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Simulation of the Thermal Process and Friction Diagnostics in a System of Nonlubricated Sliding Bearings on a Common Shaft

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Abstract—The development of a method of thermal diagnostics of friction allowing evaluating friction torque in a system of sliding bearings on a common shaft by temperature data under the assumption of uniform temperature distributions on the cross section of a shaft and along the length of bearings in the case of dependence of thermal properties of materials on temperature is described. The resistance of the obtained solutions to errors in temperature data is shown.

Keywords: system of sliding bearings, friction torque, heat generation capacity, thermal diagnostics of friction, mathematical model, direct and inverse problem, conjugate gradient method, iterative regularization, computational experiment

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INTRODUCTION

The problem of increasing the information content of tests and development of new methods for experimental data processing becomes especially relevant during benchmark and performance testing of machines and mechanisms in extreme conditions, which possess inherent high material costs and are often of unique character. The compactness of practical tribocoupling does not allow placing the necessary measuring devices for direct measuring of the friction torque in cylindrical coupling, in sliding bearings in particular. In this case, the method of heat friction diagnostics, which yields a solution to the heat exchange boundary inverse problem for evaluating the power of frictional heating based on temperature data, is an effective method of determining friction moments [1]. Friction moment is determined as the ratio of heat generation output to the angular velocity of rotation. Problems of determining frictional heating functions in various friction pairs are shown in works [2–5].

Theoretical investigations of friction heat diagnostics in the bearing system on the common shaft were carried out in [6]. A mathematical model of the heat process was used under the assumption of constant thermal properties during determination of friction moments based on temperature data. At the same time, the method of the iterative regularization, used in solution of the inverse problem, has been applied in the case of dependence of the thermal properties on temperature [7]. This has been confirmed by computational experiments conducted during identification of frictional heating in one friction bearing contained in [8]. However, when determining the frictional heating and, consequently, friction moments in the bearings system, solution of the inverse problem is less resistant to errors as concerns temperature data as several parallel functions are determined in one bearing and temperature errors superimpose on the temperature errors in other bearings. In addition, convergence of the successive approximations of the solution significantly reduces. The aforementioned peculiarities of solving inverse heat exchange problems require investigation of heat frictional functions in the bearing system in the case of dependence of materials properties on temperature.

The purpose of the work is to study the theoretical possibility of evaluating the power of friction heat generation in a bearing based on the temperature data using a simplified three-dimensional mathematical model of the thermal process in the form of a one-dimensional heat equation for the shaft and the two-dimensional equations for taking into account dependences of materials' thermal properties on the temperature.

FORMULATION OF PROBLEM

Let us consider a system of N self-lubricating sliding bearings (Fig. 1). Sliding occurs on the contact surface of the polymer bushes with a metal shaft. Contact angles are certain and unchangeable during the whole test period. Let us make assume that the load is evenly distributed along the length of bearings; heat exchange at the ends of the bearings and housings is negligibly low; the temperature distribution is homo-

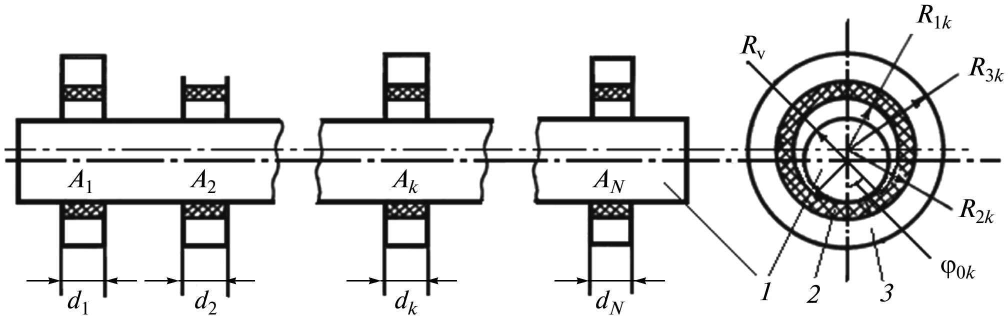

Fig. 1. Scheme of the bearing system on the common shaft. (*1*) Shaft, (*2*) bush, and (*3*) holder.

geneous along the length of bearings; and the shaft speed is above 1 rps, which allows the temperature distribution to be taken over the friction surface and over a rather thin cross-sectional view of the shaft as homogeneous.

