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The Cauchy Problem of the Moment Elasticity Theory in $\mathbf{R}^m$

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Abstract—In this paper we consider the problem of analytical continuation of the solution to the system of equations of the moment theory of elasticity in spatial many-dimensional domain. We give an explicit formula of restoring of solution inside the domain by values of sought-for solution and values of strains on part of the boundary of this domain.

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1. INTRODUCTION

In many real problems a part of boundary is inaccessible for measuring of displacements and tensions or only integral characteristics are known. In an experimental investigation of tensely deformed state of nature constructions the measuring can be made on accessible regions, i.e., components of displacements and tensions are given on a part of boundary, only. Therefore there arises the necessity to consider the problem of continuation of a solution to the system of equations of the elasticity theory in the domain using values of displacements and tensions on a part of boundary.

The system of equations for the moment elasticity theory is elliptic. The Cauchy problem for such systems is ill-posed. A solution uniquely exists, but it is not stable with respect to a small change of data. In ill-posed problems the existence of solution and its belonging to the class of correctness [1] is assumed a priori. The uniqueness of solution follows from the general Holmgren's theorem [2]. After the establishment of uniqueness in theoretical investigations of ill-posed problems, the important questions of obtaining an estimate of conditional stability and constructing regularizing operators arise.

In twenties of the previous century T. Carleman has constructed a formula, which connects the values of an analytical function with respect to a complex variable at points of the analyticity domain with its values on a part of boundary of this domain. Based on the Carleman formula, M. M. Lavrent'ev [1] introduced the notion of the Carleman function of the Cauchy problem for the Laplace equation and gave a mode of its construction for several cases. The structure of the Carleman function gives the possibility to construct the regularization and obtain the estimate of conditional stability in these problems. The Carleman function for the Laplace equation has been constructed in [3].

In the present paper, based in the Carleman function and papers [4—10], we construct a regularized solution of the Cauchy problem for the system of equations of the moment elasticity theory for domains of a special form.

Let $x = (x_1, \dots, x_m)$ and $y = (y_1, \dots, y_m)$ be points of a real Euclidian space R^m, D be a bounded simply connected domain in R^m with the piecewise smooth boundary ∂D (∂D is composed of differential manifolds of dimension $m-1$) and S be the smooth part of ∂D with the smooth edge.

Let a $2m$-component vector-function $U(x) = (u_1(x), \dots, u_m(x), w_1(x), \dots, w_m(x)) = (u(x), w(x))$ satisfy the system of equations of the moment elasticity theory in the domain D [11]:

$$\begin{aligned} &(\mu+\alpha)\Delta u + (\lambda+\mu-\alpha)\,\mathrm{grad}\,\mathrm{div}\,u + 2\alpha\,\mathrm{rot}\,w + \rho\theta^2 u = 0, \\ &(\nu+\beta)\Delta w + (\varepsilon+\nu-\beta)\,\mathrm{grad}\,\mathrm{div}\,w + 2\alpha\,\mathrm{rot}\,u - 4\alpha w + j\theta^2 w = 0, \end{aligned} \tag{1.1}$$

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where coefficients λ, μ, ν, β, ε, and α, which characterize the medium, satisfy the conditions $\mu > 0$, $3\lambda + 2\mu > 0$, $\alpha > 0$, $\varepsilon > 0$, $3\varepsilon + 2\nu > 0$, $\beta > 0$, $j > 0$, $\rho > 0$, $\theta \in R^1$.

We call a vector-function $U(y)$ regular in D, if it is continuous together with its partial derivatives of the second order in D and of the first order on $\overline{D} = D \bigcup \partial D$.

Problem definition. It is required to find the regular solution U of system (1.1) in the domain D, based on its Cauchy conditions given on S:

$$U(y) = f(y), \quad T(\partial_y, n(y))U(y) = g(y), \quad y \in S, \tag{1.2}$$

where $T(\partial_y, n(y))$ is the tension operator defined by the equality

$$T(\partial_y, n(y)) = \begin{Vmatrix} T^{(1)}(\partial_y, n) & T^{(2)}(\partial_y, n) \\ T^{(3)}(\partial_y, n) & T^{(4)}(\partial_y, n) \end{Vmatrix},$$

$$T^{(i)}(\partial_y, n) = \left\| T^{(i)}_{kj}(\partial_y, n) \right\|_{m\times m}, \quad i = 1, \dots, 4,$$

