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Modeling of Thermal Processes in the Bearing System on a Common Shaft Taking into Consideration Its Rotational Speed

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Abstract—A mathematical model of the thermal process in the bearing system on a common shaft is given that represents the system of two- and three-dimensional of heat equation with convection term taking into account the rotation of the shaft. The results of determining the time step in the numerical solution of the problem are presented, the calculated and experimental temperature data are compared, and studies of the mutual influence of temperature fields in the bearing system and the determination of the rotational speed above which the mathematical model can be simplified are determined.

Keywords: sliding bearing, the mathematical model, friction, thermal conductivity, temperature, heat equation, numerical solution

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INTRODUCTION

The existing methods of directly measuring the frictional moment apply special elastic elements [1, 2]. The application of these methods at friction conjunctions of test benches and real machines is highly difficult. Thus, it is necessary to determine the work spent by friction based on other values. The most convenient values are the temperature data, since they require no complex or bulky measuring equipment. Temperature registration near the friction zone, which develops an adequate mathematical model of the conjunction of thermal processes and can be used to solve the corresponding inverse boundary problem, yields data on the rate of specific heat generation (heat flow) under friction. It is known that the majority of mechanical work at friction is converted into heat while the heat consumption of other components is insignificant compared to the amount of heat generated [3]. Thus, the value of the rate of specific heat generation under friction is approximately equal to the powers of specific friction and can be used to determine the frictional moment. This method is called *heat-friction diagnostics* [4].

The problem of conducting direct measurements of frictional moment is even more complicated for friction systems, in particular for systems of bearings on a common shaft (Fig. 1). Measurements of frictional moments at each sliding bearing using strain gauges or other devices are both difficult and inaccurate.

Heat-friction diagnostics for a bearing system presupposing a uniform temperature distribution within the cross section of the shaft due to rather high rotational speeds (more than 5 rad/s) and regarding the shaft as a one-dimensional rod is studied in the works [2–7]. In these cases, the coefficient of heat transfer in a heat-transfer model takes into account the rotational speed. The rotational speed of the shaft in sliding bearings, at which calculations of temperature fields must take into account the rotation of the shaft, are determined in [8]. In work [9], heat-friction diagnostics that take into account the influence of the rotation of the shaft on the temperature field is studied for one sliding bearing in the case of a plane.

Successful heat-friction diagnostics is greatly determined by the adequate mathematical model of the thermal process in the studied friction conjunction. This paper considers the problem of determining temperature-field dynamics in a bearing system on a common shaft with a low rotational speed, which requires that the rotation of the shaft be taken into account.

MODEL OF HEAT PROCESSES

The mathematical model of thermal processes in a bearing system on a common shaft is a generalization of the one-bearing model. To obtain convenient formulas for calculating the temperature field in a sliding bearing, it usually accepted that the coefficient of

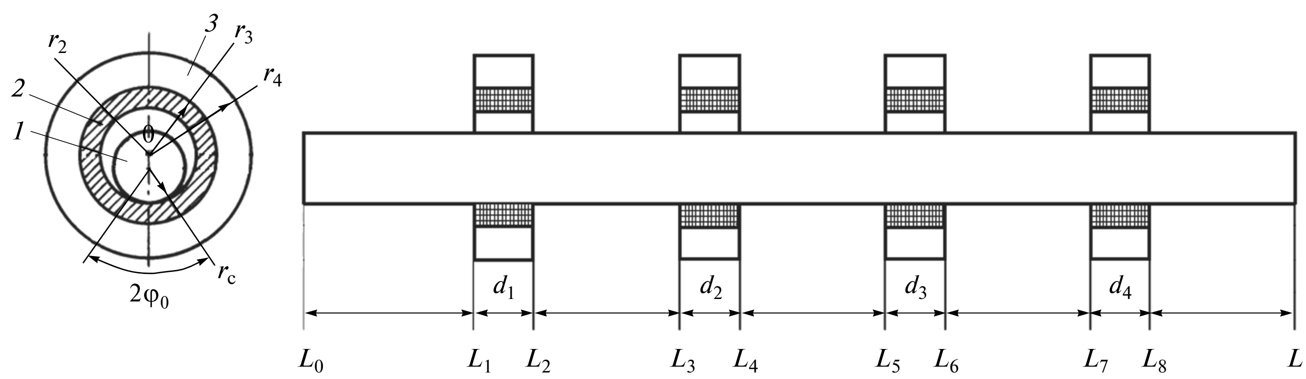

Fig. 1. Calculation scheme for a system of sliding bearings: (*1*) shaft; (*2*) sleeve; (*3*) race.

heat-flow separation on the contact surface of a shaft-bearing is constant. Under these conditions, the heat-transfer problem is considered in [10–12].