Then unsteady temperature field for the shaft, generally with a variable cross section, is a one-dimensional equation, and the bearing elements are described by two-dimensional equations in cylindrical coordinates. With the above assumptions in place, we give a simplified quasi-three-dimensional mathematical thermal model for a bearing system [1]. Heat generation with $Q_k(t)$ power takes place in the A_k contact area of the shaft with the kth bearing.

Temperature distribution $T_k = T_k(r, \varphi, t)$ in the kth bush with a bracket (in the kth bearing) is described by two-dimensional heat generation equations:

$$C_k(T_k)\frac{\partial T_k}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_k(T_k)\frac{\partial T_k}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\varphi}\left(\lambda_k(T_k)\frac{\partial T_k}{\partial\varphi}\right), \quad (1)$$
$$0 < t \le t_m, \quad R_{1k} < r < R_{3k},$$
$$0 < \varphi < \pi, \quad k = 1, 2, \ldots, N.$$

Coefficients $C_k(T_k)$, $\lambda_k(T_k)$ have simple discontinuities at $r = R_{2k}$.

Conditions of the convective transfer are set on free surfaces:

$$\lambda_k(T_k)\frac{\partial T_k(r,\varphi,t)}{\partial r}\bigg|_{r=R_{1k}} = \alpha_{1k}[T_k(R_{1k},\varphi,t) - T_s],$$
$$|\varphi| > \varphi_{0k}; \quad \lambda_k(T_k)\frac{\partial T_k(r,\varphi,t)}{\partial r}\bigg|_{r=R_{3k}} \quad (2)$$
$$= -\alpha_{3k}[T_k(R_{3k},\varphi,t) - T_s], \quad 0 < \varphi < \pi.$$

Due to the symmetry of the temperature fields in the bearings, against vertical lines passing through their axes, the following conditions are true:

$$\frac{\partial T_k(r,\varphi,t)}{\partial\varphi}\bigg|_{\varphi=0} = \frac{\partial T_k(r,\varphi,t)}{\partial\varphi}\bigg|_{\varphi=\pi} = 0, \quad R_{1k} \le r \le R_{3k}. \quad (3)$$

Distribution of the temperature $U = U(z, t)$ in the shaft is described by a one-dimensional equation connected with two-dimensional temperature fields of bearings:

$$C_v(U)S(z)\frac{\partial U}{\partial t} = \frac{\partial}{\partial z}\left[S(z)\lambda_v(U)\frac{\partial U}{\partial z}\right] - P(z)\alpha_v(U - T_a)$$
$$+ \Theta_k(z)\left[\frac{Q_k(t)}{d_k} + 2R_{1k}\int_0^{\varphi_{ok}}\lambda_k(T_k)\frac{\partial T_k}{\partial r}\bigg|_{r=R_{1k}} d\varphi\right], \quad (4)$$
$$U = U(z,t), \quad 0 < z < L, \quad 0 < t \le t_m,$$

where

$$\Theta_k(z) = \begin{cases} 1, & z \in \bigcup_{k=1}^{N} A_k, \\ 0, & z \notin \bigcup_{k=1}^{N} A_k. \end{cases}$$

Let us assume the temperature in bushes within their contact areas as averaged as against shaft temperature of A_k with length d_k because the temperature distribution in the bearings along their length is considered homogeneous:

$$T_k(R_{1k},\varphi,t) = \frac{1}{d_k}\int_{A_k} U(z,t)dz, \quad |\varphi| \le \varphi_{ok}. \quad (5)$$

Conditions of the first and third class are recorded at the ends of the shaft:

$$U(0,t) = U_1(t), \quad (6)$$

$$\lambda_v(U)\frac{\partial U(z,t)}{\partial z}\bigg|_{z=L} = -\alpha_v[U(L,t) - T_s], \quad (7)$$

The initial condition in the friction unit is considered homogeneous:

$$T_k(r,\varphi,0) = U(z,0) = T_0. \quad (8)$$

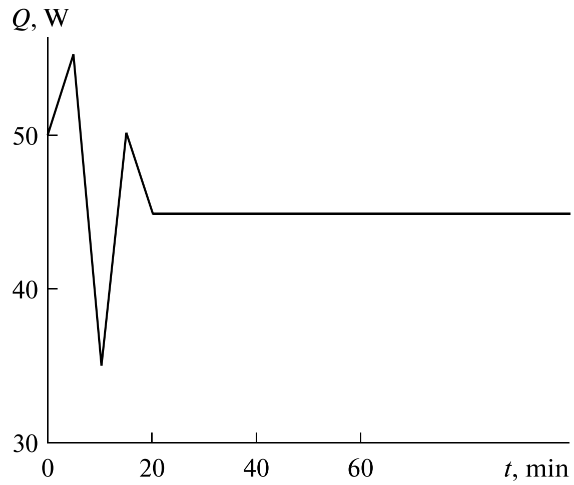

Fig. 2. Dependence of the heat generation friction capacity on time in the system bearings.

