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Effect of Dimensions of Pad and Disk on the Temperature and Duration of Braking

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Abstract—A solution to the axisymmetric nonstationary thermal problem of friction for a disk–pad tribosystem has been obtained using the finite element method taking into account the interrelation between the temperature and velocity of sliding in the course of braking. The calculations are carried out for an FMK-11 ceramics pad and a ChNMKh cast iron disk at the coefficient of friction that depends on the temperature. The effect of the dimensions of the pad and disk on the temperature and duration of braking is studied provided that the volumes of the pad and disk remain unchanged.

Keywords: frictional heating, temperature, braking, finite element method

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INTRODUCTION

Mathematical models of nonstationary heat generation during braking, which are based on analytical solutions of one-dimensional thermal problems of friction, are presented in monographs [1–4]. An advantage of this modeling is that it yields to exact formulas for the temperature of friction contact, while its failure is that real dimensions of the friction elements are difficult to account for. This drawback can be overcome using numerical methods, in particular the finite element method (FEM). A review of works devoted to the use of the FEM in solving frictional heat problems for a disk–pad system is presented in [5]. Depending on some input parameters of the tribosystem, such as the initial velocity of braking, the relative dimensions of the pad and disk, and the coefficient of mutual overlap, the temperature is determined using 2D (axisymmetric) [6–8] or 3D (spatial) [9] FEM calculation models.

One of the most important output parameters in modeling the frictional heating of brakes is the duration of braking, which is determined from the solution of the equation of motion. In the majority of numerical models, which are related to frictional heat generation in braking systems, the equation of motion is first integrated to find the variation in the velocity in the course of braking and then a functional equation (nonlinear in the general case) is derived from the condition of stoppage for determining the duration of braking. Subsequently, the latter characteristic is used as an input parameter when solving the corresponding boundary heat-conduction problem [10, 11].

The mutual effect of the velocity and the temperature during braking was considered when constructing numerical-analytical solutions of the system of equation of friction thermal dynamics. We got sight of the fact that the coefficient of friction is the parameter that relates the equation of motion (the force of friction) to the thermal problem of friction (the specific power of friction). Since the temperature dependence of the coefficient of friction should be taken into account in studying thermal conditions of the operation of heavy-duty brakes, variations in the velocity of sliding and the temperature in the course of braking are interrelated. Using this idea, we proposed the axisymmetric FEM-based calculation model for determining the temperature and duration of braking [12].

The aim of this work was to study the effect of the design dimensions of the friction elements of a disk–pad braking system on the maximum temperature and the duration of braking with account for the temperature dependence of the coefficient of friction.

PROBLEM FORMULATION

Let a four-wheel vehicle with mass m move with the constant linear velocity V_0. Each wheel of the vehicle is equipped with a disk–pad braking system that includes two pads, each of which has the thickness δ_p and is in contact with the two working surfaces of the disk with thickness $2\delta_d$. At a moment of time assumed to be the initial moment of time $t = 0$, the stationary pads are pressed to the working surfaces of the rotating disk by the constant pressure p_0. Because of the friction, the mechanical energy of the system transforms to heat, which is accompanied by a decrease in the velocity and the heating of the friction elements. At the moment of time $t = t_s$, the vehicle stops. Here, we make the following assumptions:

(1) The coefficient of mutual overlap in the disk–pad pair is equal to 1.

(2) The total force of friction is uniformly distributed over the braking systems.

(3) The materials of the friction elements are homogeneous and have constant thermal characteristics.

(4) The coefficient of friction f depends on the temperature T.

(5) The thermal contact of each pad with the disc is perfect; i.e., the temperatures of the working surfaces of the pad and disk are the same, and the sum of the heat flux densities that are directed inside each component in normal direction to these surfaces is equal to the specific power of friction.

(6) On the free surfaces of the pads and the disk, convective heat transfer to the environment occurs with the constant heat transfer coefficient h.

