is related to the cylindrical system of coordinates $r\theta z$ (Fig. 1a).

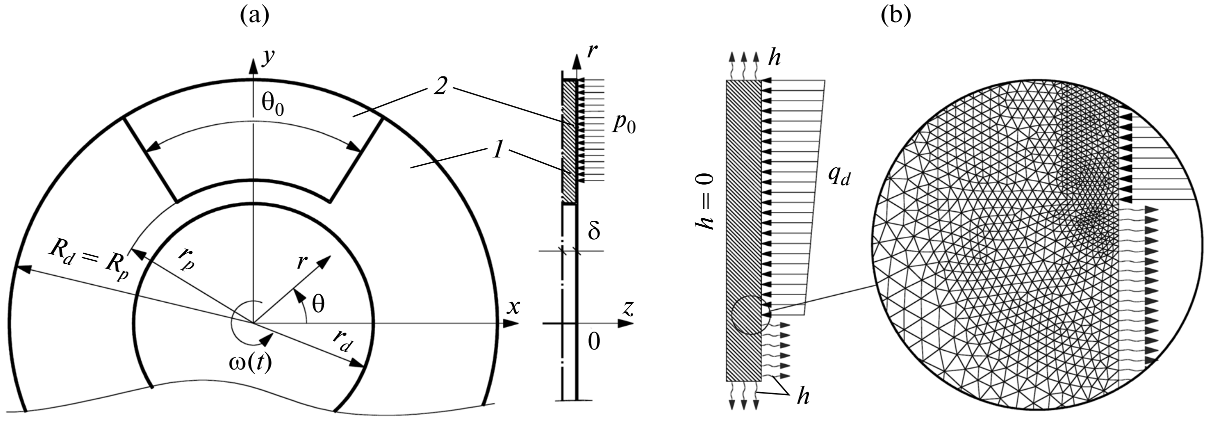

Fig. 1. Schematics of (a) contact of pad with disk and (b) heating and convection cooling of disk with FEM network.

The temperature field $T(r, z, t)$ in the disk can be determined from solving the following axisymmetric boundary heat-conduction problem:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k_d}\frac{\partial T}{\partial t}, \quad r_d \le r \le R_d, \quad -\delta \le z \le 0, \quad t > 0, \tag{1}$$

$$K_d \left.\frac{\partial T}{\partial z}\right|_{z=0} = \begin{cases} q_d(r,t), & r_p \le r \le R_p, \quad 0 < t \le t_{\text{end}}, \\ h[T_0 - T(r,0,t)], & r_d \le r \le r_p, \quad t > 0, \end{cases} \tag{2}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=-\delta} = 0, \quad r_d \le r \le R_d, \quad t > 0, \tag{3}$$

$$K_d \left.\frac{\partial T}{\partial r}\right|_{r=r_d} = h[T(r_d, z, t) - T_0], \quad -\delta \le z \le 0, \quad t > 0, \tag{4}$$

$$K_d \left.\frac{\partial T}{\partial r}\right|_{r=R_d} = h[T_0 - T(R_d, z, t)], \quad -\delta \le z \le 0, \quad t > 0, \tag{5}$$

$$T(r,z,0) = T_0, \quad r_d \le r \le R_d, \quad -\delta \le z \le 0, \tag{6}$$

where, in accordance with the accepted mode of braking, the intensity of the frictional heat flow directed into the disk along the normal to the friction surface can be written as follows:

$$q_d(r,t) = \gamma\eta q(r,t), \quad q(r,t) = f p_0 p^*(t) r \omega_0 \omega^*(t), \quad rp \le r \le R_p, \quad 0 < t \le t_{\text{end}}, \tag{7}$$

$$p^*(t) = \begin{cases} 1, & it_{sc} \le t < it_{sc} + t_s, \\ 0, & it_{sc} + t_s \le t < (i+1)t_{sc} \wedge (n-1)t_{sc} - \\ - t_c \le t \le t_{\text{end}}, & i = 0,1,\ldots,n-1, \end{cases} \tag{8}$$

$$\omega^*(t) = \begin{cases} 1 - (t - it_{sc})/t_s, & it_{sc} \le t < it_{sc} + t_s, \\ i = 0,1,\ldots,n-1, \\ [t - (it_{sc} + t_s)]/t_c, & it_{sc} + t_s \le t < (i+1)t_{sc}, \\ i = 0,1,\ldots,n-2, \\ 0, & (n-1)t_{sc} - t_c \le t \le t_{\text{end}}. \end{cases} \tag{9}$$

Here, $\gamma = \theta_0/(2\pi)$ is the overlapping factor [9] and $\eta = K_d\sqrt{k_p}/(K_d\sqrt{k_p} + K_p\sqrt{k_d})$ is the heat-partitioning factor [10].