$$T^{(1)}_{kj}(\partial_y, n) = \lambda n_k \frac{\partial}{\partial y_j} + (\mu - \alpha) n_j(y) \frac{\partial}{\partial y_k} + (\mu + \alpha)\delta_{kj} \frac{\partial}{\partial n(y)}, \quad k, j = 1, \dots, m,$$

$$T^{(2)}_{kj}(\partial_y, n) = -2\alpha \sum_{p=1}^{3} \varepsilon_{kjp} n_p(y), \quad T^{(3)}_{kj}(\partial_y, n) = 0, \quad k, j = 1, \dots, m,$$

$$T^{(4)}_{kj}(\partial_y, n) = \varepsilon\, n_k(y) \frac{\partial}{\partial y_j} + (\nu - \beta) n_j(y) \frac{\partial}{\partial y_k} + (\nu + \beta) \frac{\partial}{\partial n(y)}, \quad k, j = 1, \dots, m,$$

$n(y) = (n_1(y), \dots, n_m(y))$ is the outer unit normal vector to the surface ∂D at a point y, where $f(y) = (f_1(y), \dots, f_m(y))$ and $g(y) = (g_1(y), \dots, g_m(y))$ are given continuous vector-functions on S, δ_{kj} is the Kronecker symbol, $\varepsilon_{k\,j\,p}$ is a so-called ε-tensor or the Levi-Civita symbol defined by the equalities

$$\varepsilon_{k\,j\,p} = \begin{cases} 0, & \text{if at least two of three indexes } k, j, p \text{ are equal;} \\ 1, & \text{if } (k, j, p) \text{ contains an even number of permutations of numbers } (1,2,3); \\ -1, & \text{if } (k, j, p) \text{ contains an odd number of permutations of numbers } (1,2,3). \end{cases}$$

2. THE CONSTRUCTION OF THE CARLEMAN MATRIX FOR DOMAINS OF THE CAP TYPE AND THE REGULARIZATION OF THE PROBLEM SOLUTION

It is known [8] that for a regular solution to system (1.1) the integral presentation holds true

$$U(x) = \int_{\partial D} \left(\Psi(y, x) \left\{ T(\partial_y, n) U(y) \right\} - \left\{ T(\partial_y, n) \Psi(y, x) \right\}^* U(y) \right) ds_y, \quad x \in D, \tag{2.1}$$

where "*" is the transposition operation, and $\Psi(y, x)$ is the matrix of fundamental solutions of the statics of the moment elasticity theory,

$$\Psi(y, x) = \begin{Vmatrix} \Psi^{(1)}(y, x) & \Psi^{(2)}(y, x) \\ \Psi^{(3)}(y, x) & \Psi^{(4)}(y, x) \end{Vmatrix}.$$

Here

$$\Psi^{(i)}(y, x) = \left\| \Psi^{(i)}_{k\,j}(y, x) \right\|_{m\times m}, \quad i = 1, \dots, 4,$$

$$\Psi^{(1)}_{k\,j}(y, x) = \sum_{l=1}^{4} \left(\delta_{k\,j} \alpha_l + \beta_l \frac{\partial^2}{\partial\, x_k \partial\, x_j} \right) \varphi_m(i k_l r), \quad k, j = 1, \dots, m,$$

$$\Psi^{(2)}_{kj}(y,x)=\Psi^{(3)}_{kj}(y,x)=\frac{2\alpha}{\mu+\alpha}\sum_{l=1}^{4}\sum_{p=1}^{n}\varepsilon_l\varepsilon_{kjp}\frac{\partial}{\partial x_p}\varphi_m(ik_lr),\quad k,j=1,\dots,m,$$

$$\Psi^{(4)}_{kj}(y,x)=\sum_{l=1}^{4}\left(\delta_{kj}\gamma_l+\delta_l\frac{\partial^2}{\partial x_k\partial x_j}\right)\varphi_m(ik_lr)\quad k,j=1,\dots,m,$$

$r=|x-y|$, φ_m is the fundamental solution of the Helmholtz equation,

$$\varphi_m(\lambda r)=A_m\left(\frac{\lambda}{2}\right)^{\frac{m}{2}-1}K_{\frac{m}{2}-1}(\lambda r),\ \ A_{2k}=(-1)^k\cdot 2^{k-1},\ \ A_{2k+1}=(-1)^k\cdot 2^{-k+\frac{1}{2}},$$

