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Development of a Mathematical Model of Hydroerosive Wear in Piston Couples of Hydraulic Machines: Part 2

O. P. Budarova* and S. V. Boldyrev

Naberezhnye Chelny Institute of Kazan Federal University, pr. Syuyumbike 10A, Naberezhniye Chelny, Republic of Tatarstan, 423812 Russia

**e-mail: smoljakova.olgapetrovna@yandex.ru*

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Abstract—The paper presents a definition of hydroerosive wear that occurs in hydraulic machines due to the presence of water or air in a fluid that was used in the mathematical model. As is indicated by the complex of equations obtained by conversions, the relationship between the velocity of the hydroerosive wear of piston couples and the presence of water in fluid is exponential. In addition, the wear velocity is determined to be directly proportional to the presence of air inside of the hydraulic system and inversely proportional to the pressure achieved by the pump.

Keywords: math model, hydroerosive wear, hydraulic machines, water, air, fluids, piston couples, hydraulic machines

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INTRODUCTION

Volumetric hydrodrives are widely used not only in contemporary automatic transfer lines of machine manufacturing enterprises, but also in various mobile technics. Due to its quite complicated construction and fluid kinematics, volumetric hydraulic pumps are the most fragile elements of hydrodrives. Statistically, the most frequent pump failures are connected with the pollution by fluids such as water, air, and mechanical impurities. Presently, hydrosystems of manufacturing equipment use filters with a degree of filtration no greater than 5 μm, which allows one to avoid failures caused by the blockage of narrow channels and abrasive wear. At the same time, water may often penetrate hydrosystems through damaged heat exchange devices, as well as with fragments of lubricating-cooling fluids. Water is also present in atmospheric air, which permanently exists in hydrosystems of ground facilities in amounts of 5–9% [1]. As practice has shown, at the same time, frequent emergency failures connected with jamming or the extreme wear of the piston, plunger, and other friction pairs take place. It is necessary to note that water-in-oil or oil-in-water emulsions are applied for hydrodrives that operate at high pressures. The wear forecasting of such highly loaded machines is also a rather important problem.

Based on the above-said, is seems important to obtain a reasoned physicomathematical model of hydroerosive wear of piston friction pairs of hydromachines that operate under the of watery fluid.

STATEMENT OF THE PROBLEM

Part 1 [2] of the present work suggested a model of hydroerosive wear based on the hydropercussive impact of vapor bubbles that formed due to the adiabatic compression of humid air bubbles in the piston chamber with a corresponding temperature increment. Formula (10), which expresses the speed at which evaporated water droplets hit the inside of the vapor bubble by the friction surface was obtained based on the main equations of thermal processes, as well as the equation by D. Bernoulli. Material strength parameter with respect to pulse load depending on the ultimate bending strength, character of fatigue curve and Poisson coefficient was also defined. At this point, it is necessary to estimate the load at hydropercussive impact and pass to the wear rate of piston couples as a function of water and air in oil, as well as to define the duration of the incubation period.

DEFINITION OF HYDROEROSIVE WEAR RATE

If we call an area on the surface equal to the region where evaporated water droplets hit as a *spot* (Fig. 1, part 1), then, during the incubation period, the number of hits on a unit area will be n_i and the number of droplet hits per spot is n_i^* as follows:

$$n_i^* = n_i \pi d^2 / 4, \qquad (30)$$

where d is the diameter of the spot.

Along with (27), Eqs. (29) and (30) imply the following:

$$n_i^* = a_1(S/P). \quad (31)$$

The results show that the number of hits per spot needed to initiate destruction is proportional to the ratio between the strength characteristic of the material S and the stress P created by droplets hits. Considering that Eq. (31) is based on the fatigue properties of materials during bending, it can be assumed that n_i^* depends on S/P; however, one cannot conclude that there is a linear dependence between these parameters. Thus, we attempt the following:

$$n_i^* = a_1(S/P)^{a_2}, \quad (32)$$

where a_1 and a_2 are unknown constants.

