ISSN 1063-4541, Vestnik St. Petersburg University, Mathematics, 2018, Vol. 51, No. 2, pp. 192–200. © Pleiades Publishing, Ltd., 2018.
Original Russian Text © A.H. Mammadli, 2018, published in Vestnik Sankt-Peterburgskogo Universiteta: Matematika, Mekhanika, Astronomiya, 2018, Vol. 63, No. 2, pp. 331–342.

ASTRONOMY

The Evolution of Jupiter's Orbit in the Case of a Stellar Approach to the Solar System

A. H. Mammadli

Batabat Astrophysical Observatory, Nakhchiivan Department of the Azerbaijan National Academy of Sciences, Nakhchivan, 7000 Azerbaijan

e-mail: azad_mammadli@yahoo.com

Received July 15, 2017; in final form, September 21, 2017

Abstract—The spatial motion of a passively gravitating body is investigated within the restricted three-body problem. The exact expression of the force function without expansion in series is used. The influence of the perturbing star as it approaches the Sun on the orbit of Jupiter is investigated. It is shown that a star of one to five solar masses that approaches the Solar System in a hyperbolic orbit within a minimum distance of 50 to 100 AU significantly affects the size and shape of Jupiter's orbit only in the case when the sample star is at the perihelion, and Jupiter is in conjunction or in opposition to it. The results are shown in the form of figures and tables.

Keywords: celestial mechanics, restricted three-body problem, force function, orbital elements of Jupiter.

DOI: 10.3103/S1063454118020061

1. STATEMENT OF THE PROBLEM. EXPRESSION OF THE DISTURBING FUNCTION

The planar averaged three-body problem is considered in [1, 2] by Mamedov, and the doubly averaged parabolic three-body problem is considered by Mammadli [3]. It is shown that in the case of a moderate approach of the disturbing body to the central body, the dimensions and shape of the orbit of the central body remain constant, and only its orientation changes. The disturbing body is taken to be a solar-mass star; the planetary orbits are studied in the case of its approach toward the Sun.

Kholshevnikov and Mishchuk [4] considered a restricted hyperbolic three-body problem and estimated the influence of a solar-mass star on the orbits of the planets in the case of its approach to the Sun at a distance q' of 100 to 1152 AU. It was shown that under a moderate encounter of such a star with the Sun, the dimensions of the planetary orbits do not exhibit changes. If the star approaches the Sun at a distance $q' \geq 100$ AU, rather minor changes in the inclination, eccentricity, longitude of the ascending node, and argument of the perihelion of the planetary orbits are observed.

The present study is concerned with the evolution of Jupiter's orbit during stellar encounters with the Solar System within the restricted hyperbolic three-body problem.

Let the disturbing body be a star P' with a mass m', which moves relative to the central body, the Sun P_0 with a mass m_0, in a hyperbolic orbit. The motion of the passively gravitating body, Jupiter P with a mass m, needs to be studied.

Let us choose a rectilinear Cartesian coordinate system with the origin at the center of the body P_0. In this coordinate system, the differential equations of motion of the passively gravitating body P will be written as follows [1–3, 5]:

$$\frac{d^2x}{dt^2} = \frac{\partial U}{\partial x}, \quad \frac{d^2y}{dt^2} = \frac{\partial U}{\partial y}, \quad \frac{d^2z}{dt^2} = \frac{\partial U}{\partial z}, \tag{1}$$

where the force function $U = U(x, y, z, x', y', z')$ depends on the coordinates x, y, and z and x', y', and z' of the bodies P and P' and is determined by the equation

$$U = U_0 + R, \quad U_0 = \frac{G(m + m_0)}{r}, \quad R = \frac{Gm'}{r'^2}\left(\frac{r'^2}{\Delta} - r\cos\theta\right). \tag{2}$$

Here, G is the gravitational constant, U_0 is the force function of the undisturbed motion, and R is the disturbing function. Therefore, the system of equations (1) at $U = U_0$, or, equivalently, $R = 0$, is the system of equations of the undisturbed motion. Additionally, r is the radius-vector of the body P, and r' and Δ are the distances of the disturbing body from the central body P_0 and from the point P:

$$r^2 = x^2 + y^2 + z^2, \quad \text{and} \quad r'^2 = x'^2 + y'^2 + z'^2, \tag{3}$$

$$\Delta^2 = (x - x')^2 + (y - y')^2 + (z - z')^2 = r^2 + r'^2 - 2rr'\cos\theta. \tag{4}$$

Here, θ is the angle between the radius-vectors r and r', and the cosine of this angle is determined by the equation

$$\cos\theta = \frac{xx' + yy' + zz'}{rr'} = \alpha\alpha' + \beta\beta' + \gamma\gamma'. \tag{5}$$