A picture of heat distribution in a bearing can only be adequate if the spatial distribution of friction heat is taken into account. Heat-friction diagnostics using a full three-dimensional model of thermal process needs a preset temperature on a plane near the friction zone, which is almost impossible. Thus, quasi-three-dimensional mathematical model can be developed.

The proposed model is based on supposition that temperature distribution in uniform along the bearing and the body, for heat transfer at their end surfaces is insignificant. Calculations using full three-dimensional heat transfer model in a sliding bearing show that convectional heatsink from end surfaces of sleeve and race is less than 0.5% of heat release rate at friction. Thus, the sleeve and the body can be regarded as plane while the shaft is considered to be three-dimensional.

The nonstationary temperature field in the bearings is described by two-dimensional quasilinear heat equations for sleeve and bodies:

$$C_{ik}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_{ik}\frac{\partial T_k}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(\lambda_{ik}\frac{\partial T_k}{\partial \varphi}\right),$$
$$R_{2k} < r < R_{4k}, \quad -\pi < \varphi < \pi, \quad 0 < t \le t_m,$$
$$k = 1\ldots N, \quad i = 2,3. \tag{1}$$

For the shaft, this field is described by the following three-dimensional equation with a convection term to take into account the rotation of the shaft:

$$C_1\frac{\partial U}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_1\frac{\partial U}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(\lambda_1\frac{\partial U}{\partial \varphi}\right)$$
$$+\Omega C_1\frac{\partial U}{\partial \varphi} + \frac{\partial}{\partial z}\left(\lambda_1\frac{\partial U}{\partial z}\right), \tag{2}$$
$$0 < r < R_1, \quad -\pi < \varphi < \pi, \quad 0 < t \le t_m.$$

In shaft-sleeve friction zones, the conditions of heat release at friction are set as follows:

$$\lambda_1\frac{\partial U(r,\varphi,z,t)}{\partial r}\bigg|_{r=R_1} - \lambda_2\frac{\partial T_k(r,\varphi,t)}{\partial r}\bigg|_{r=R_{2k}}$$
$$= Q_k(\varphi,t), \quad |\varphi| \le \varphi_0, \tag{3}$$

$$\frac{1}{d_k}\int_{z_{k-1}}^{z_k} U(R_1,\varphi,z,t)dz = T_k(R_{2k},\varphi,t). \tag{3'}$$

On spare surfaces of the shaft, sleeves, and races, the conditions of convective heat transfer are set as

$$\lambda_1\frac{\partial U(r,\varphi,z,t)}{\partial r}\bigg|_{r=R_1} = -\alpha_1(U(R_1,\varphi,z,t) - T_0), \tag{4}$$

$$\lambda_{2k}\frac{\partial T_k(r,\varphi,t)}{\partial r}\bigg|_{r=R_2} = \alpha_2\left(T_k(R_{2k},\varphi,t) - T_0\right),$$
$$|\varphi| > \varphi_0, \tag{5}$$

$$\lambda_{3k}\frac{\partial T_k(r,\varphi,t)}{\partial r}\bigg|_{r=R_{4k}} = -\alpha_3(T_k(R_{4k},\varphi,t) - T_0),$$
$$-\pi < \varphi \le \pi. \tag{6}$$

At the ends of the shaft, the first- and third-type conditions are set as follows:

$$\lambda_1\frac{\partial U(r,\varphi,z,t)}{\partial z}\bigg|_{z=L} = -\alpha_1(U(r,\varphi,L,t) - T_0),$$
$$U(r,\varphi,0,t) = T_0. \tag{7}$$

In the center of the shaft, the boundedness condition of heat flow is set as

$$\lim_{r\to 0}\left(r\lambda_1\frac{\partial U}{\partial r}\right) = 0. \tag{8}$$