Coefficient a_1 was defined based on recommendations from S.P. Kozyrev [3] depending on the velocity of the hits and corresponds to the value $a_1 = 6.986 \times 10^{-12}$. According to G.S. Springer [4], the index of power a_2 equals 5.7.

Combining Eqs. (30) and (32) and expressing the diameter of the spot via the radius of the vapor bubble R'', we get

$$n_i^* = \frac{2.225 \times 10^{-12}}{(R'')^2}\left(\frac{S}{P}\right)^{5.7}. \quad (33)$$

In this form, Eq. (33) is correct under the condition that $n_i^* > 1$. This is required by fatigue theory in accordance with which the load must be applied several times before the material starts to deconstruct.

Stress P created by droplet hit in Eq. (33) can be defined using the Galler formula s follows:

$$P = \frac{\rho_o a_o W_\delta}{1 + \dfrac{\rho_o a_o}{\rho_{st} a_{st}}}, \quad (34)$$

where ρ_o and a_o are the oil density and sonic speed in oil, respectively, and ρ_{st}, a_{st} are the steel density and sonic speed in steel (or the same for another wearable material).

The duration of the incubation period t_i is defined based on the assumption that erosion is caused mainly by hits directed normally to the surface of the material according to the following dependence:

$$t_i = \frac{n_i}{Z_d W_\delta}, \quad (35)$$

where Z_d is the amount of water droplets per one air bubble.

Let us assume that, after some period of time, the erosion of material surface under the influence of water droplets occurs at an almost constant rate (linear approximation). In order to calculate the rate of erosion, it is necessary to draw an analogy between the behavior of material hit by water droplets and that of samples subjected to bending fatigue. It is accepted that erosion is uniform along the whole surface area. According to the linear approximation, the removal of mass from a unit area is defined as

$$m = \alpha(n - n_i), \quad (36)$$

where α is the rate of mass removal, kg/hit; n is the number of hits on a unit area, $1/m^2$, which can be defined by the formula

$$n = (W_\delta \cos\theta) Z'_d \ \tau \quad (37)$$

where θ is the droplet's angle of incidence (in our case, this is the angle that defines the direction of the hit); and is the time of droplet impact.

The probability that the sample would be destroyed during bending tests between the minimal and arbitrary life times l can be defined from the Weibull distribution. The life time (or number of loading cycles) can be replaced by the number of hits per spot.

In dimensionless form, m can be expressed as

$$m^* = m/\rho_{st} d_{v.b.}, \quad (38)$$

where $d_{v.b.}$ is the diameter of the vapor bubble. Then, we can obtain

$$\frac{m}{\rho_{st} d_{v.b.}} = a_3 \left(\frac{n^* - n_i^*}{n_a^*}\right)^{a_4\beta}, \quad (39)$$

where n_a^* is the characteristic life time; a_3, a_4 are the coefficients; and β is the Weibull inclination.

Replacing n by n^* in Eq. (30) transforms expression (39) as follows:

$$\frac{m}{\rho_{st} d_{v.b.}} = \frac{\alpha}{\rho_{st}\pi d_{v.b.}^3/4}\left(n^* - n_i^*\right). \quad (40)$$

Equating the right sides of Eqs. (39) and (40) we get

$$\frac{\alpha}{\rho_{st}\pi d_{v.b.}^3/4} = a_3 \frac{1}{\left(n_a^*\right)^{a_4\beta}} \frac{\left(n^* - n_i^*\right)^{a_4\beta}}{n^* - n_i^*}. \quad (41)$$

Since it was accepted during the development of the model that the rate of mass removal is constant after the end of the hidden period, then a_4 and β are equal to 1 and Eq. (41) will be rewritten as

$$\frac{\alpha}{\rho_{st}\pi d_{v.b.}^3/4} = a_3 \frac{1}{n_a^*}. \quad (42)$$

The characteristic life time is connected with the minimal life time. Therefore, the connection between n_a^* and n_i^* can be expressed by the dependence

$$n_a^* = a_5 n_i^{*a_6}, \quad (43)$$

where a_5 and a_6 are constants.