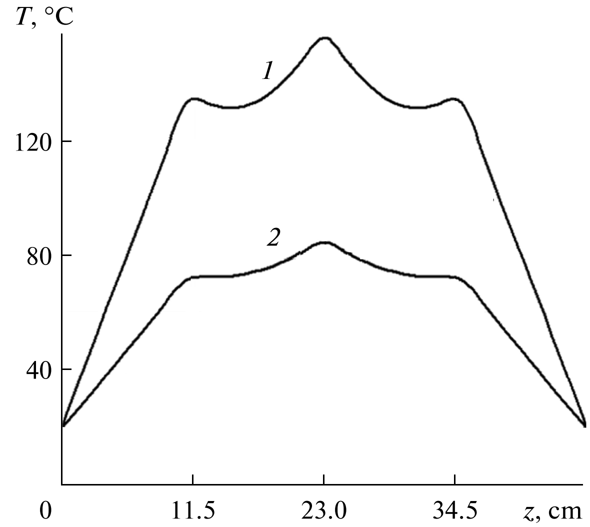

Fig. 3. Temperature distribution along the shaft length: (*1*) at the time moment of 17 min; (*2*) within the period of the set mode.

SOLUTION OF THE DIRECT PROBLEM

At given functions of heat generation capacity $Q_k(t)$, the nonsteady temperature field is determined by solution of the direct problem (1)–(8) by the finite-difference method.

Computational experiment was carried out to investigate the algorithm performance on the basis of the qualitative analysis of the non-steady temperature field in the sliding bearings system made of polymer composite materials. Model problems were solved for the three-bearing system on a common shaft with radius $R_v = 0.0145$ and length $L = 0.46$ m. The distances between bearings and length of the free shaft ends are 0.1 m, and the length of the bearing is $d_k = 0.02$ m, k = 1, 2, 3. The contact angles with shaft bushings are $2\varphi_{0k} = 28°$, $k = 1, 2, 3$.

Steel is the material of the shaft and the bearing race and F4K20-filled PTFE is the material for bushes. The dependences of the thermal and physical properties on temperature for the filled PTFE are as follows:

$$\lambda_k = 0.07(T - 100)/150 + 0.35 \left(\frac{\text{W}}{\text{m °C}}\right),$$

$$C_k = \left[6 \times 10^{-3}(T - 30) + 3\right] \times 10^6 \left(\frac{\text{J}}{\text{m}^3 \text{ °C}}\right),$$

$$k = 1, 2, 3.$$

For steel:

$$\lambda_v = 30.5(T - 100)/150 + 55.5 \left(\frac{\text{W}}{\text{m °C}}\right),$$

$$C_v = \left[1.2 \times 10^{-3}(T - 30) + 3.7\right] \times 10^6 \left(\frac{\text{J}}{\text{m}^3 \text{ °C}}\right).$$

The same conditions of the first class are given at the ends of the shaft, which should ensure symmetry of the temperature fields relative to the middle of the system by setting similar functions of the thermal generation power at extreme bearings.

Let similar thermal generation power resulting from friction act on all three bearings. Dependences are given in Fig. 2. Results of calculations given in Fig. 3 show that the temperature distribution along the shaft is symmetrical against the center. In addition, the temperature values in the second bearing are considerably higher than the temperatures on the neighboring extreme bearings. This is because of the reciprocal influence of the bearings temperature fields. Temperatures of two extreme bearing exert similar influence on the temperature field in the central bearing. The influence of the temperature of the central bearing on the temperature field of one of the extreme bearings is much more prominent that of the temperature of another extreme bearing, off the first one.

Results of the calculations show that the developed algorithm allows evaluating the temperature distribution along the shaft length and takes into account the mutual temperature influence in sliding bearings through the metal shift.