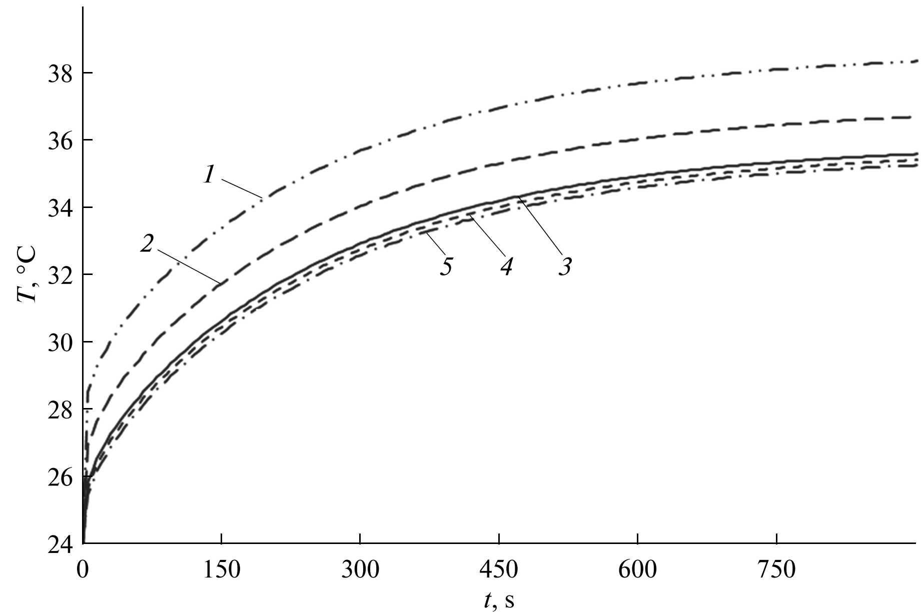

Fig. 2. Calculated dependences of maximum temperatures in the friction zone at various Courant numbers γ: (*1*) $\gamma = 36$; (*2*) 12; (*3*) 2; (*4*) 1; (*5*) 1.8.

Based on the angular coordinate, the following periodicity conditions are met:

$$\left.\frac{\partial T_k(r,\varphi,t)}{\partial \varphi}\right|_{\varphi=-\pi} = \left.\frac{\partial T_k(r,\varphi,t)}{\partial \varphi}\right|_{\varphi=\pi}, \quad (9)$$

$$T_k(r,-\pi,t) = T_k(r,\pi,t),$$

$$\left.\frac{\partial U(r,\varphi,z,t)}{\partial \varphi}\right|_{\varphi=-\pi} = \left.\frac{\partial U(r,\varphi,z,t)}{\partial \varphi}\right|_{\varphi=\pi}, \quad (10)$$

$$U(r,-\pi,z,t) = U(r,\pi,z,t).$$

The starting temperature distributions in the friction conjunction elements are taken to be equal and uniform as shown below:

$$T_k(r,\varphi,0) = U(r,\varphi,z,0) = T_0. \quad (11)$$

NUMERICAL SOLUTION

Problems (1)–(11) are solved by the finite difference method and are reduced to a set of one-dimensional heat equations. The presence of a convection term in heat equation (2) to take into account the rotation of the shaft leads to certain difficulties in solving the problem numerically. Using monotonic and locally one-dimensional difference equations to approximate the sum allows one to meet the maximum principle, i.e., at any τ and h_φ steps, an approximated solution can be found for the time and angular variable. In the case when the time step tends to zero, among the multiple approximate solutions, one should choose the solution that meets the condition of sticking. The algorithm developed for the numerical determination of the temperature field should be used for to solve the boundary problem by solving the inverse heat-transfer problem in order to determine friction heat transfer. Thus, in the numerical solution of the direct problem, the machine time needed to solve the inverse problem depends on the time step. If the step is too small, the machine time can be impracticable. So, the maximum possible time step at the preset sticking criterion should be chosen.

Based on in silico experiments that use a detailed spatial network, a time step that provides a convergence solution was determined at different values of the Courant number $(\gamma = \tau\upsilon/h_\varphi,\ \upsilon = R_1\Omega)$ that characterize the relation of time step based on the angular variable with the rotational speed of the shaft and time step. Since the rotating shaft is common for all of the sliding bearings, it is enough to consider the case of one bearing. The time step found can be used for temperature calculation for a system of several bearings.

Results of temperature calculations vs. the Courant number are given for the following geometric dimensions: $R_{1k} = 12$ mm, $R_{2k} = 13$ mm, $R_{3k} = 16$ mm, $R_{4k} =$ 30 mm, and $k = 1$ (Fig. 2). The material of the shaft and the race is steel, while the sleeve is made of F4K20-filled PTFE. The rotational speed of the shaft π rad/s and contact angle is 30°. The intensity of the specific heat release is constant at $Q = 67$ kWt/m^2. The convergence solution is found at $\gamma < 1$. For practical calculations, the time step can be determined from the condition $\gamma = 2$, since, at $\gamma < 2$, the temperature values change within a 1° interval.