Dependence of duration of incubation on water content in oil at an air content of 5% (by volume) and its relative humidity $\varphi = 0.4$

C_w, %	0	0.2	0.4	0.8	1.0
t_i, c	1.23×10^9	2.68×10^3	1.34×10^3	5.71×10^2	5.37×10^2

After introducing the dimensionless rate of mass removal

$$\alpha^* = \frac{\alpha}{\rho_{st}\pi d_{v.b.}^3/4} \quad (44)$$

and replacing ratio a_3/a_5 with a_7, Eqs. (42)–(44) are rewritten as

$$\alpha^* = a_7 \frac{1}{\left(n_i^*\right)^{a_6}}. \quad (45)$$

According to the recommendations [4], the index of power a_6 in (45) was taken to be 0.7.

Changes in the rate of mass removal can be denoted in the following form:

$$\frac{\partial m}{\partial \tau} = \frac{\partial m}{\partial n}\frac{\partial n}{\partial \tau}. \quad (46)$$

As a result of differentiation, we obtain

$$\frac{\partial m}{\partial \tau} = \alpha Z_d' \left(W_\delta \cos\theta\right). \quad (47)$$

Below, we accept that $\theta = 0$.

Parameter α can be expressed via Eqs. (33), (44), and (45) as follows:

$$\alpha = \frac{\rho_{st}\pi d_{v.b.}^3}{4}\left\{6.986 \times 10^{-12}\left[a_7 (S/P)^{5.7}\right]^{-0.7}\right\}. \quad (48)$$

Introducing the volume of the vapor bubble V'' and considering that 1 m^3 of oil contains a certain amount of air bubbles Z_{bub}rate of mass removal from a unit area per a unit of time can be denoted as follows:

$$\alpha = a_8 a_7^{-0.7} (P/S)^4 \rho_{st} V'' W_\delta Z_{bub}, \quad (49)$$

where a_8 is constant.

Product $a_8 a_7^{-0.7}$, which is constant at a given water and air content, will be denoted as

$$a_8 a_7^{-0.7} = \left(Z_d/Z_{lim}\right)^n, \quad (50)$$

where Z_{lim} is the limiting amount of droplets inside the air bubble and n is the index of power, $0 < n < 1$.

Considering equations that express the experimental dependences of the wear rate of axial-piston pump pistons of the water content in oil [5], index of power n was taken to be 0.2.

Finally, taking into account (50), Eq. (49) will be denoted as

$$\alpha = (P/S)^4 \left(Z_d/Z_{lim}\right)^{0.2} \rho_{st} V'' W_\delta Z_{bub}. \quad (51)$$

Equations (51) and (10) imply that the rate of metal removal from the unit area in contact with oil is directly proportional to the amount of air and water in oil and inversely proportional to pressure developed by the pump; it also depends on the physical properties of wearable material and oil.

RESULTS AND DISCUSSION

After a series of transformations, we obtained the dependence of the rate of mass removal from the surfaces of pump pistons on processing medium parameters, such as the pressure in the piston chamber, the concentration of water and atmospheric-air pollution in the oil, and sonic speed in oil and its density, as well as on the properties of the wearable surface, including Poisson's coefficient, sonic speed, density, and parameter β of the fatigue curve. Figure 2 shows the dependence of rate of mass removal (α) on the content of water in oil (C_w) and on the volume of air in 1 m^3 of oil (V_{air}). The dependence of the rate of mass removal on the water content in oil represents an exponential increasing function approaching saturation. Dependence of rate of mass removal on content of air in oil is described by an increasing linear function.