Figure 4 shows solution results of the model problem with simulation of the successive short-term changes of the friction torque (heat generation capacity) in sliding bearings in the system. At the beginning, all heat generation capacities in bearings are the same, equal to 40 W, and remain at this level for 10 min. Over the following 10 min, the heat generation capacity changes in the first bearing, while the heat generation values in the other two bearings stay unchanged. The heat generation

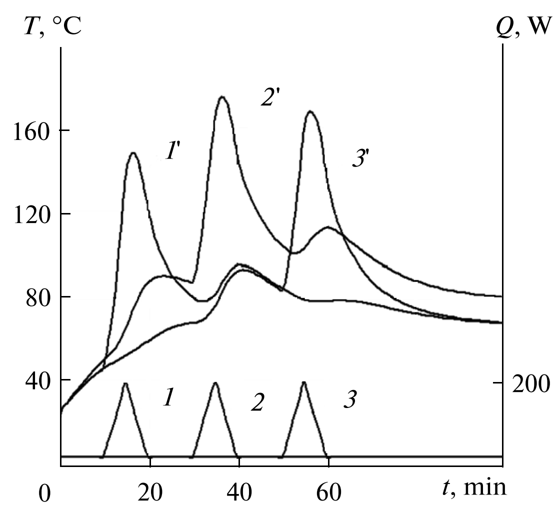

Fig. 4. Time changes of the temperature in bushes (*1'*, *2'*, *3'*) at the distance of 0.2 mm from the friction zone along the load axis at the set dependences of the heat generation capacities (*1*, *2*, *3*) in the contact zones with the corresponding bearings.

capacities in the second and third bearings then change every 10 min alternatively with the same dynamics as in the first bearing. Dependences of time variation of heat generation capacities are shown in Fig. 4.

In the initial 10-min interval, the friction torques are equal on all sliding bearings; however, approximately after 5 min, the temperature in the second bearing becomes higher than the others. The heat generation capacities in sliding bearings have a combined effect on the maximum temperature value found in the system over the whole test interval. As expected, the maximum temperature is higher on the second bearing in spite of the integrally equal heat generation in each bearing.

The temperature lags to decrease in a steady mode in all sliding bearings during reduction of the heat generation in a linear fashion within 5 min and, therefore, it continues to drop further. Reduction proceeds with some delay up to heat generation increase in one of the bearings. In addition, the obtained dependences of the temperature time variation in sliding bearings show that temperature change in one bearing exerts considerable influence on temperature values in tandem bearings. Thus, for example, with an increase of the temperature in the second bearing by 80°C over 30–35 min, the temperature in the first bearing increases approximately by 25°C, while in the third bearing it increases by 35°C. Uneven temperature growth in bearings is connected with the fact that temperature decrease takes place in the first bearing up to the moment of temperature growth and, in the third bearing, the temperature shows a tendency to increase.

The dynamic of temperature change in the first bearing has an effect on the dynamic of temperature change in the third bearing and vice versa. As was expected, the mutual temperature effects of the extreme bearings are weaker than the effect of the temperature dynamic in the central bearing.

The speed of the algorithm and capacity for proper consideration of the characteristics of the heat process allows using it for evaluating the heat generation capacity (friction torque) in the sliding bearing system.

AN ITERATIVE ALGORITH FOR SOLVING THE INVERSE PROBLEM

Let temperature measurements are set at points on a circle with angular coordinates φ_{kj}, $k = 1, 2, \ldots, N$, $j = 1, 2, \ldots M_k$ in the kth sliding bearing bush at fixed radius R_k:

$$T_k(R_k, \varphi_{kj}, t) = f_k(\varphi_{kj}, t). \tag{9}$$

The inverse problem for determining functions of the heat generation capacities $Q(t) = \{Q_1(t), Q_2(t), \ldots, Q_N(t)\}$ based on a priori temperature information (9) is reduced to minimize the residual functional

$$J[Q(t)] = \frac{1}{2}\sum_{k=1}^{N}\sum_{j=1}^{M_k}\int_0^{t_m}[T_k(R_k, \varphi_{kj}, t) - f_k(\varphi_{kj}, t)]^2 dt \tag{10}$$

on solving simultaneous equations (1)–(8). The vector function of heat generation capacity $Q(t)$ acts as control.

In order to solve the inverse problem by the conjugate gradient method, it is necessary to determine the gradient of the residual functional (10). The effective method to determine the gradient is consideration of the conjugate boundary problem [7, 9]. Let us give the vector function $Q(t)$ of the increments $\Delta Q_k(t)$ to all the components; in addition, temperatures $T_k(r, \varphi, t)$ and $U(z, t)$ obtain increments $v_k(r, \varphi, t)$ and $w(z, t)$, respectively. The following system of equations is obtained by taking into account Eqs. (1)–(8) at $Q_k(t)$ and $Q_k(t) + \Delta Q_k(t)$ control for linear parts of the increments $v_k(r, \varphi, t)$ and $w(z, t)$:

$$\frac{\partial(C_k v_k)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(\lambda_k v_k)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2(\lambda_k v_k)}{\partial \varphi^2},$$
$$0 < t \le t_m, \quad R_{1k} < r < R_{3k}, \quad 0 < \varphi < \pi, \quad k = 1, 2, \ldots, N, \tag{11}$$

$$\left.\frac{\partial(\lambda_k v_k)}{\partial r}\right|_{r=R_{1k}} = \alpha_{1k} v_k(R_{1k}, \varphi, t), \quad |\varphi| > \varphi_{ok},$$
$$\left.\frac{\partial(\lambda_k v_k)}{\partial r}\right|_{r=R_{3k}} = -\alpha_{3k} v_k(R_{3k}, \varphi, t), \quad 0 < \varphi < \pi, \tag{12}$$

$$\left.\frac{\partial(\lambda_k v_k)}{\partial \varphi}\right|_{\varphi=0} = \left.\frac{\partial(\lambda_k v_k)}{\partial \varphi}\right|_{\varphi=\pi} = 0, \tag{13}$$

$$S(z)\frac{\partial(C_v w)}{\partial t} = \frac{\partial}{\partial z}\left(S(z)\frac{\partial(\lambda_v w)}{\partial z}\right) - P(z)\alpha_v w + \Theta_k(z)\left[\frac{\Delta Q_k(t)}{d_k} + 2R_{1k}\int_0^{\varphi_{ok}} \left.\frac{\partial(\lambda_k v_k)}{\partial r}\right|_{r=R_{1k}} d\varphi\right], \tag{14}$$

$$w(0,t) = 0, \tag{15}$$

$$\left.\frac{\partial(\lambda_v w)}{\partial z}\right|_{z=L} = -\alpha_v w(L,t), \tag{16}$$

$$v_k(R_{1k},\varphi,t) = \frac{1}{d_k}\int_{A_k} w(z,t)dz, \quad |\varphi| \le \varphi_{ok}, \tag{17}$$

$$v_k(r,\varphi,0) = w(z,0) = 0. \tag{18}$$

The conjugate problem of the relatively unknown $\Psi_k(r, \varphi, t)$ for bearings and $\Phi(z, t)$ for the shaft is obtained by using the Lagrange multiplier method and the condition of a stationary variation problem aiming at minimization of functional (10)

$$-C_k(T_k)\frac{\partial}{\partial t}\left(\frac{\Psi_k}{r}\right) = \frac{\lambda_k(T_k)}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{\Psi_k}{r}\right)\right) + \frac{\lambda_k(T_k)}{r^2}\frac{\partial^2}{\partial \varphi^2}\left(\frac{\Psi_k}{r}\right) + \frac{1}{r}\sum_k^N\sum_{j=1}^{M_k}[T_k(R_k,\varphi_{kj},t) - f_k(\varphi_{kj},t)] \times \bar{\delta}(r-R_k)\bar{\delta}(\varphi-\varphi_{kj}), \quad \bar{\delta}(x) = \begin{cases}1, & x = 0;\\ 0, & x \ne 0,\end{cases} \tag{19}$$

$$\lambda_k(T_k)\frac{\partial}{\partial r}\left.\left(\frac{\Psi_k}{r}\right)\right|_{r=R_{1k}} = \alpha_{1k}\frac{\Psi_k(R_{1k},\varphi,t)}{R_{1k}}, \quad |\varphi| > \varphi_{ok},$$
$$\lambda_k(T_k)\frac{\partial}{\partial r}\left.\left(\frac{\Psi_k}{r}\right)\right|_{r=R_{3k}} = -\alpha_{3k}\frac{\Psi_k(R_{3k},\varphi,t)}{R_{3k}}, \quad 0 < \varphi < \pi, \tag{20}$$

$$\left.\frac{\partial \Psi_k}{\partial \varphi}\right|_{\varphi=0} = \left.\frac{\partial \Psi_k}{\partial \varphi}\right|_{\varphi=\pi} = 0, \tag{21}$$

$$-C_v(U)S(z)\frac{\partial \Phi}{\partial t} = \lambda_v(U)S(z)\frac{\partial^2\Phi}{\partial z^2} - P(z)\alpha_v\Phi + 2\Theta_k(z)R_{1k}\int_0^{\varphi_{ok}}\left[\lambda_k(T_k)\frac{\partial}{\partial r}\left(\frac{\Psi_k}{r}\right)\right]_{r=R_1} d\varphi, \tag{22}$$