COMPARISON OF CALCULATED AND EXPERIMENTAL TEMPERATURES

To establish the adequacy of the mathematical model expressed as two- and three-dimensional heat equations

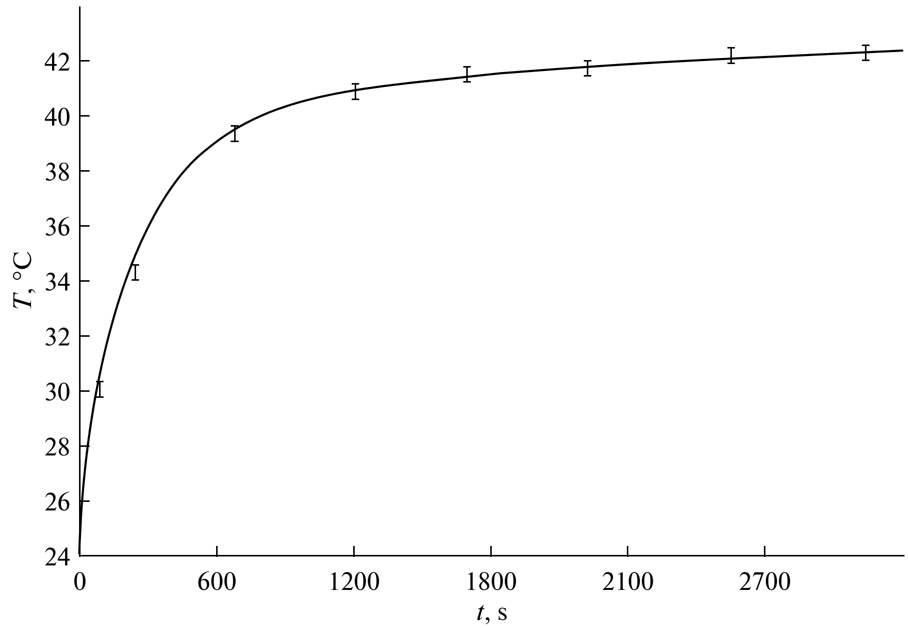

Fig. 3. Calculated dependences of temperature on time in an inner point of a sleeve at a distance of 0.5 mm from the friction zone. (I is confidence interval for experimental temperature data).

for a real thermal process in a sliding bearing, a comparison was conducted of the calculated and experimental temperatures (Fig. 3). The tests were conducted on the SMT-1 friction machine for one bearing. Temperatures were registered using a type-T thermocouple with a diameter of 0.1 mm on a Termodat multichannel device at 5 points in the sleeve at a distance of 0.5 mm from friction zone. The contact angle was 60°. The calculated dependence of temperature at $\varphi = 0$ lies within the interval of experimental data (Fig. 3). Similar results were obtained for other measurement points which proves to be adequate for describing the thermal process in the sliding bearing using the proposed model.

MODELING OF THERMAL PROCESS IN A BEARING SYSTEM

The solution algorithm was generalized for a system of sliding bearings. As an example, a system of four equal bearings (F4K20-filled PTFE) was considered. The intensity of heat generation in bearings were described as the following time functions:

$$\begin{aligned} Q_1(\varphi,t) &= 3851.656(t+1)^{\frac{1}{4}}|\cos\varphi|, \\ Q_2(\varphi,t) &= 2^{15.89-\frac{(t-600)^2}{90000}}|\cos\varphi|, \\ Q_3(\varphi,t) &= 3851.656\pi|\cos\varphi|, \\ Q_4(\varphi,t) &= 12^{4.3194-\frac{(t-600)^2}{360000}}|\cos\varphi|. \end{aligned} \tag{13}$$

Intensity functions were chosen such that the first one increases, the third one is constant, and the third and the fourth have maximum peaks at 10 min. Temperature-field dynamics in the bearings mutually influence the temperature dependence on time in neighbor bearings, which is shown by the temperature dynamics in the friction zone of the bearing (Fig. 4). As the rate of heat release in the first bearing increases, the temperature in the friction zone must also increase. Under the influence of a decrease in the rate of heat release in the second bearing after 10 min, the temperature in the first one decreases after 15 min of work. The temperature of the third bearing could be stable due to the stability of heat release rate but, under the influence of temperature decrease at the fourth bearing, it also decreases.

