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N/D Method for πK Scattering as Part of MISP 2020 Proceeding

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Abstract—In this work we show an application of the N/D method to πK $I = 1/2$ s-wave coupled channel scattering. Free parameters are constrained via matching to chiral perturbation theory. While we were able to find parameters that provide an excellent description of the scattering data, the corresponding amplitudes showed some unphysical properties like poles on the first sheet. By this we were guided to propose an alternative method to parametrise πK scattering amplitudes and form factors.

Keywords: N/D method, kaon, pion, unitarity, analyticity, scattering

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1. MOTIVATION

As the lightest hadrons, pions and kaons are abundantly occurring in processes involving net strangeness and therefore a proper treatment of their interactions is crucial for our understanding of hadron physics. A theoretically sound description of πK final state interactions require a formalism consistent with analyticity and unitarity. A proper description of such meson-meson interactions is still unclear and controversial in the higher energy regions, since in this energy range a lot of resonances are very broad and overlapping, making a parametrisation using sums of standard Breit–Wigner resonances questionable as those sums violate unitarity.

An alternative provides the N/D method, which gives the right branch cut structure and is believed to be consistent with unitarity and analyticity [1]. It was applied to πK scattering up to 1.4 GeV in reference [2]. We improve the model by adding the η'K channel as well as higher energetic resonances in the $I = 1/2$ s-wave amplitude.

2. FORMALISM OF THE N/D METHOD

Considering a $2 \to 2$ process with relative angular momentum L the most general T-matrix contains two kinds of cuts. The physical right hand cut (RHC) coming from unitarity and unphysical left hand cuts (LHC) due to crossing symmetry. The N/D method separates these two aspects of the analytic structure by splitting the T-matrix in two parts via $T = \nu^{L/2} D^{-1} N \nu^{L/2}$, where $\nu = p^2$ is given by the CMS momentum p of the initial particles. Here D contains the physical RHC and N contains the unphysical LHC's. Approximating LHC contributions as polynomials, one has the freedom to set $N = 1$ and consequently $T = \nu^{L/2} D^{-1} \nu^{L/2}$. D can be calculated dispersively employing the unitarity condition relating the imaginary part of T^{-1} along the RHC and the phase space σ via $\Im(T^{-1}) = -\sigma$ resulting in

$$D = \sum_{m=0}^{L} a_m s^m + \sum_i \frac{\gamma_i}{s - s_i} - \frac{(s - s_0)^{L+1}}{\pi} \int_{s_{\text{th}}}^{\infty} ds' \frac{\nu(s')^L \sigma(s')}{(s' - s)(s' - s_0)^{L+1}}. \quad (1)$$

Besides the dispersion integral and its subtraction polynomial, the pole terms allow for zeroes in the amplitude (e.g. Adler zeros). Equation (1) is the most general description of an elastic scattering amplitude neglecting LHC's that is consistent with unitarity. We can match the leading order part of D to chiral

Table 1. Parameters for second N/D model fit to the pseudo-data generated from [4]

Parameter	Value
M_{R}	(1353.9 ± 4.7) MeV
c_d	(−33.4 ± 2.4) MeV
c_m	(41.5 ± 4.6) MeV
$c_{\eta'}$	(−924.60 ± 240.17)
M'_{R}	(1945 ± 19) MeV
c'_d	(−15.1 ± 4.3) MeV
c'_m	(206.7 ± 15.1) MeV
$c'_{\eta'}$	(654.41 ± 1777.16)
$\tilde{a}_0^{\mathrm{SL}}$	(−0.927 ± 0.039)

perturbation theory (ChPT) contact potentials V_0 and resonance terms consistent with chiral symmetry V_R [3] via $\sum_{m=0}^{L} a_m^{\mathrm{L}} s^m + \sum_i \frac{\gamma_i^{\mathrm{L}}}{s - s_i} = (V_0 + V_R)^{-1}$. The remaining integral and sub-leading constants defined as G can be identified with the two-particle loop, which can be calculated analytically by dimensional regularisation. Using $V = V_0 + V_R$, the expression for the T-matrix can now be written as $T = (V^{-1} - G)^{-1}$, which agrees to the Bethe-Salpeter equation $T = V + VGT$.

3. APPLICATION OF THE MODEL

For the πK scattering phase shifts there are two different sources: On the one hand, there is a parametrisation of existing πK scattering data, which is constrained to satisfy forward dispersion relations. It provides phase shifts with very high accuracy up to 1.6 GeV [4]. On the other hand there are the experimental data [5, 6]. The free parameters of our model originate mainly from the resonance terms V_R depending on two universal coupling constants c_m and c_d and one scalar resonance mass M_{R} each [3]. Moreover the subleading subtraction constant $\tilde{a}^{\mathrm{SL}}$, which absorbs the scheme dependency of G and is contained within the subtraction polynomial of equation (1), is an additional free parameter. The coupling of the η'K channel to the resonances is considered with an additional coupling constant $c_{\eta'}$ in the $\eta' \mathrm{K} K_0^*$ vertex function, effectively allowing for effects sub-leading in large N_C. For the first fit these additional coupling constants are fixed to 1 while they enter as free parameters into the second fit. The fits and pseudo-data used can be seen in Fig. 1. The first fit performed well at low energies, but is unable to reproduce the given structure at higher energies. The second fit describes the reference phase with very high accuracy. Its parameters can be found in Table 1.