$$\Phi = \Phi(z,t), \quad 0 < z < L, \quad 0 < t < t_m,$$

$$\frac{\Psi_k(R_{1k},\varphi,t)}{R_{1k}} = \frac{1}{d_k}\int_{A_k}\Phi(z,t)dz, \quad |\varphi| \le \varphi_{ok}, \tag{23}$$

$$\Phi(0,t) = 0, \tag{24}$$

$$\lambda_v(U)\left.\frac{\partial \Phi}{\partial z}\right|_{z=L} = -\alpha_v\Phi(L,t), \tag{25}$$

$$\Psi_k(r,\varphi,t_m) = \Phi(z,t_m) = 0. \tag{26}$$

The formula for calculating the components of the functional gradient is obtained by solving the conjugate boundary problem (10):

$$J'_k[Q(t)] = \frac{\Psi(R_{1k},0,t)}{2d_k}, \quad k = \overline{1,N}. \tag{27}$$

According to the method of conjugate gradients, the successive approximations $Q_k^n(t)$ for the function $Q_k(t)$ are calculated by the following iteration scheme [10]:

$$Q_k^{n+1}(t) = Q_k^n(t) - \beta_k^n S_k^n(t), \quad n = 0, 1, 2, \ldots, \quad k = \overline{1,N}, \tag{28}$$

$$S_k^n(t) = J'_k[Q^n(t)] + \gamma_k^n S_k^{n-1}(t), \quad \gamma_k^0 = 0,$$
$$\gamma_k^n = \frac{\int_0^{t_m}\left(J'_k[Q^n(t)]\right)^2 dt}{\int_0^{t_m}\left(J'_k[Q^{n-1}(t)]\right)^2 dt}. \tag{29}$$

Initial approximations $Q_k^0(t)$ are set on a random basis. Compared to the standard method of the conjugate gradients, when the descent step is the same for all descent directions, the step vector is determined on the basis of the target functional minimum [11]. Descent steps β_k^n are determined on the basis of the minimum residual functional at the next iteration step:

$$J[Q^{n+1}(t)] = \min_{\beta>0}\left\{\frac{1}{2}\sum_{k=1}^N\sum_{j=1}^{M_k}\int_0^{t_m}[T_k^n(R_k,\varphi_{kj},t) - f(\varphi_{kj},t) - v_k(R_k,\varphi_{kj},t)\beta_k^n]^2 dt\right\}. \tag{30}$$

Due to the problem linearity, in the increments (11)–(18), its solution may be presented in the form of a sum:

$$v_k(r,\varphi,t) = \sum_{i=1}^N v_{ki}(r,\varphi,t), \quad k = \overline{1,N}, \tag{31}$$

where $v_{ki}(r, \varphi, t)$ is the temperature increment in the kth bearing calculated under the influence of the heat capacity increment $\Delta Q_i(t) \ne 0$ in the shaft contact zone with the ith bearing, whereas the corresponding increments are zero in other bearings. Decomposition of the temperature increment in the form of (31) allows taking into account mutual dynamic influences of the bearings' temperature fields during calculation of the descent steps β_k^n. During decomposition of all increments $v_k(r, \varphi, t)$ in the form of (31), the descent steps $\beta_k^n, k = \overline{1,N}$ are determined by the solution of linear equations system being the condition of functional minimum (30):

$$\beta_1^n \sum_{k=1}^{N} \sum_{j=1}^{M_k} \int_0^{t_m} \nu_{ki}(R_k, \varphi_{kj}, t) \nu_{k1}(R_k, \varphi_{kj}, t) dt$$

$$+ \beta_2^n \sum_{k=1}^{N} \sum_{j=1}^{M_k} \int_0^{t_m} \nu_{ki}(R_k, \varphi_{kj}, t) \nu_{k2}(R_k, \varphi_{kj}, t) dt$$

$$+ \ldots + \beta_N^n \sum_{k=1}^{N} \sum_{j=1}^{M_k} \int_0^{t_m} \nu_{ki}(R_k, \varphi_{kj}, t) \nu_{kN}(R_k, \varphi_{kj}, t) dt \quad (32)$$

$$= \sum_{k=1}^{N} \sum_{j=1}^{M_k} \int_0^{t_m} [T_k^n(R_k, \varphi_{kj}, t) - f(\varphi_{kj}, t)] \nu_{ki}(R_k, \varphi_{kj}, t) dt,$$

$$i = \overline{1, N}.$$

The algorithm of the successive approximations to solve the boundary inverse problem based on determination of the vector function of the heat generation capacity $Q(t)$ in the sliding bearing results in the following steps:

1. Using the known nth approximation $Q_k^n(t)$ of the $Q(t)$ function, the direct problem (1)–(8) is solved;

2. The conjugated boundary problem (11)–(18) is solved by substitution of the temperature values and components $J'_k[Q^n(t)]$of the residual functional gradient are calculated by formula (27).