$K_\nu(\lambda)$ is the Macdonald function, $\alpha_l=\frac{(-1)^l(\sigma_2^2-k_l^2)(\delta_{3l}+\delta_{4l})}{2\pi(\mu+\alpha)(k_3^2-k_4^2)}$, $\beta_l=-\frac{\delta_{1l}}{2\pi\rho\theta^2}+\frac{\alpha_l}{k_l^2}$, $\sum\limits_{l=1}^{4}\beta_l=0$,

$$\gamma_l=\frac{(-1)^l(\sigma_1^2-k_l^2)(\delta_{3l}+\delta_{4l})}{2\pi(\beta+\nu)(k_3^2-k_4^2)},\quad \delta_l=-\frac{\delta_{2l}}{2\pi(j\theta^2-4\alpha)}+\frac{\gamma_l}{k_l^2},\quad \sum_{l=1}^{4}\beta_l=0,$$

$$\varepsilon_l=\frac{(-1)^l(\delta_{3l}+\delta_{4l})}{2\pi(\beta+\nu)(k_3^2-k_4^2)},\quad \sum_{l=1}^{4}\varepsilon_l=0,\quad k_1^2=\frac{\rho\theta^2}{\lambda+2\mu},\quad k_2^2=\frac{j\theta^2-4\alpha}{\varepsilon+2\nu},$$

$$k_3^2+k_4^2=\sigma_1^2+\sigma_2^2+\frac{4\alpha^2}{(\mu+\alpha)(\beta+\nu)},\quad k_3^2k_4^2=\sigma_1^2\sigma_2^2.$$

It is not difficult to verify that with $u=\Psi^{(1)}_j(y,x)$, $w=\Psi^{(3)}_j(y,x)$ or $u=\Psi^{(2)}_j(y,x)$, $w=\Psi^{(4)}_j(y,x)$ the homogeneous equations of the statics of the moment elasticity theory are fulfilled, where $\Psi^{(i)}_j(y,x)$ is the j-vector-row of the ith matrix.

Following [3], we adduce the next definition.

Definition. The Carleman matrix of problem (1.1), (1.2) is a $(2m\times 2m)$-matrix $\Pi(y,x,\tau)$, which depends on two points y and x and a positive numeric parameter τ and satisfies the following two conditions:

$$1)\quad \Pi(y,x,\tau)=\Psi(y,x)+G(y,x,\tau),$$

where the matrix $G(y,x,\tau)$ satisfies system (1.1) with respect to the variable y everywhere in the domain D, $\Psi(y,x)$ is the matrix of fundamental solutions to system (1.1);

$$2)\quad \int_{\partial D\setminus S}(|\Pi(y,x,\tau)|+|T(\partial_y,n)\Pi(y,x,\tau)|)\,ds_y\le\varepsilon(\tau),$$

where $\varepsilon(\tau)\to 0$ with $\tau\to\infty$, $|\Pi|$ is the Euclidean norm of the matrix $\Pi=\|\Pi_{ij}\|_{2m\times 2m}$, i.e., $|\Pi|=\Big(\sum\limits_{i,j=1}^{2m}\Pi_{ij}^2\Big)^{1/2}$, in particular, $|U|=\Big(\sum\limits_{k=1}^{m}(u_k^2+w_k^2)\Big)^{1/2}$.

In papers [4, 5] the following theorem was proved.

Theorem 1. *Each regular solution $U(x)$ to system* (1.1) *in the domain D is defined by the formula*

$$U(x)=\int_{\partial D}(\Pi(y,x,\tau)\{T(\partial_y,n)U(y)\}-\{T(\partial_y,n)\Pi(y,x,\tau)\}^*U(y))ds_y,\quad x\in D,\tag{2.2}$$

where $\Pi(y,x,\tau)$ is the Carleman matrix.

Using the Carleman matrix, it is easy to deduce the estimate for the stability of solution to the Cauchy problem (1.1), (1.2), and indicate the method of efficient solving this problem.