If the temperature distribution is known, the components σ_r, σ_θ, σ_z, and σ_{rz} of the stress tensor can be determined from solving the following quasistatic boundary thermoelasticity problem:

$$(1 - 2\nu_d)\nabla^2\mathbf{u} + \nabla \operatorname{div}\mathbf{u} = 2\alpha_d(1 + \nu_d)\nabla T, \quad r_d \le r \le R_d, \quad -\delta \le z \le 0, \quad t > 0, \tag{10}$$

$$\sigma_z(r,0,t) = \sigma_{rz}(r,0,t) = 0, \quad r_d \le r \le R_d, \quad t > 0, \tag{11}$$

$$u_z(r,-\delta,t) = \sigma_{rz}(r,-\delta,t) = 0, \quad r_d \le r \le R_d, \quad t > 0, \tag{12}$$

$$\sigma_r(r_d,z,t) = \sigma_{rz}(r_d,z,t) = 0, \quad -\delta \le z \le 0, \quad t > 0, \tag{13}$$

$$\sigma_r(R_d,z,t) = \sigma_{rz}(R_d,z,t) = 0, \quad -\delta \le z \le 0, \quad t > 0, \tag{14}$$

where $\mathbf{u} = \{u_r, u_z\}$ is the displacement vector and ∇ is the Hamiltonian in the cylindrical system of coordinates. The stresses and displacements in Navier equation (10) and homogeneous boundary conditions (11)–(14) are related by the following Duhamel–Neumann relations:

$$\sigma_r = 2\mu\varepsilon_r + \lambda e - \beta T, \quad \sigma_\theta = 2\mu\varepsilon_\theta + \lambda e - \beta T, \quad \sigma_z = 2\mu\varepsilon_z + \lambda e - \beta T, \quad \sigma_{rz} = 2\mu\varepsilon_{rz}, \tag{15}$$

$$\varepsilon_r = \frac{\partial u_r}{\partial r}, \quad \varepsilon_\theta = \frac{u_r}{r}, \quad \varepsilon_z = \frac{\partial u_z}{\partial z}, \quad \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), \tag{16}$$

$$e = \varepsilon_r + \varepsilon_\theta + \varepsilon_z, \quad \mu = E_d/[2(1 + \nu_d)], \quad \lambda = 2\nu_d\mu/(1 - 2\nu_d), \quad \beta = \alpha_d(3\lambda + 2\mu). \tag{17}$$

NUMERICAL SOLUTION AND ANALYSIS

Boundary heat-conduction problem (1)–(9) and boundary thermoelasticity problem (10)–(17) were successively solved using the FEM, which was implemented in the MSC.Patran software package and the MSC.Nastran computation module [11]. The calculation model consisted of 33193 nodes and 65243 axisymmetric triangle CTRIAX6 elements. The density of the elements near the zone of frictional heating in which high gradients of the temperature and the thermal stresses occurred was increased (Fig. 1b). The minimum size of an element was 0.02 mm, while the maximum size was 0.2 mm. The numerical solution of the heat-conduction problem (1)–(9) was found with the step $\Delta t = 0.001$ s during braking and $\Delta t = 0.01$ s during the acceleration of the disk. The obtained values of the temperature at the nods of the spatial network were used as the input data for solving thermoelasticity problem (10)–(17). The calculations took a long time since, during the determination of the temperature field at every tenth time step, 7860 boundary thermoelasticity problems of type (11)–(17) had to be successively solved in order to achieve the time t_{end}. Therefore, in order to automate the process of recording temperature values at the nods of the finite-element network at each time step and to subsequently use these values as the input parameters for the numerical solution of the thermoelasticity problem, the MSC.Nastran module was added with a program written in Python language. One more author program written in the same language admitted to carry out simultaneous computations for four time steps, while the third program made it possible to process the results obtained at each time step in separate files.