The following expressions are used for the rectangular coordinates x, y, and z [5–7]:

$$\begin{aligned} x &= r\alpha, \quad \alpha = \cos u\cos\Omega - \cos i\sin u\sin\Omega, \\ y &= r\beta, \quad \beta = \cos u\sin\Omega + \cos i\sin u\cos\Omega, \\ z &= r\gamma, \quad \gamma = \sin u\sin i. \end{aligned} \tag{6}$$

If all variables in (6) are primed, we obtain similar expressions for the coordinates x', y', and z' [5, 7]. Here, $u = v + \omega$ and $u' = v' + \omega'$ are the arguments of latitude, Ω and Ω' are the longitudes of the ascending node, i and i' are the orbital inclinations of the bodies P and P' to the main plane, ω and ω' are the arguments of pericenters (for Jupiter's orbit, the argument of perihelion), and v and v' are the true anomalies of their orbits.

It should be noted that Eq. (6) is the solution of equation system (1) for the undisturbed motion, i.e., at $U = U_0$ (or $R = 0$) [5]. For the disturbed motion ($R \neq 0$), the solution of equation system (1) is also represented as (6), under the condition that the orbital elements u', Ω', i', a', and e' of the disturbing body are considered known, and the orbital elements u, Ω, i, a, and e of the body P are determined from differential equations, such as the Lagrange equations (see the next section), for the osculating elements [5, 7].

Now let us express the disturbing function R via the orbital elements. For this we will use the orbital equation of the body P:

$$r = \frac{p}{1 + e\cos v}, \quad p = a(1 - e^2) \quad (0 < e < 1). \tag{7}$$

For the hyperbolic motion of the disturbing body P', we have

$$r' = \frac{p'}{1 + e'\cos v'}, \quad p' = a'(e'^2 - 1) = q'(1 + e') \quad (e' > 1). \tag{8}$$

Thus, the disturbing function R from (4), using the abovementioned formulas (5)–(8), is expressed via the orbital elements as follows:

$$R = \frac{Gm'}{r'^2}\left(\frac{r'^2}{\sqrt{r^2 + r'^2 - 2rr'\cos\theta}} - r\cos\theta\right). \tag{9}$$

For brevity, expressions (7) and (8) for r and r', as well as expression (5) for $\cos\theta$ in (9), are not substituted. This substitution is performed in the computer numerical integration of the Lagrange equations for osculating elements.

2. LAGRANGE EQUATIONS FOR OSCULATING ELEMENTS

Let us write out the Lagrange equations for Keplerian osculating elements in the restricted three-body problem (see [5, 7]) in the new independent variable v'. In these equations, the equation relative to v will be considered instead of the equation relative to the mean anomaly M. For this, the following equations can be used [7]:

$$\mu = G(m_0 + m) = n^2 a^3, \quad \mu' = G(m_0 + m'), \tag{10}$$

$$\frac{\partial R}{\partial M} = \frac{\partial R}{\partial v}\frac{\partial v}{\partial M}, \quad \frac{dv}{dt} = \frac{\partial v}{\partial M}\frac{dM}{dt} + \frac{\partial v}{\partial e}\frac{de}{dt}, \quad r'^2\, dv' = \sqrt{\mu' p'}dt,$$
$$\frac{\partial v}{\partial M} = \frac{\partial v}{\partial M_0} = \frac{a^2\sqrt{1-e^2}}{r^2}, \quad \text{and} \quad \frac{\partial v}{\partial e} = \frac{a\sin v}{r}\left(1 + \frac{p}{r}\right), \tag{11}$$

where the radius-vectors r and r', as well as focal parameters p and p', are determined earlier with Eqs. (7) and (8). Let as write out the Lagrange equations as

$$\frac{da}{dv'} = \frac{2a\delta^2}{\sqrt{p}(1-e^2)}\frac{\partial \tilde{R}}{\partial v}, \quad \delta = 1 + e\cos v,$$
$$\frac{de}{dv'} = \frac{1}{e\sqrt{p}}\left(\delta^2\frac{\partial \tilde{R}}{\partial v} - (1-e^2)\frac{\partial \tilde{R}}{\partial \omega}\right),$$
$$\frac{di}{dv'} = \frac{1}{\sin i\sqrt{p}}\left(-\frac{\partial \tilde{R}}{\partial \Omega} + \cos i\frac{\partial \tilde{R}}{\partial \omega}\right), \tag{12}$$
$$\frac{d\Omega}{dv'} = \frac{1}{\sin i\sqrt{p}}\frac{\partial \tilde{R}}{\partial i},$$
$$\frac{d\omega}{dv'} = \frac{1}{\sqrt{p}}\left(-\frac{\cos i}{\sin i}\frac{\partial \tilde{R}}{\partial i} + \frac{1-e^2}{e}\frac{\partial \tilde{R}}{\partial e}\right)$$