To find an approximate solution to problem (1.1), (1.2) we construct the Carleman matrix as follows:

$$\Pi(y,x,\tau)=\begin{Vmatrix}\Pi^{(1)}(y,x,\tau) & \Pi^{(2)}(y,x,\tau)\\ \Pi^{(3)}(y,x,\tau) & \Pi^{(4)}(y,x,\tau)\end{Vmatrix}, \tag{2.3}$$

$$\Pi^{(i)}(y,x,\tau)=\left\|\Pi^{(i)}_{kj}(y,x,\tau)\right\|_{m\times m},\quad i=1,\dots,4,$$

$$\Pi^{(1)}_{kj}(y,x,\tau)=\sum_{m=1}^{4}\left(\delta_{kj}\alpha_m+\beta_m\frac{\partial^2}{\partial y_k\partial y_j}\right)\Phi_\tau(y,x,i\lambda_m),\quad k,j=1,\dots,m,$$

$$\Pi^{(2)}_{kj}(y,x,\tau)=\Pi^{(3)}_{kj}(y,x,\tau)=\frac{2\alpha}{\mu+\alpha}\sum_{m=1}^{4}\sum_{s=1}^{3}\varepsilon_m\varepsilon_{kjs}\frac{\partial}{\partial x_s}\Phi_\tau(y,x,i\lambda_m),\quad k,j=1,\dots,m,$$

$$\Pi^{(4)}_{kj}(y,x,\tau)=\sum_{m=1}^{4}\left(\delta_{kj}\gamma_m+\delta_m\frac{\partial^2}{\partial y_k\partial y_j}\right)\Phi_\tau(y,x,i\lambda_m),\quad k,j=1,\dots,m,$$

where

$$C_mK(x_m)\Phi(y,x,\lambda)=\frac{\partial^{k-1}}{\partial s^{k-1}}\int_0^\infty \operatorname{Im}\left[\frac{K(i\sqrt{u^2+s}+y_m)}{i\sqrt{u^2+s}+y_m-x_m}\right]\frac{\psi(\lambda u)\,du}{\sqrt{u^2+s}}, \tag{2.4}$$

$\psi(\lambda u)=\begin{cases}uJ_0(\lambda u), & m=2k,\ k\geq 1;\\ \cos\lambda u, & m=2k+1,\ k\geq 1,\end{cases}$ $J_0(u)$ is the Bessel function of zero order, $s=(y_1-x_1)^2+\dots+(y_{m-1}-x_{m-1})^2$, $C_2=2\pi$,

$$C_m=\begin{cases}(-1)^k2^{-m}(m-2)\pi\omega_m(k-2)!, & m=2k;\\ (-1)^k2^{-m}(m-2)\pi\omega_m(k-1)!, & m=2k+1,\end{cases}$$

$K(\omega)$, $\omega=u+iv$ (u and v are real) is an integer function, which takes real values on the real axis and satisfies the conditions

$$K(u)\neq 0,\ \sup_{v\geq 1}|\exp\nu\,|\operatorname{Im}\lambda|\,K^{(p)}(\omega)|=M(p,u)<\infty,\ \ p=0,\dots,m,\ \ u\in R^1.$$

Lemma 1 ([3]). *The function $\Phi(y,x,\lambda)$ can be represented in the form*

$$C_m\Phi(y,x,\lambda)=\varphi_m(i\lambda r)+g_m(y,x,\lambda),\quad r=|y-x|,$$

φ_m is the fundamental solution to the Helmholtz equation, $g_m(y,x,\lambda)$ is a regular function with respect to x and y, satisfying the equation $\Delta(\partial_y)g_m-\lambda^2g_m=0$.

In (2.4) we set $K(\omega)=\exp(\tau\omega)$. Then

$$\Phi(y,x,\lambda)=\Phi_\tau(y-x,\lambda),$$

$$C_m\Phi_\tau(y-x,\lambda)=\frac{\partial^{k-1}}{\partial s^{k-1}}\int_0^\infty \operatorname{Im}\left[\frac{\exp\,\tau(i\sqrt{u^2+s}+y_m-x_m)}{i\sqrt{u^2+s}+y_m-x_m}\right]\frac{\psi(\lambda u)\,du}{\sqrt{u^2+s}}$$
$$=\exp\tau(y_m-x_m)\,\frac{\partial^{k-1}}{\partial s^{k-1}}\int_0^\infty\left[-\cos\tau\sqrt{u^2+\alpha^2}+(y_m-x_m)\frac{\sin\tau\sqrt{u^2+s}}{\sqrt{u^2+s}}\right]\psi(\lambda u)du. \tag{2.5}$$

Now in formulas (2.3), (2.4), and (2.5) we set $\Phi(y,x,\lambda)=\Phi_\tau(y-x,\lambda)$ and obtain the matrix $\Pi(y,x)=\Pi(y,x,\tau)$.