Based on a study of temperature fields in sliding bearings, it was determined that distances of 10 cm between the bearings would exclude a mutual influence on the temperature-field dynamics of the bearings. These results can also be used, e.g., to develop a multiposition test bench for friction and wear testing materials.

In the heat-friction diagnostics based on solving the inverse boundary problem heat transfer, the machine time for solving the direct problem plays a key role. Faster calculations are possible if the mathematical model is simplified. The mathematical model of thermal process in the bearing system can be simplified by assuming that the temperature distribution is uniform in the cross section of the shaft. Thus, the shaft can be regarded as one-dimensional so that its rotational speed can only be taken into account in the coefficient of heat transfer of its surface with the environment. To justify the supposition of uniform temperature distribution and analyze the temperature dis-

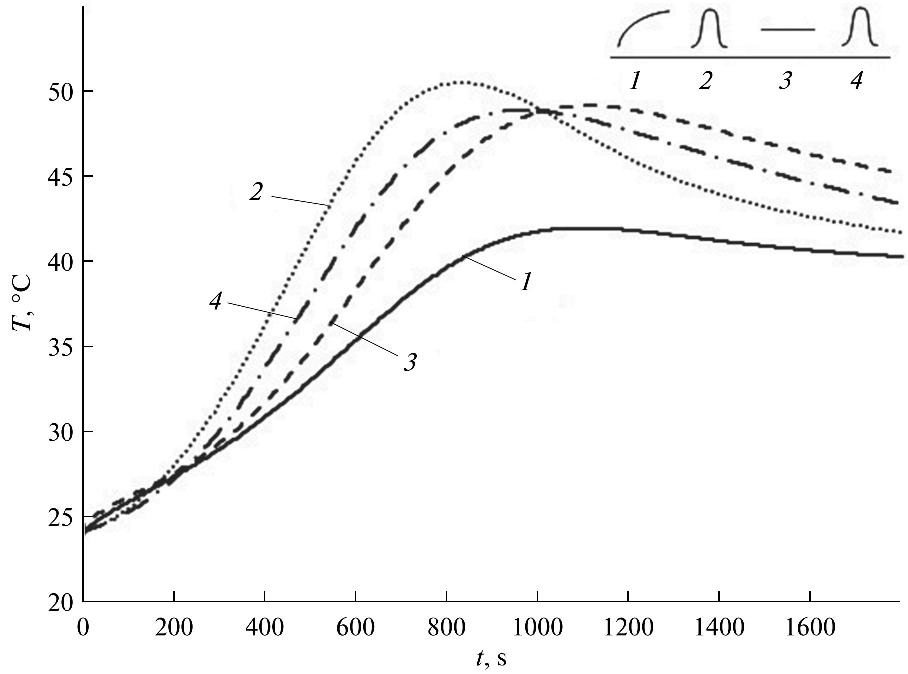

Fig. 4. Dependences of maximum temperatures in the contact zone of sliding bearings on time: (*1*) first bearing; (*2*) second bearing; (*3*) third bearing; (*4*) fourth bearing.

tribution by radial variable, it is necessary to study the temperature dynamics in the shaft within the circle. The accepted intensity of heat release and the diameter and thermophysical properties of the shaft allow one to consider the temperature distribution by radial variable to be uniform. As the rotational speed of the shaft increases, the temperature distribution within the circle will tend to be uniform. Hence, it is necessary to determine the rotational speed of the shaft above which temperature distribution can be regarded as uniform.

At a small distance from a sliding bearing (2–4 mm) by the axial variable, the temperature distribution in the shaft by circle becomes uniform. So, to study the uniformity of temperature distribution by a circle variable in a system of sliding bearings, one bearing that has the initial characteristics given above was considered. In the calculations at various rotational speeds, specific the function of the intensity of heat generation remained unchanged due to the condition $pR_1\Omega = pV = \text{const}$. The maximum and minimum values of the temperature in the shaft friction zone are observed at the beginning and end of contact. The calculated dependences of the surface temperature of the shaft on the angular coordinate are presented (Fig. 5). Despite the increase in the coefficient of heat transfer, upon an increase in the rotational speed of the shaft after the end of a contact the shaft surface is not cooled enough, so minimum temperature at contact increases. Furthermore, due to the decrease in the contact time, the maximum temperature on the shaft surface decreases. Thus, an increase in the rotational speed leads to uniform temperature distribution along the circumference.