The calculations were carried out for a ChNMKh cast-iron disk and an FMK-11 metal-ceramic pad. The thermal and mechanical characteristics of ChNMKh cast iron were as follows [2]: $K_d = 51$ W/(m K), $k_d = 14.4$ m^2/s, $\alpha_d = 0.108 \times 10^{-6}$ K^{-1}, $E_d = 99.97$ GPa, and $\nu_d = 0.29$. In the accepted calculation model, the effect of the temperature and, therefore, thermal stresses in the disk was taken into account by introducing the heat-partitioning factor η; that factor could be calculated if the thermal characteristics of the FMK-11 material were known, which were taken as follows [2]: $K_p = 34.4$ W/(m K) and $k_p = 14.64$ m^2/s. The dimensions of the disk and the pad were taken the same as those in [8], i.e., $r_d = 66$ mm, $r_p = 76.5$ mm, $R_d = R_p = 113.5$ mm, $\delta = 5.5$ mm, and $\theta_0 = 64.5°$. The remaining input parameters were as follows: $p_0 =$ 1.47 MPa, $\omega_0 = 88.46$ s^{-1}, $f = 0.5$, $t_s = 3.96$ s, $t_c = 10$ s, $t_{cn} = 300$ s, and $n = 10$.

A total duration of ten cycles of frictional heating in the course of braking and cooling in the course of subsequent acceleration, including the duration of the convective cooling of the disk after the last (10th) stop, was $t_{\text{end}} = 10(3.96\text{ s} + 10\text{ s}) - 10\text{ s} + 300\text{ s} = 429.6$ s. In accordance with this sequence of the cooling and

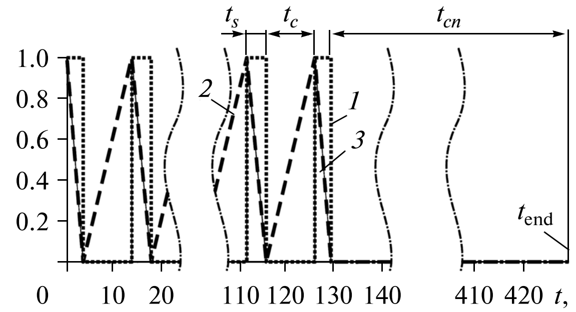

Fig. 2. Variations in dimensionless (*1*) pressure $p^*(t)$, (*2*) angular velocity $\omega^*(t)$ and (*3*) intensity of frictional heat flow $p^*(t)\omega^*(t)$ with time t.

heating of the disk, the time variations in dimensionless pressure $p^*(t)$ (8) and angular velocity $\omega^*(t)$ (9), as well as of their product $\omega^*(t)p^*(t)$, which describes the time profile of intensity of the frictional heat flow $q_d(r, t)$ (7), are shown in Fig. 2.

The variations of the temperature and thermal stresses at the inner ($r = r_d$) and the outer ($r = R_d$) edges of the working surface of the disk ($z = 0$) with time t were studied. In accordance with the aim of this work, the calculations were carried out for the values of the coefficient of heat transfer h that varied from zero in the case of the thermal insulation of the free surfaces of the disk to the maximum value of 100 W/(m^2 K) in the case of the forced air cooling of those surfaces.

With an increase in the number of brakings, the temperature of the surface of the disk increases (Fig. 3). The time profiles of the temperature for the selected values of the radial variable are different. At $h = 0$, the temperature of the inner edge of the disk increases nearly monotonously with time and reaches the steady value of 583°C after the last (10th) braking. The temperature of the outer edge of the disk at this value of the coefficient of heat transfer fluctuates; i.e., it increases during each braking and decreases during the subsequent acceleration of the car to a value that somewhat exceeds the initial value and eventually reaches the same steady value of 583°C obtained at the inner edge. In the case of the thermally insulated free surfaces of the disk, the maximum value of the temperature of 624°C is achieved during the last braking. The consideration of convective cooling (h = 100 W/(m^2 K)) leads to a general decrease in the temperature of the disk and to fluctuations in the temperature at its inner edge. In this case, the maximum temperature at the outer edge of the disk during the last cycle is equal to 441.6°C, while the minimum temperature, which is achieved at the end of the time interval under consideration (at $t = t_{\text{end}}$), is equal to 77.3°C.