From Lemma 1 it follows

Lemma 2. *The matrix $\Pi(y,x,\tau)$ given by formulas* (2.3), (2.4), *and* (2.5) *is the Carleman matrix of problem* (1.1), (1.2).

Proof. From (2.3)–(2.5) and Lemma 1 we have

$$\Pi(y,x,\tau)=\Psi(y,x)+G(y,x,\tau),$$

where

$$G(y,x,\tau)=\left\|\begin{matrix} G^{(1)}(y,x,\tau) & G^{(2)}(y,x,\tau)\\ G^{(3)}(y,x,\tau) & G^{(4)}(y,x,\tau)\end{matrix}\right\|,$$

$$G^{(i)}(y,x,\tau)=\left\|G^{(i)}_{k\,j}(y,x,\tau)\right\|_{m\times m},\quad i=1,\dots,4,$$

$$G^{(1)}_{k\,j}(y,x,\tau)=\sum_{l=1}^{4}\left(\delta_{k\,j}\alpha_l+\beta_l\frac{\partial^2}{\partial\,x_k\partial\,x_j}\right)g_m(y,x,k_l,\tau),\quad k,j=1,\dots,m,$$

$$G^{(2)}_{k\,j}(y,x,\tau)=G^{(3)}_{k\,j}(y,x,\tau)=\frac{2\alpha}{\mu+\alpha}\sum_{l=1}^{4}\sum_{p=1}^{n}\varepsilon_l\varepsilon_{k\,j\,p}\frac{\partial}{\partial x_p}g_m(y,x,k_l,\tau),\quad k,j=1,\dots,m,$$

$$G^{(4)}_{k\,j}(y,x,\tau)=\sum_{l=1}^{4}\left(\delta_{k\,j}\gamma_l+\delta_l\frac{\partial^2}{\partial\,x_k\partial\,x_j}\right)g_m(y,x,k_l,\tau),\quad k,j=1,\dots,m.$$

By direct calculation we can ascertain that the matrix $G(y,x,\tau)$ with respect to variable y satisfies system (1.1) everywhere in D. It is not difficult to prove that

$$\int_{\partial D\setminus S}\left(|\Pi(y,x,\tau)|+|T(\partial_y,n)\Pi(y,x,\tau)|\right)\,ds_y\le C_1(x)\,\tau^m\exp(-\tau\,x_m),\tag{2.6}$$

$C_1(x)$ is a bounded function inside D. □

We set

$$U_\tau(x)=\int_S[\Pi(y,x,\tau)\{T(\partial_y,n)U(y)\}-\{T(\partial_y,n)\Pi(y,x,\tau)\}^*\,U(y)]ds_y.\tag{2.7}$$

The following theorem takes place.

Theorem 2. *Let $U(x)$ be a regular solution to Eq. (1.1) in the domain D and satisfy on $\partial D\setminus S$ the boundary condition*

$$|U(y)|+|T(\partial_y,n)U(y)|\le M,\quad y\in\partial D\setminus S.\tag{2.8}$$

Then for $\tau\ge 1$ the estimate holds

$$|U(y)-U_\tau(y)|\le MC_m(x)\tau^m\exp(-\tau\,x_m),$$

where $C_m(x)=C_m(\rho)\int\limits_{\partial D_\rho}\frac{ds_y}{r^m}$.

Proof. From formulas (2.2) and (2.7) we have

$$|U(x)-U_\tau(x)|=\left|\int_{\partial D\setminus S}[\Pi(y,x,\tau)\{T(\partial_y,n)U(y)\}-\{T(\partial_y,n)\Pi(y,x,\tau)\}^*\,U(y)]\,ds_y\right|$$
$$\le\int_{\partial D\setminus S}\left(|\Pi(y,x,\tau)|+|T(\partial_y,n)\Pi(y,x,\tau)|\right)\left(|U(y)|+|T(\partial_y,n)U(y)|\right)ds_y.$$