3. Assuming that the increments of the heat capacities functions equal the corresponding components of the functional gradient, i.e., $\Delta Q_k(t) = J'_k[Q^n(t)]$, N of the boundary problems are solved successively for increments (19)–(26) relative to their components $\nu_{ki}(r, \varphi, t)$ at decomposition (31).

4. Descent steps $\beta_k^n, k = \overline{1, N}$ are determined by solving system (32).

5. The next approximation $Q_k^{n+1}(t)$ I s determined by Eqs. (28) and (29).

6. The process of solution refinement is stopped after meeting the iteration regularization conditions [5]:

$$J[Q(t)] < \delta^2.$$

COMPUTATIONAL EXPERIMENTS

A system consisting of three bearings on a common shaft with a radius of 0.0125 m and a length of 0.2 m was considered. The distance between bearings was 0.05 m. At such a small distance between bearings, the mutual influence of their temperature fields will be more significant than for the case considered for the solution of the direct problem. The geometrical dimensions of bushes and holders were considered to be the same: $R_{1k} = 0.013$, $R_{2k} = 0.016$, $R_{3k} = 0.029$ m. Bearing lengths are $d_k = 0.02$ m, $k = 1, 2, 3$. The contact angles of bushes with the shaft were $2\varphi_{0k} = 24°$, $k = 1, 2, 3$. The dependences of the thermal properties on temperature for the filled PTFE are the same as for the direct problem solution.

Boundary problems were solved by the finite-difference method during solution of the inverse problem for determination of frictional function of the heat generation in the bearing system.

Computational experiments investigate the influence of computational errors and temperature measurement errors on determination of the heat generation capacities in the bearing system. Those computational experiments were developed in the following way. Model functions of the heat generation capacities Q_{f1}, Q_{f2}, and Q_{f3}were set and the direct problem (1)–(8)was solved. Results of the calculated temperature at fixed points in each bush were stored and used as the measured temperature data for solving the inverse problem determining the heat generation capacities. Computational experiments were carried out by varying locations of the measuring points by radius, angle and by varying of their amount. Calculations show that the quality of determination of the heat generation capacities deteriorates with distance of the measuring point from the friction area both by radius and angle. It was revealed that temperature measuring in one point located as close as possible to the friction area of each bearing is sufficient for the qualitative evaluation of the heat generation capacities. Functions of the heat generation capacities are not restored by the temperature data obtained in the points located at the boundary of bushes with holders or in the holders' points.

The same functions of the heat generation capacities were set for all calculations in extreme bearings ($Q_{f1} = Q_{f3}$). Two points were taken in each bearing as the measuring point: $R_1 = R_2 = R_3 = 0.0125$ m, $\varphi_{11} = \varphi_{21} = \varphi_{31} = 0°$, $\varphi_{12} = \varphi_{22} = \varphi_{32} = 9°$. Figure 5a shows the results of a numerical experiment with precise temperature data. Results of calculations show high accuracy of the determination of the functions of heat generation capacities and the possibility of stopping the iteration process of the refinement of a solution based on the "sticking" condition of the successive approximation.

The precise temperature data perturbed in the following way during stability investigation of the problem for evaluation of the heat generation capacities toward the temperature errors:

$$\overline{f_k}(\varphi_j, t) = f_k(\varphi_{kj}, t) + 2\Delta(\sigma(t) - 0.5),$$

$$0 \le t \le t_m, \quad k = \overline{1, N},$$

where $\sigma(t)$ is a random function evenly distributed on the segment [0, 1] used for solving the inverse problem of heat generation.

Figure 5 shows results concerning evaluation of the heat generation capacities at different error levels Δ.