and

$$\frac{dv}{dv'} = \frac{\delta^2\mu}{p\sqrt{p}}\frac{r'^2}{\sqrt{\mu\mu' p'}}$$
$$+ \frac{1}{ep\sqrt{p}}\left[-2a^2\delta^2 e\frac{\partial \tilde{R}}{\partial a} - p\delta^2\frac{\partial \tilde{R}}{\partial e} + a\delta^2(1+\delta)\sin v\frac{\partial \tilde{R}}{\partial v} - p(1+\delta)\sin v\frac{\partial \tilde{R}}{\partial \omega}\right]. \tag{13}$$

Here, a, e, and p are the semimajor axis, eccentricity, and focal parameter of the orbit of the body P, while i, Ω, and ω are the inclination to the main plane xy, longitude of the ascending node, and the argument of perihelion, respectively.

In system of equations (12), the function $\tilde{R}$ is related to the disturbing function R from (9), as follows:

$$\tilde{R} = \frac{r'^2 R}{\sqrt{\mu\mu' p'}} = A\left(\frac{r'^2}{\sqrt{r^2 + r'^2 - 2rr'\cos\theta}} - r\cos\theta\right), \tag{14}$$

where

$$A = \frac{m'}{\sqrt{p'(m_0 + m)(m_0 + m')}}. \tag{15}$$

The function $\tilde{R}$ is expressed in the orbital elements by substituting expressions (7) and (8) for r and r', as well as expressions (5) and (6) for $\cos\theta$, into (14). This allows the partial derivatives of the function $\tilde{R}$ with respect to the orbital elements to be calculated. For brevity, no such substitution is not shown here, although it is performed in the computer numerical integration of the Lagrange equations for osculating elements.

Thus, solving the system of equations (12) and (13) using numerical integration, we find the osculating elements

$$a = a(v'), \quad e = e(v'), \quad i = i(v'), \quad \Omega = \Omega(v'), \quad \omega = \omega(v'), \quad v = v(v'),$$

and calculate the mean motion $n = n(v')$ and mean anomaly $M = M(v')$ with the formulas

$$n = \sqrt{\frac{\mu}{a^3(v')}}, \quad M = M_0 + n(t - t_0).$$

Further, we find the relation between the independent variable v' and time t. In the case of a hyperbolic orbit ($e' > 1$) of the disturbing body, this relation is established by the equation [5]

$$\begin{gathered} t - t_0 = \frac{q'\sqrt{q'}}{\sqrt{\mu'(e'-1)^3}}\left[e'\tan F - \ln\tan\left(\frac{F}{2} + \frac{\pi}{4}\right)\right], \\ \tan\frac{F}{2} = \sqrt{\frac{e'-1}{e'+1}}\tan\frac{v'}{2}, \end{gathered} \tag{16}$$

where the mass parameter μ' is determined by Eq. (10).

3. SPECIAL CASES OF THE LAGRANGE EQUATIONS

In the case of small inclinations, it is convenient to use the Lagrange variables $\tilde{p}$ and $\tilde{q}$ instead of the elements i and Ω [7]:

$$\tilde{p} = \tan i \sin\Omega, \quad \tilde{q} = \tan i \cos\Omega, \quad i = a\tan\sqrt{\tilde{p}^2 + \tilde{q}^2}, \quad \Omega = \arctan\frac{\tilde{p}}{\tilde{q}}; \tag{17}$$

at small eccentricities, the Lagrange variables $\tilde{h}$ and $\tilde{k}$ should be introduced instead of the elements e and ω in the following formula [7]:

$$\tilde{h} = e\sin\omega, \quad \tilde{k} = e\cos\omega, \quad e = \sqrt{\tilde{h}^2 + \tilde{k}^2}, \quad \omega = \arctan\frac{\tilde{h}}{\tilde{k}}. \tag{18}$$

As a rule, the variables $\tilde{h}$ and $\tilde{k}$ are introduced instead of the elements e and $\tilde{\omega} = \omega + \Omega$. Since we are interested in the variations in the elements e and ω, we use Eq. (18).

Now the disturbing function $\tilde{R}$ from (14), which is involved in the system of equations (12) and (13), should be expressed through the Lagrange variables:

$$\tilde{R} \equiv \tilde{R}(v, \tilde{h}, \tilde{k}, \tilde{p}, \tilde{q}, u', i', \Omega') = A\left(\frac{r'^2}{\sqrt{r^2 + r'^2 - 2rr'\cos\theta}} - r\cos\theta\right). \tag{19}$$

To do this, it is sufficient to replace the orbital elements e, ω, Ω, and i for $\cos\theta$ and r in expressions (5), (6), and (7) with the Lagrange variables due to Eqs. (17) and (18). Then, we can calculate the partial derivatives of the function $\tilde{R}$ with respect to the Lagrange variables. For brevity, such a substitution is not shown here.