On the other hand, in the combined consideration of Eqs. (10) and (51), one can assume that the dependence of the wear rate on pressure in the piston chamber of the pump is described by a decreasing exponential function. According to preliminary calculations, at high pressures in hydrosystem (from 15 to 20 MPa), the integral dependence of hydroerosive wear of pistons constitutes magnitudes is in the range of 0.2×10^{-12} to 0.5×10^{-12}, which is ten times less than at pressures of 4–10 MPa. In our opinion, the given result justifies the use of water-containing emulsions, including both water-in-oil and oil-in-water types, at pressures greater than 15 MPa. It is necessary to emphasize that the question is only about hydroerosive wear during cavitation without taking into account other types of wear.

According to (35), the higher the water content in oil and the rate of water droplets hits on the surface of the wearable material, the shorter the duration of the incubation period. The table shows the results of calculations of the incubation period depending on the water content in oil (in percents).

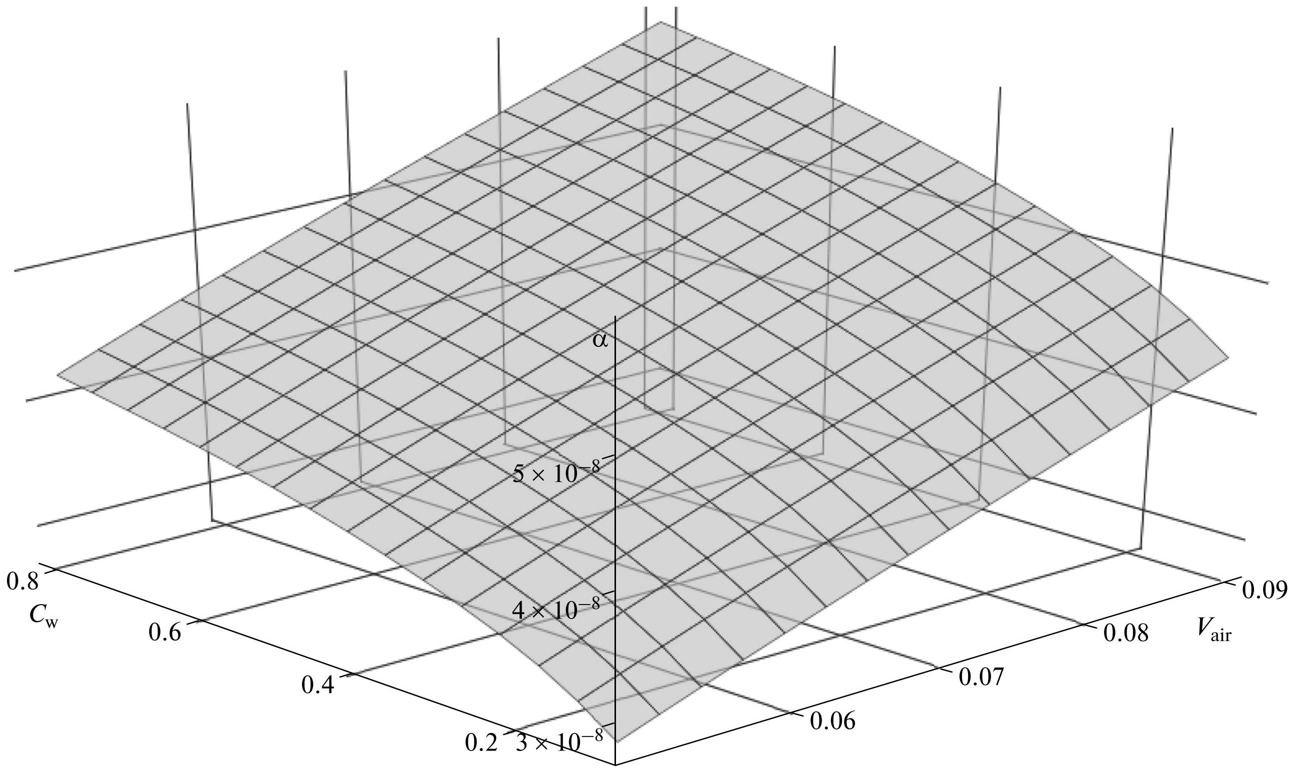

Fig. 2. Dependence of rate of mass removal (α) on content of water (C_w) and air (V_{air}) in oil.