The suppositions that allow one to simplify the mathematical model of thermal process in a bearing system are only acceptable if a satisfactory level of accuracy is achieved in the description of the process. Differences between the maximum and minimum temperatures on the shaft surface vs. the rotational speed of the shaft (Fig. 6) allow one to determine the

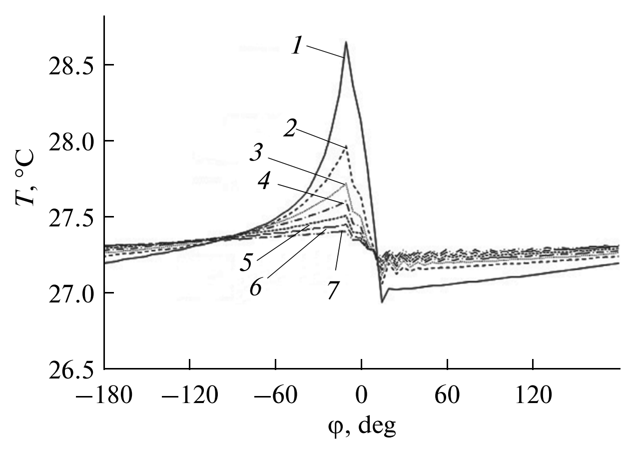

Fig. 5. Temperature distribution on the shaft surface at various rotational speed values: (*1*) 0.1π rad/s; (*2*) 0.3π; (*3*) 0.5π; (*4*) 0.7π; (*5*) π; (*6*) 1.4 π; (*7*) 2π.

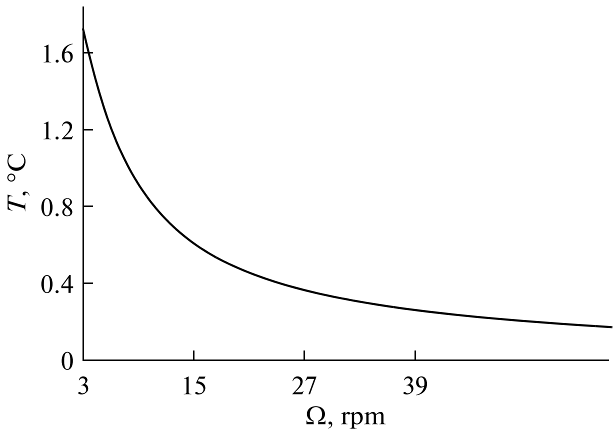

Fig. 6. Dependence of difference between maximum and minimum shaft surface temperatures on the rotational speed of the shaft at a moment of time $t = 1$ min.

rotational speed of the shaft above which the temperature distribution can be regarded as uniform.

CONCLUSIONS

—A mathematical model of the thermal processes in a system of sliding bearings that takes into account the rotational speed of the shaft and a practicable calculation method to determine time step based on in silico experiments have been proposed.

—By comparing calculate and experimental temperature data, the adequacy of the mathematical model proposed for a real thermal process in sliding bearings has been established.

—Based on a study of the temperature fields in sliding bearings, the distances between bearings excluding their mutual influence have been determined and the rotational speed above which the mathematical model can be simplified has been established.

—The proposed mathematical model can be applied to determine the friction forces in each bearing of the system based on the temperature data.

NOTATION

Q_k—rate of specific heat release in the contact zone of the kth bearing; U—temperature of the shaft; T_k—temperature of the kth bearing; T_0—temperature of environment; k, N—indices and number of bearings; R_1—radius of the shaft; R_{2k}, R_{3k}—inner and outer radii of the kth bearing sleeve; R_{4k}—outer radius of the kth bearing race; d_k—length of the kth bearing; L—length of a shaft; r, φ, z—cylindrical coordinates; z_{k-1}, z_k—beginning and end axial coordinates of the kth bearing position; $2\varphi_0$—contact angle; Ω—angular speed; t—time; t_m—testing time; C_1—volume heat capacity of the shaft material; C_{2k}, C_{3k}—volume heat capacity of materials of the kth bearing sleeve and race, respectively; α_1, α_2, α_3—coefficients of heat transfer on the surfaces of the shaft, sleeve, and race, respectively; λ_1—coefficient of thermal conductivity of the shaft material; λ_{2k}, λ_{3k}—coefficients of thermal conductivity of materials of kth bearing sleeve and race, respectively; p—pressure; V—linear speed.

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