Taking into account these assumptions and because of the geometric symmetry of the contact between the pads and the disk, to determine the temperature field, it is sufficient to consider the following axisymmetric (in the cylindrical system of coordinates r, θ, z) thermal problem of friction for the system that consists of a pad and a disk with the thickness δ_d that has the adiabatic end surface $z = -\delta_d$ as follows (Fig. 1):

$$K_p\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)=\rho_p c_p\frac{\partial T}{\partial t},\quad r_p<r<R_p,\ 0<z<\delta_p,\ 0<t\le t_s, \tag{1}$$

$$K_d\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)=\rho_d c_d\frac{\partial T}{\partial t},\quad r_d<r<R_d,\ -\delta_d<z<0,\ 0<t\le t_s, \tag{2}$$

$$K_d\left.\frac{\partial T}{\partial z}\right|_{z=0^-}-K_p\left.\frac{\partial T}{\partial z}\right|_{z=0^+}=q(r,t),\quad r_p\le r\le R_p,\ 0\le t\le t_s, \tag{3}$$

$$T(r,0^+,t)=T(r,0^-,t),\quad r_p\le r\le R_p,\ 0\le t\le t_s, \tag{4}$$

$$K_p\left.\frac{\partial T}{\partial z}\right|_{z=\delta_p}=h[T_a-T(r,\delta_p,t)],\quad r_p\le r\le R_p,\ 0\le t\le t_s, \tag{5}$$

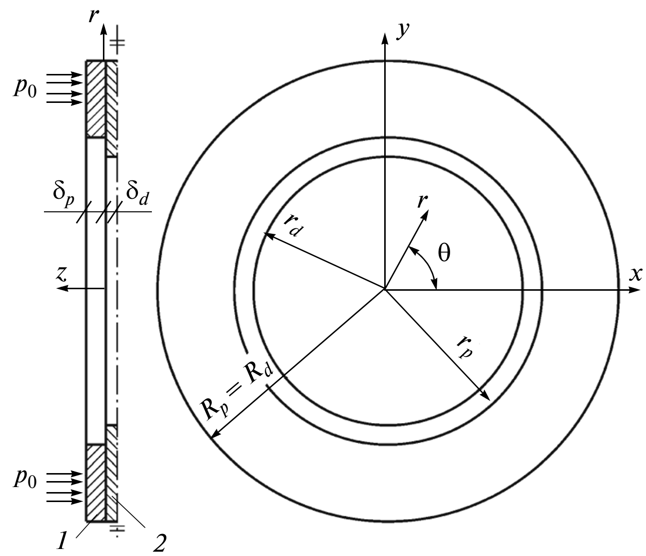

Fig. 1. Schematic of disk–pad friction pair: (*1*) pad and (*2*) disk.

$$K_p\left.\frac{\partial T}{\partial r}\right|_{r=r_p}=h[T(r_p,z,t)-T_a],\quad 0\le z\le\delta_p,\ 0\le t\le t_s, \tag{6}$$

$$K_p\left.\frac{\partial T}{\partial r}\right|_{r=R_p}=h[T_a-T(R_p,z,t)],\quad 0\le z\le\delta_p,\ 0\le t\le t_s, \tag{7}$$

$$K_d\left.\frac{\partial T}{\partial z}\right|_{z=0^-}=h[T_a-T(r,0^-,t)],\quad r_d\le r\le r_p,\ 0\le t\le t_s, \tag{8}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=-\delta_d}=0,\quad r_d\le r\le R_d,\ 0\le t\le t_s, \tag{9}$$

$$K_d\left.\frac{\partial T}{\partial r}\right|_{r=r_d}=h[T(r_d,z,t)-T_a],\quad -\delta_d\le z\le 0,\ 0\le t\le t_s, \tag{10}$$

$$K_d\left.\frac{\partial T}{\partial r}\right|_{r=R_d}=h[T_a-T(R_d,z,t)],\quad -\delta_d\le z\le 0,\ 0\le t\le t_s, \tag{11}$$

$$T(r,z,0)=T_a,\quad r_p\le r\le R_p,\ 0\le z\le\delta_p, \tag{12}$$

$$T(r,z,0)=T_a,\quad r_d\le r\le R_d,\ -\delta_d\le z\le 0. \tag{13}$$

The specific power of friction in boundary condition (3) is as follows [21]:

$$q(r,t)=f[T(r,0,t)]p_0 rR_w^{-1}V(t),\quad r_p\le r\le R_p,\ 0\le t\le t_s, \tag{14}$$