Now, based on (2.6) and (2.8), we obtain the desired inequality. □

Let us adduce the result, which allows us to calculate $U(x)$ approximately, when on the surface S instead of $U(y)$ and $T(\partial_y, n)U(y)$ their continuous approximations are given $f_\delta(y)$ and $g_\delta(y)$:

$$\max_S |f(y) - f_\delta(y)| + \max_S |T(\partial_y, n)U(y) - g_\delta(y)| \leq \delta, \ \ 0 < \delta < 1. \tag{2.9}$$

Let us define a function

$$U_{\tau\,\delta}(x) = \int_S [\Pi(y, x, \tau) g_\delta(y) - \{T(\partial_y, n)\Pi(y, x, \tau)\}^* f_\delta(y)] ds_y, \tag{2.10}$$

where

$$\tau = \frac{1}{x_m^0} \ln \frac{M}{\delta}, \quad x_m^0 = \max_D x_m, \quad x_m > 0.$$

Theorem 3. *Let $U(x)$ be the regular solution to system* (1.1) *in the domain D, which satisfies* (2.8) *in $\partial D \setminus S$. Then the estimate is fulfilled*

$$|U(x) - U_{\tau\,\delta}(x)| \leq C(x) \delta^{x_m/x_m^0} \left(\ln \frac{M}{\delta} \right)^m, \quad x \in D,$$

where $C(x)$ is a function bounded inside the domain and depending on ρ and the space dimension.

Proof. From formulas (2.2) and (2.10) we have

$$\begin{aligned} U(x) - U_{\tau\,\delta}(x) = \int_S &[\Pi(y, x, \tau)\{T(\partial_y, n)U(y) - g_\delta(y)\} \\ &- \{T(\partial_y, n)\Pi(y, x, \tau)\}^* (U(y) - f_\delta(y))] ds_y \\ &+ \int_{\partial D \setminus S} [\Pi(y, x, \tau)\{T(\partial_y, n)U(y)\} - \{T(\partial_y, n)\Pi(y, x, \tau)\}^* U(y)] ds_y. \end{aligned}$$

Now from the condition of Theorem and inequalities (2.4), (2.8), and (2.9) for any $x \in D$ we obtain

$$\begin{aligned} |U(x) - U_{\tau\,\delta}(x)| &= C'(x)\delta\tau^m \exp\tau(x_m^0 - x_m) + C''(x)\tau^m \exp(-\tau\, x_m) \\ &\leq C(x)\tau^m (M + \delta \exp \tau\, x_m^0) \exp(-\tau\, x_m). \end{aligned}$$

Since $\tau = \frac{1}{x_m^0} \ln \frac{M}{\delta}$, from the latter inequalities the assertion of Theorem follows. □

From these theorems we can obtain the estimate of stability.

Theorem 4. *Let $U(x)$ be the regular solution to system* (1.1) *in the domain D, satisfying conditions*

$$|U(y)| + |T(\partial_y, n)U(y)| \leq M, \ \ y \in \partial D \setminus S,$$
$$|U(y)| + |T(\partial_y, n)U(y)| \leq \delta, \quad 0 < \delta < 1, \quad y \in S.$$

Then

$$|U(x)| \leq C(x) \delta^{x_m/x_m^0} \left(\ln \frac{M}{\delta} \right)^m,$$

where $C(x) = \widetilde{C} \int\limits_{\partial D} \frac{1}{r^m} ds_y$, $\widetilde{C}$ is a constant, which depends on λ, μ, ε, β, ν.

Proof. From Theorem 2 it follows

$$|U(y)| \leq |U_\tau(y)| + MC_m(x)\tau^m \exp(-\tau x_m),$$

$$\begin{aligned} |U_\tau(x)| &\leq \int_S (|\Pi(y, x, \tau)| + |T(\partial_y, n)\Pi(y, x, \tau)|) (|U(y)| + |T(\partial_y, n)U(y)|) \, ds_y \\ &\leq \delta \int_S (|\Pi(y, x, \tau)| + |T(\partial_y, n)\Pi(y, x, \tau)|) \, ds_y \leq \left| \delta C_m(x) \tau^m \exp(-\tau x_m) \int_{\partial S} |r^{-m}| ds_y \right|, \end{aligned}$$

where $x_m^0 = \max\{x_m : x \in \overline{D}\}$. Hence with $\tau = \frac{1}{x_m^0} \ln \frac{M}{\delta}$ we obtain the assertion of Theorem 4. □

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