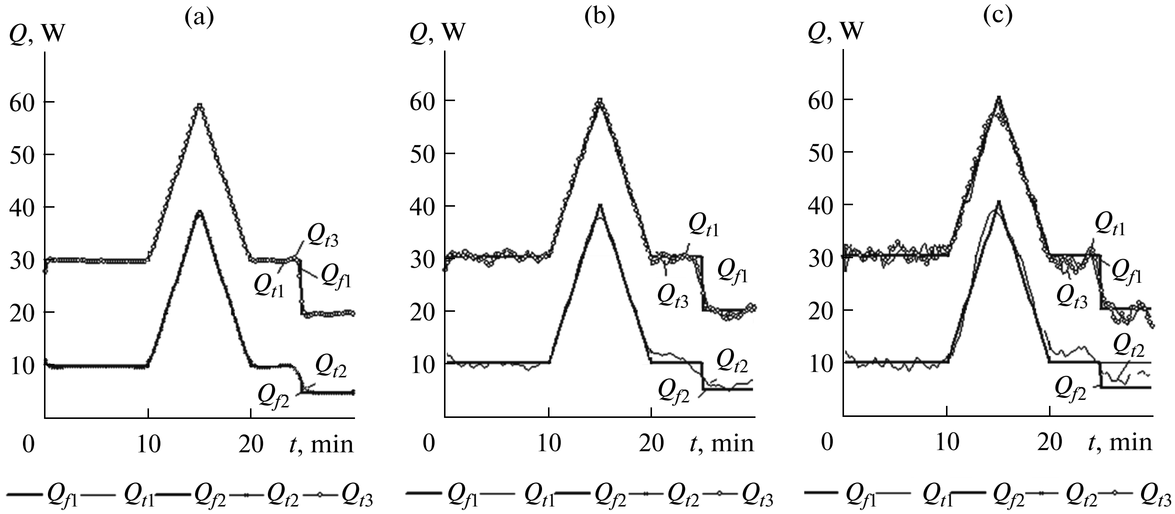

Fig. 5. Evaluation of the heat generation capacities in the system of three bearings on the common shaft: (a) with precise temperature data; (b and c) with perturbed temperature data, respectively, with error levels Δ equal 1 and 3°C. Q_{f1}–Q_{f3} ($Q_{f1} = Q_{f3}$) are model functions; Q_{t1}–Q_{t3} are calculated functions of the heat generation capacities.

Calculations show that evaluation precision of the heat generation functions is comparable with the accuracy of the temperature data settings. Calculations showed that, with an increase of error level Δ, the number of iterations, required for getting solution, decreased. Thus, for example, at the error level of $\Delta = 1°C$ the number of iterations is 32. At $\Delta = 3°C$ the iteration process stops at the 22nd iteration. Such behavior of the iteration process is explained by the different levels of the allowable residual δ^2. In the first case, iterations cease at a residual value of 0.08. In the second case, iterations cease at a residual value of 0.75, which is obtained for a smaller number of iterations.

CONCLUSIONS

Computational experiments show that a simplified three-dimensional mathematical model of the thermal process in the sliding bearing system on the common shaft proposed for thermal friction diagnostics effectively takes into account the mutual influence of the thermal field dynamics in bearings.

An algorithm for evaluation of the heat generation capacities suitable for practical application and correspondingly the friction moments in the bearing system on the common shaft based on temperature data using mathematical models taking into account dependences of the thermal and physical properties of materials on temperature was developed.

NOTATION

A_k, $k = 1, 2, ..., N$	shaft contact zone with the kth bearing
$Q_k(t)$	function of the heat generation capacity from friction in the shaft contact zone with the kth bearing
R_{1k}, R_{1k}, R_{1k}	radii of the elements of the kth bearing
$T_k = T_k(r, \varphi, t)$	temperature of the kth bearing
r, φ	cylindrical coordinates
t, t_m	current and calculated time
$C_k(T_k)$, $\lambda_k(T_k)$	coefficients of the specific thermal capacity and thermal conductivity of the kth bearing
α_{1k}, α_{3k}	coefficients of the convective transfer from the internal surface of the bush and from the holder external surface of the kth bearing
$U = U(z, t)$	the shaft temperature
z	coordinate along the shaft axis
$C_v(U)$, $\lambda_v(U)$	coefficients of the specific thermal capacity and thermal conductivity of the shaft
$S(z)$, $P(z)$	area and perimeter of the cross section of the shaft
L	is the shaft length
$2\varphi_{0k}$	angle of the shaft contact with the bush of the kth bearing
α_v	coefficient of the convective transfer with the external surface of the shaft
v_k	temperature increment of the kth bearing
w	increment of the shaft temperature

$\Psi_k(r, \varphi, t)$	variable of the conjugate problem for the kth bearing
$\Phi(z, t)$	variable of the conjugate problem for the shaft
J, J', J'_k	residual functional, its gradient and gradient component
f_k	measured temperature in the bush of the kth bearing
δ^2	residual allowable level
Δ	error level in temperature data

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