Table 1. Dimensions of pad and disk

Variant no.	pad			disk		
	R_p, mm	r_p, mm	δ_p, mm	R_d, mm	r_d, mm	δ_d, mm
1	99.5	62.5	11.7	99.5	52	6.5
2	106.5	69.5	10.8	106.5	59	6.0
3	113.5	76.5	10.0	113.5	66	5.5
4	120.5	83.5	9.3	120.5	73	5.1
5	127.5	90.5	8.7	127.5	80	4.8

Table 2. Thermal characteristics of materials

Characteristic	FMK-11	ChNMKh
K, W/(m K)	34.3	51
c, J/(kg K)	500	500
ρ, kg/m^3	4700	7100

and variations in the velocity $V(t)$ of the vehicle in the course of braking is found by solving the following equation of motion:

$$m\frac{dV(t)}{dt} = -Af(t), \quad 0 < t \le t_s, \tag{15}$$

where

$$A = \frac{8p_0}{R_w}, \quad f(t) = \int_0^{2\pi}\int_{r_p}^{R_p} f[T(r,0,t)]r^2 dr d\theta, \text{ and} \tag{16}$$

$$V(0) = V_0. \tag{17}$$

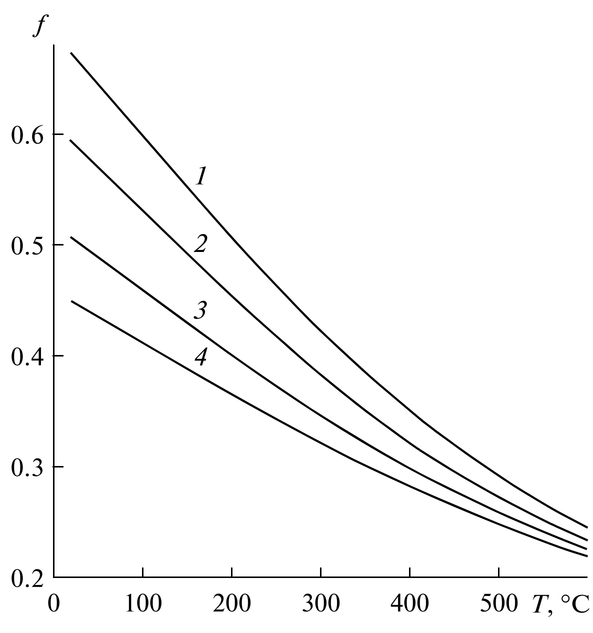

Fig. 2. Temperature dependences of coefficient of friction f of FMK-11–ChNMKh pair at four different contact pressures: (*1*) p_0 = 0.59 MPa; (*2*) 0.78; (*3*) 1.18; and (*4*) p_0 = 1.47 MPa.

The integration of Eq. (15) taking into account formula (16) and initial condition (17) yields

$$V(t) = V_0 - Am^{-1}F(t), \quad F(t) = \int_0^t f(t)d\tau, \quad 0 \le t \le t_s. \tag{18}$$

Under the condition $V(t_s) = 0$, the following functional equation for determining the duration of braking t_s can be derived from solution (18):

$$F(t_s) = A^{-1}mV_0. \tag{19}$$

Thus, we have formulated the mathematical model of the process of frictional heating in the course of braking in which the temperature, the velocity, and the duration of braking are interrelated. The connecting element of the model is the joint consideration of the temperature dependence of the coefficient of friction both in boundary condition (3) of boundary heat-conduction problem (1)–(14) and in equation of motion (15)–(17).

NUMERICAL SOLUTION AND ANALYSIS

The solution of boundary heat-conduction problem (1)–(14) was obtained by the FEM using the Comsol Multiphysics 4.4 software package [13]. The area occupied by the pad was divided into 440 axisymmetric quadrilateral elements and the area occupied by the disk was divided into 280 elements, which resulted in 2997 degrees of freedom. Taking into account the fairly small difference in the dimensions of the pad and the disk for all five variants considered below, the number of elements in the radial (22 elements for the pad and 28 elements for the disk) and the axial (20 elements for the pad and 10 elements for the disk) directions were retained unchanged. From the viewpoint of the accuracy of the calculations, the consideration of the maximum temperature gradients along the normal to the friction surface was also significant. Therefore, the axial size of the finite elements was increased with increasing their distance from the friction surface; the ratio of the smallest to the largest size was retained unchanged and equal to 0.2.