The variations in the maximum temperature $T_{\max}$ and the temperature T_{end} achieved at the moment of

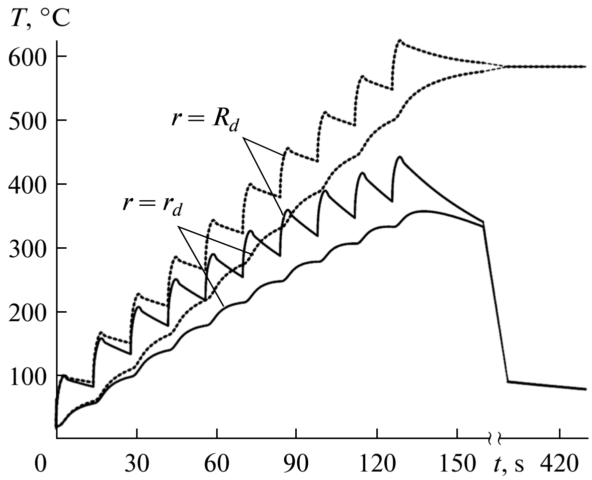

Fig. 3. Evolution of temperature T at inner ($r = r_d$) and outer ($r = R_d$) edges of heated surface ($z = 0$) of disk at (dashed lines) $h = 0$ and (solid lines) $h = 100$ W/(m^2 K).

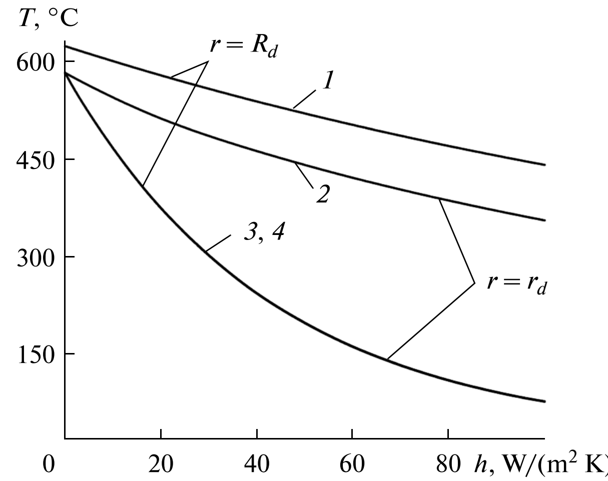

Fig. 4. Dependences of (*1*, *2*) maximum temperature and (*3*, *4*) temperature achieved at $t = t_{end}$ at inner ($r = r_d$) and outer ($r = R_d$) edges of heated surface ($z = 0$) of disk on coefficient of hear transfer h.

time $t = t_{end}$ with an increasing coefficient of heat transfer h from 0 to 100 W/(m^2 K) are shown in Fig. 4. Since at a constant value of h the specific friction power reaches the maximum value at the outer edge of the disk (at $r = R_d$), the temperature at this edge also takes on the maximum value. The sharpest drop in the temperature with intensifying convection cooling occurs at the last moment of time $t = t_{end}$; the temperature drops from 583°C in the case of the thermal insulation of the free surfaces of the disk to 77.3°C in the case of $h = 100$ W/(m^2 K).

The fluctuations in the temperature during repeated intermittent braking (Fig. 3) lead to the fluctuations in the thermal stresses at both inner and outer edges of the heated surface of the disk (Fig. 5). In accordance with boundary conditions (11), the radial σ_r and the circumferential σ_θ components of the stress tensor take on nonzero values on the working surface of the disk. The positive radial stress σ_r at the inner edge of the disk has only a pronounced value when the intensive cooling of the free surfaces of the disk occurs (Fig. 5a). At the outer edge of the disk, the sign of the stress σ_r changes from negative during braking to positive during the acceleration of the disk. The absolute value of the radial stress during the entire ten cycles does not exceed 1.2 MPa. At the same time, the circumferential stress σ_θ has the positive sign at the inner and the negative sign at the outer edges of the heated surface of the disk (Fig. 5b). The maximum values of $|\sigma_\theta|$ increase with an increasing number of brakings and reach the extreme values during the last (10th) cycle. Convection heating has no pronounced effect on the value of $\sigma_\theta > 0$ at $r = r_d$ and leads to an increase in the value of $|\sigma_\theta|$ at the outer edge of the disk (at $r = R_d$) by ~30%. Upon completing the last cycle, the circumferential stress at the inner edge of the disk monotonously decreases with time, but continually remains tensile. During this period of time, the sign of the stress σ_θ at the outer edge of the disk changes from negative to positive; i.e., a high tensile circumferential stress arises. The stress state of the working surface of the disk was assessed using the generalized Huber–Mises–Henky stress (the stress intensity) $\sigma_{HMH} = \sqrt{\sigma_r^2 + \sigma_\theta^2 - \sigma_r\sigma_\theta}$. At $h = 0$, the maximum value of this stress $\sigma_{HMH} = 73$ MPa is achieved at the inner edge of the disk at the end of the last (10th) cycle, i.e., at moment of time $t = 128.54$ s (Fig. 5c). The maximum values of the stress σ_{HMH} achieved in each cycle increase during a few initial cycles and then reach a steady value. Thus, an increase in the number of cycles of the braking and acceleration of the disk does lead to a monotonous rise in the stress intensity in contrast with the temperature.