It can be seen from the table that the duration of the incubation period upon change in C_w from 0 to 0.2% decreases by six orders of magnitude and constitutes 2680 s.

The suggested model of hydroerosive wear takes into account thermodynamic processes with significant temperature increments that occur cavitation conditions, as well as fatigue processes on steel surface connected with the multiple-hit impact of exploding overheated water molecules. The model provides an explanation for extreme wear and the appearance of pitting on the internal and external surfaces of pump pistons operating under the conditions of watery oil, plastic deformations in places of local temperature increments caused by the closing of humid air bubbles under the influence of operating medium pressure with the formation of vapor bubbles hitting the wearable surface.

CONCLUSIONS

It is necessary to note that the question regarding the most dangerous range pressures in terms of the appearance of hydroerosive wear (0.15–15 MPa) requires additional research. To a great extent, wear depends on different combinations of exploitation factors, such as pressure on suction and pressure lines of pumps, properties of fluid and its pollutions, material of friction pairs, etc. Since, under the conditions of watery fluid, hydroerosive wear depends on many factors and an analysis of the obtained equations is complicated, it is necessary to perform several series of calculations using the suggested model, as well as to carry out experimental research.

The algorithm for calculating the defining wear rate of pistons block of axial-piston pump was developed. However, crucially, it can be applied to piston and plunger couples of other type of machines or, with some additions, can be used to calculate the wear rate of laminose and gear pairs of hydromachines. Apparently, hydroerosive wear according to the model under consideration is also possible in bladed hydromachines.

NOTATION

R—radius, m; Z—amount of droplets of air bubbles in 1 m^3; m—mass, kg; ρ_w—density, kg/ m^3; W_δ—water droplets hit rate, m/s; a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8—constants; ν—Poisson coefficient; n—number of droplets hits; S—strength characteristic of material, Pa; d—diameter, m; P—pressure created by droplet hit, Pa; V—volume, m^3; t_i—duration of incubation period, s; α—rate of mass removal, kg/hit; θ—droplet incidence angle, °; τ—droplets impact time, s; β—constant (Weibull inclination); m^*—dimensionless mass removal from unit area; α^*—dimensionless rate of mass removal.

REFERENCES

1. Kondakov, L.A., Nikitin, G.A., Prokof'ev, V.N., Skritskii, V.Ya., and Sosonkin, V.L., *Mashinostroitel'nyi gidroprivod* (Engineering Hydraulics), Prokof'ev, V.N., Ed., Moscow: Mashinostroenie, 1978.
2. Budarova, O.P. and Boldyrev, S.V., Development of mathematical model of hydroerosion wear of piston pairs of hydraulic machines. Part 1, *J. Frict. Wear,* 2014, vol. 35, no. 5 (in press).
3. Kozyrev, S.P., *Gidroabrazivnyi iznos metallov pri kavitatsii* (Hydroabrasive Wear of Metals at Cavitation) Moscow: Mashinostroenie, 1971.
4. Springer, G.S., *Erosion by Liquid Impact* New York: Wiley, 1976; Moscow: Mashinostroenie, 1981.
5. Razvalyaev, S.V. and Budarova, O.P., Exploitation factor effect on the hydroerosion wear of axial-piston hydromachines, *Sbornik dokladov 16 Vserossiiskoi nauch.-tekhn. konferentsii studentov i aspirantov* (Proc. 16th All-Russ.Sci.-Tech. Conf. Students and Postgraduated Students) Moscow: Mos. Ener. Inst., 2012.

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