The frictional heating of the disk–pad system was studied for the case of the single braking of a vehicle with the mass m = 5671.9 kg, which had wheels with the radius R_w = 0.314 m and moved with the velocity V_0 = 100 km/h. It was assumed that the initial temperatures of the pad and the disk were equal to the ambient temperature of T_a = 20°C and the heat transfer coefficient was constant and equal to h = 60 W/(m^2 K). The calculations were carried out for an FMK-11 ceramics pad and a ChNMKh cast iron disk. Five variants of the dimensions of the pad and the disk were considered (Table 1) provided that the volumes of the pad and the disk remained unchanged. The thermal characteristics of the materials of the pad and the disk are presented in Table 2 [2]. Figure 2 shows the temperature dependences of the coefficient of friction plotted for the

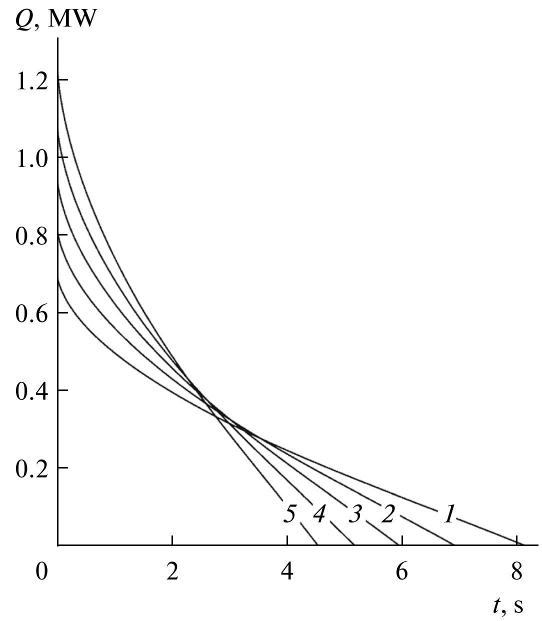

Fig. 3. Variations in power of friction Q in the course of braking at $p_0 = 1.47$ MPa and different outer radii of disk: (*1*) $R_d = 99.5$ mm; (*2*) 106.5; (*3*) 113.5; (*4*) 120.5; and (*5*) $R_d = 127.5$ mm.

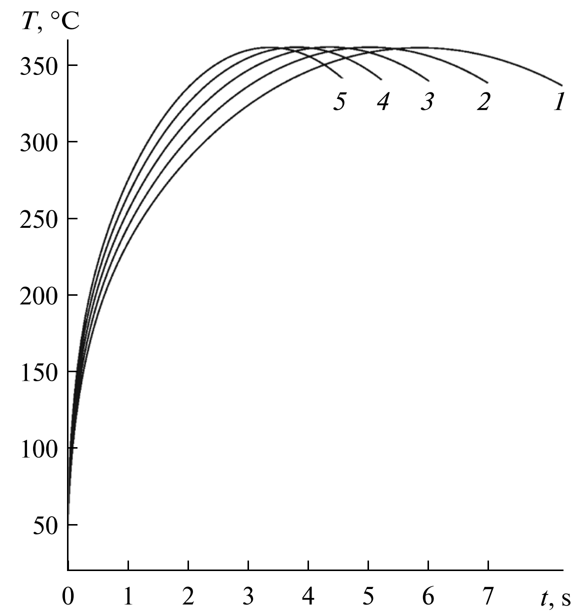

Fig. 4. Time dependences of temperature T on surface of disk–pad contact at $p_0 = 1.47$ MPa and different outer radii of disk: (*1*) $R_d = 99.5$ mm; (*2*) 106.5; (*3*) 113.5; (*4*) 120.5; and (*5*) $R_d = 127.5$ mm.

FMK-11/ChNMKh friction pair based on the obtained experimental data at four values of the contact pressure.