Figure 6 shows the effect of the intensity of the cooling of the disk on the stress intensity. The graphs are plotted for the maximum values of the stress σ_{HMH} and the values of this stress achieved at the moment of time $t = t_{end}$ at the inner ($r = r_d$) and the outer ($r = R_d$) edges of the working surface of the disk. With an increase in the coefficient of heat transfer, the maximum stresses at the inner (outer) edges of the disk somewhat increase (decrease) from 73 MPa (54.1 MPa) at $h = 0$ to 74.3 MPa (46.2 MPa). The effect of the parameter h on the stress intensity σ_{HMH} at $t = t_{end}$ is more pronounced, but the values of this stress do not exceed 2 MPa.

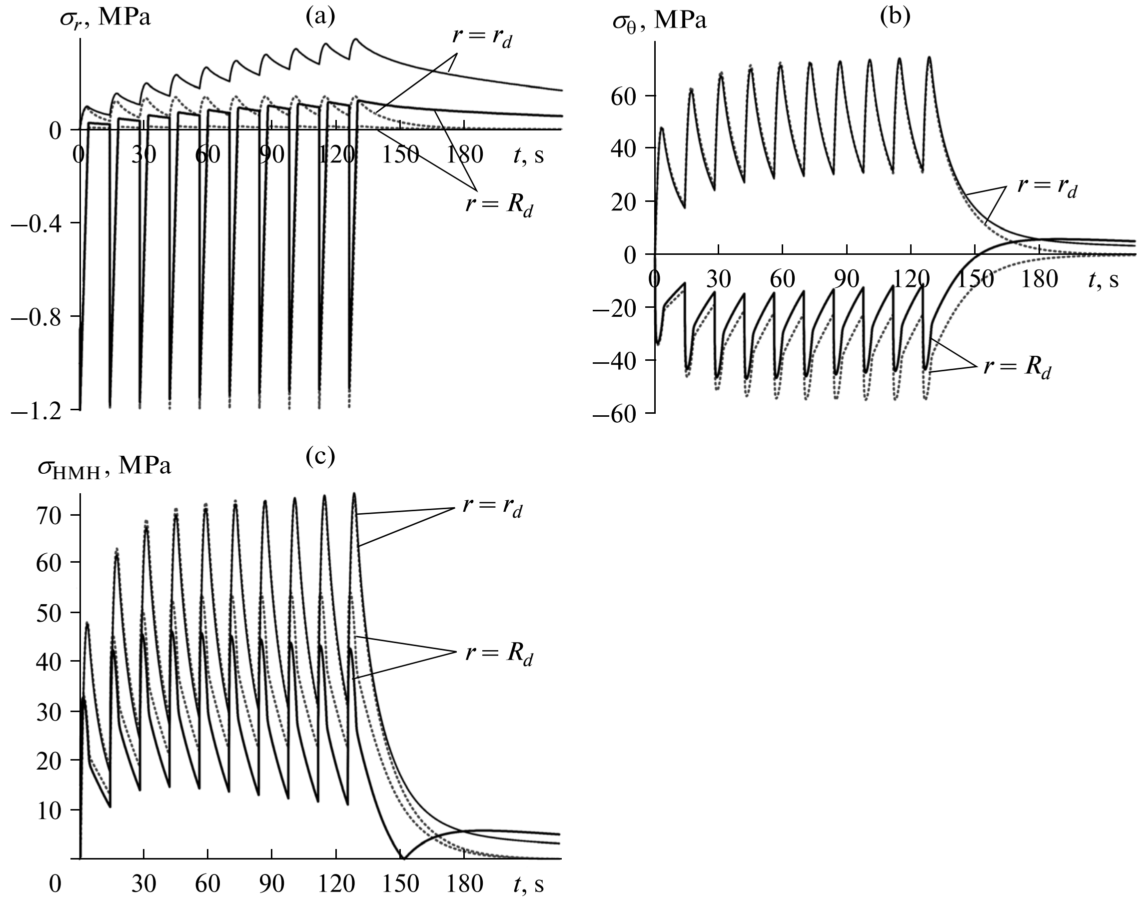

Fig. 5. Evolution of (a) radial stress σ_r, (b) circumferential stress σ_θ, and (c) Huber–Mises–Henky stress σ_{HMH} at inner ($r = r_d$) and outer ($r = R_d$) edges of heated surface ($z = 0$) of disk at (dashed lines) $h = 0$ and (solid lines) $h = 100$ W/(m^2 K).

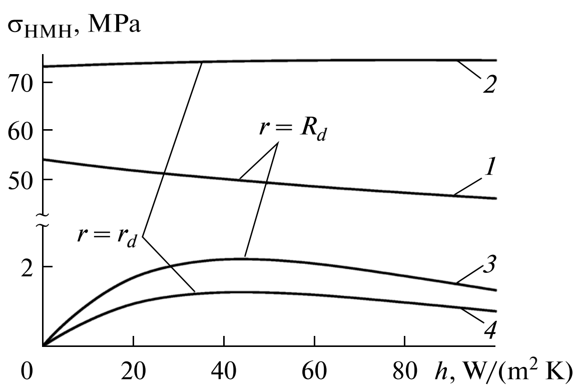

Fig. 6. Dependences of Huber–Mises–Henky stress σ_{HMH} at inner ($r = r_d$) and outer ($r = R_d$) edges of heated surface ($z = 0$) of disk on coefficient of hear transfer h: (*1*, *2*) maximum value and (*3*, *4*) value achieved at $t = t_{end}$.

CONCLUSIONS

The numerical solutions to the axisymmetric heat frictional problem for the pad–disk system and the corresponding quasistatic thermoelasticity problem have been obtained using the FEM. The temperature field and the thermal stress state of the disk during repeated intermittent braking have been calculated for the FMK-11 metal-ceramic pad–ChNMKh cast iron disk friction pair. The effect of the coefficient of heat transfer from the free surfaces of the disk on the evolution of the temperature and the thermal stresses during ten cycles of the braking and the subsequent acceleration of the disk has been