While the second fit provides an excellent description of the data, the corresponding amplitude turns our to have three problems:

- It contains a very narrow pole-like structure at around 1.34 GeV, which seems to be needed within this approach to generate the steep rise in the phase in this energy region. Moreover the fit parameters are highly fine-tuned such that even slight variations thereof destroy the structure.
- The physical Riemann sheet contains spurious poles, which spoil the analytic structure. These appear to be generated by the higher polynomial terms present in the leading order ChPT contact amplitudes V_0, which are free of parameters for our fit. Hence their existence seems to be deeply rooted in the formalism. A plot showing these poles of the first sheet can be seen in Fig. 2.
- the chiral amplitudes tend to generate a sizeable inelasticity to the ηK channel at odds with the analysis of reference [4].

4. DISCUSSION AND OUTLOOK

The results presented in this talk should be taken as a warning: There is no a priori guarantee that amplitudes constructed from the N/D method do not contain unphysical singularities. When employing

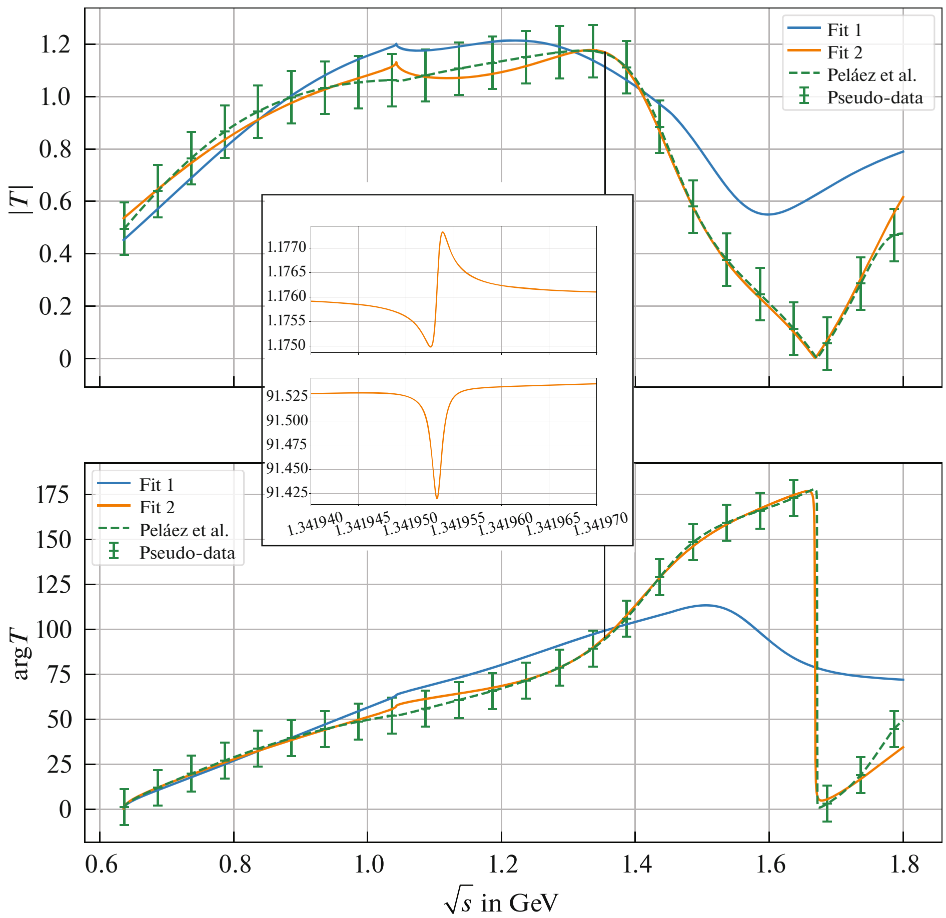

Fig. 1. N/D model fits to the pseudo-data generated from [4]. The upper (lower) panel shows the modulus (phase) of the T-matrix. The inlay is a zoom into the region where the additional singularity appears as described in the main text.

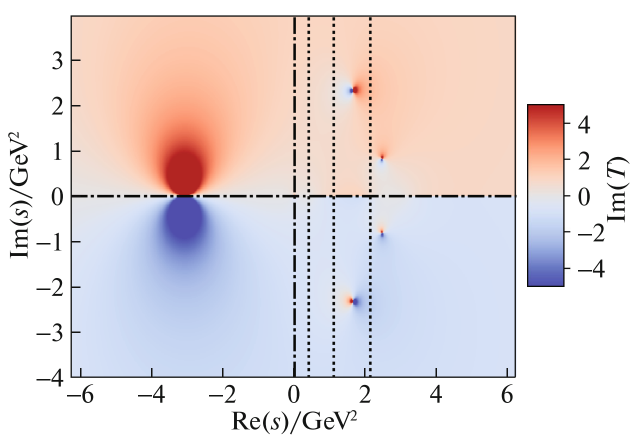

Fig. 2. Imaginary part of the T-matrix for complex energy values employing the parameters from Table 1.

this formalism avoiding poles on the physical sheet in the relevant energy range should be included as a fitting constraint in order to allow for physically sensible conclusions.

A formalism that avoids these complications was introduced in [7] in a study of the pion vector form factor and applied to ππ-KK interactions in [8]. This new framework allowed us to obtain a theoretical sound description of πK scattering up to 2.3 GeV consistent with unitarity and analyticity. The results will be presented elsewhere [9].

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