A comparative analysis of the temperature and the duration of braking for the different dimensions of the pad and the disk was carried out at the constant power of friction $Q(t) = 2\pi\int_{r_p}^{R_p} rq(r,t)dr$, where the integrand $q(r,t)$ was calculated using formula (14). Figure 3 shows variations in $Q(t)$ in the course of braking for different values of the outer radius of the disk at the pressure $p_0 = 1.47$ MPa. It can be seen that the areas between each curve and the time axis are the same.

Since the maximum value of specific power of friction (14) is achieved at the outer radius $r = R_d = R_p$ of the ring zone of contact, the temperature in this zone has also the maximum value. Figure 4 shows time variations in this temperature in the course of braking for the five configurations under study at the used maximum contact pressure $p_0 = 1.47$ MPa. The maximum value of the temperature $T_{\max} = 361.6$°C is obtained for the disk having the largest outer radius $R_d = 0.1275$ m and the smallest thickness $\delta_d = 0.0048$ m. In this case, the duration of braking is the shortest ($t_s = 4.57$ s). A decrease in the outer radius of the disk and an increase in its thickness lead to a decrease in the temperature in the zone of contact and to a rise in the duration of braking ($T_{\max} = 325.3$°C and $t_s = 8.2$ s at $R_d = 0.0995$ m and $\delta_d = 0.0065$ m). This is more pronounced in Fig. 5, which shows the dependences of the maximum temperature on the outer radius of the disk at four values of the contact pressure. It can be seen that, at a constant value of the outer radius of the disk, the duration of braking increases as the pressure and, hence, temperature of the friction surface decreases.

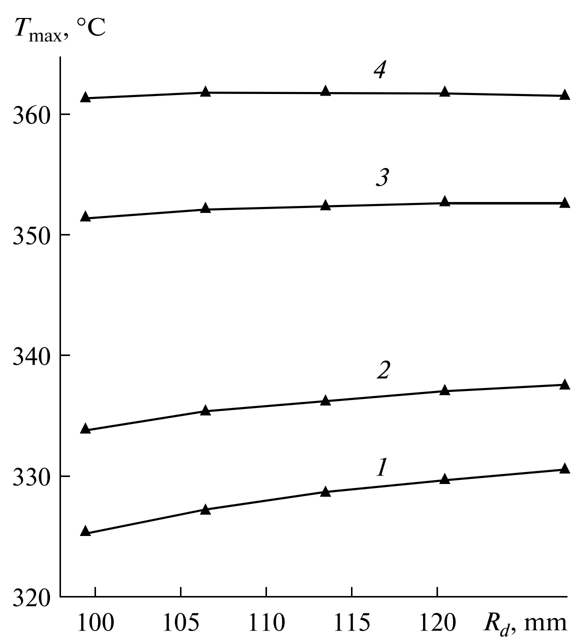

Fig. 5. Dependences of maximum temperature on outer radius R_d of disk at different contact pressures: (*1*) $p_0 = 0.59$ MPa; (*2*) 0.78; (*3*) 1.18; and (*4*) $p_0 = 1.47$ MPa.

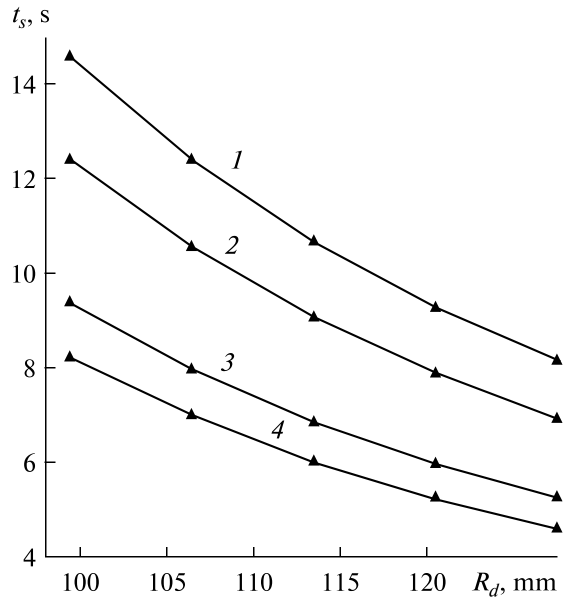

Fig. 6. Dependences of duration of braking t_s on outer radius R_d of disk at different contact pressures: (*1*) $p_0 = 0.59$ MPa; (*2*) 0.78; (*3*) 1.18; and (*4*) $p_0 = 1.47$ MPa.