studied. It has been found that each new cycle leads to an increase in the maximum temperature of the working surface of the disk. At the end of the last (tenth) cycle, the maximum temperature at the outer edge of the disk is equal to 624°C. The intensification of convective cooling leads to a decrease in the temperature, which is sharpest at the end of the time interval under consideration. The fluctuations in the temperature during each cycle determine the time variations in the thermal stresses. Near the inner edge of the friction surface of the disk, high (≈70 MPa) tensile circumferential stresses arise, which have a decisive effect on the value and character of the changes in the stress intensity (the Huber–Mises–Henky stress). This conclusion agrees with the experimental data obtained in work [12], in which it is shown that the origination of radial cracks is only possible near the inner edge of the disk, which leads to the deterioration of the frictional characteristics of the brake. The free convection cooling of the disk has no pronounced effect on the thermal stress intensity.

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NOTATION

f	coefficient of friction
E	modulus of elasticity
h	coefficient of heat transfer
K	thermal conductivity
k	thermal diffusivity
n	number of cycles
p_0	pressure
r	radial coordinate
$r_{p,d}$	inner radius
$R_{p,d}$	outer radius
t	time
t_s	duration of braking
t_c	duration of acceleration
t_{cn}	duration of cooling of disk upon completing last cycle
t_{end}	total duration of process
T	temperature
T_0	initial temperature
u_r, u_z	displacements
z	axial coordinate
α	coefficient of linear thermal expansion
δ	thickness
ν	Poisson's ratio
θ	angular coordinate
θ_0	angle of contact
σ	stress
ω	angular velocity
ω_0	initial angular velocity

Subscript

p	means values related to pad
d	means to disk

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