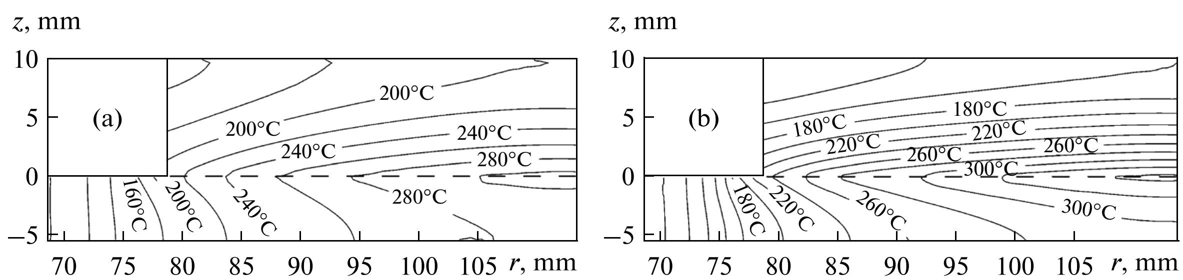

Fig. 7. Isotherms (r, z) at moment of time $t = 0.5t_s$ at $R_d = 113.5$ mm and two different contact pressures: (a) $p_0 = 0.59$ MPa and (b) $p_0 = 1.47$ MPa.

Figure 6 shows the dependences of the duration of braking on the outer radius of the disk, which correspond to the time dependences of the temperature presented in Fig. 4. At a specified pressure, the duration of braking decreases with increasing dimensions of the disk. On the contrary, at constant dimensions of the disk, the duration of braking decreases as the pressure increases.

Figure 7 shows the isotherms of the temperatures of the pad and the disk at the constant time $t = 0.5t_s$, which is approximately equal to the time of attaining the maximum temperature. As follows from boundary condition (4), the temperatures of the pad and the disk on the surface of contact are the same. With increasing distance from this surface, the temperature decreases. An increase in the pressure and, hence, the power of heat generation leads to a rise in the temperature, which is especially noticeable in the vicinity of the friction surface.

CONCLUSIONS

The thermal problem of friction for the disk–pad system is formulated, which takes into account the mutual effect of the velocity and the temperature in the course of friction via the temperature dependence of the coefficient of friction. The solution of the problem is obtained for the FMK-11 metal-ceramic pad–the ChNMKh cast iron disk friction pair using FEM. The effect of the relative dimensions of the pad and the disk on the temperature and the duration of braking is studied provided that the masses of the pad and the disk remain unchanged. It has been found that at the contact pressure that varies in the range 0.59 – 1.47 MPa the increase in the outer radius of the disk from 0.0995 to 0.1275 m (by 28.1%) leads to the decrease in the duration of braking by 44%, while the maximum temperature rises only slightly. The largest increase in the maximum temperature (1.6%) has been found at the minimum pressure $p_0 = 0.59$ MPa.

Thus, during braking under conditions that are close to the conditions we consider, including the input parameters of braking, as well as the dimensions and the thermal characteristics of the friction elements, changes in the relative dimensions of the pad and the disk will not lead to a substantial increase in the maximum temperature and its gradient in the axial direction provided that the masses of the pad and the disk remain unchanged; rather, it will favor a decrease in the duration of braking and, hence, the braking distance.

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NOTATION

c—specific heat; f—coefficient of friction; h—heat transfer coefficient; K—thermal conductivity; m—mass; p_0—pressure; Q—power of friction; q—specific power of friction; r—radial coordinate; R_w—radius of wheel; t—time; t_s—duration of braking; T—temperature; T_a—initial temperature; V—velocity; V_0—initial velocity; z—axial coordinate; δ—thickness; θ—angular coordinate; and ρ—specific density; subscript p denotes pad and subscript d denotes disk.

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