Thus, the Lagrange equations (12) and (13) in the new variables will have the form

$$\begin{aligned} \frac{da}{dv'} &= \frac{2a\delta^2}{\sqrt{p}(1 - \tilde{h}^2 - \tilde{k}^2)}\frac{\partial\tilde{R}}{\partial v}, \\ \frac{d\tilde{h}}{dv'} &= \frac{\tilde{h}\delta^2}{(\tilde{h}^2 + \tilde{k}^2)\sqrt{p}}\frac{\partial\tilde{R}}{\partial v} + \frac{1 - \tilde{h}^2 - \tilde{k}^2}{\sqrt{p}}\frac{\partial\tilde{R}}{\partial\tilde{k}} - \frac{\tilde{k}(1 + \tilde{p}^2 + \tilde{q}^2)}{\sqrt{p}(\tilde{p}^2 + \tilde{q}^2)}\left(\tilde{p}\frac{\partial\tilde{R}}{\partial\tilde{p}} + \tilde{q}\frac{\partial\tilde{R}}{\partial\tilde{q}}\right), \\ \frac{d\tilde{k}}{dv'} &= \frac{\tilde{k}\delta^2}{(\tilde{h}^2 + \tilde{k}^2)\sqrt{p}}\frac{\partial\tilde{R}}{\partial v} - \frac{1 - \tilde{h}^2 - \tilde{k}^2}{\sqrt{p}}\frac{\partial\tilde{R}}{\partial\tilde{h}} + \frac{\tilde{h}(1 + \tilde{p}^2 + \tilde{q}^2)}{\sqrt{p}(\tilde{p}^2 + \tilde{q}^2)}\left(\tilde{p}\frac{\partial\tilde{R}}{\partial\tilde{p}} + \tilde{q}\frac{\partial\tilde{R}}{\partial\tilde{q}}\right), \\ \frac{d\tilde{p}}{dv'} &= \frac{\tilde{p}(1 + \tilde{p}^2 + \tilde{q}^2)}{\sqrt{p}(\tilde{p}^2 + \tilde{q}^2)}\left(\tilde{k}\frac{\partial\tilde{R}}{\partial\tilde{h}} - \tilde{h}\frac{\partial\tilde{R}}{\partial\tilde{k}}\right) + \frac{\sqrt{(1 + \tilde{p}^2 + \tilde{q}^2)^3}}{\sqrt{p}}\frac{\partial\tilde{R}}{\partial\tilde{q}}, \\ \frac{d\tilde{q}}{dv'} &= \frac{\tilde{q}(1 + \tilde{p}^2 + \tilde{q}^2)}{\sqrt{p}(\tilde{p}^2 + \tilde{q}^2)}\left(\tilde{k}\frac{\partial\tilde{R}}{\partial\tilde{h}} - \tilde{h}\frac{\partial\tilde{R}}{\partial\tilde{k}}\right) - \frac{\sqrt{(1 + \tilde{p}^2 + \tilde{q}^2)^3}}{\sqrt{p}}\frac{\partial\tilde{R}}{\partial\tilde{p}}, \end{aligned} \tag{20}$$

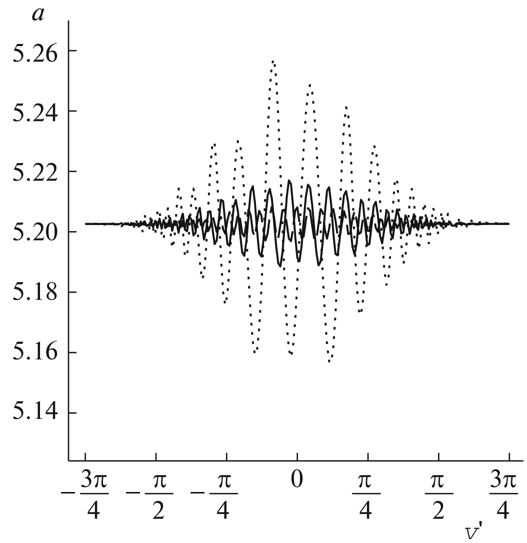

Fig. 1. Variations in the semimajor axis a of Jupiter's orbit depending on the true anomaly v' of the sample star moving in a hyperbolic orbit with $e' = 1.15$ and $m' = 5M_\odot$: dotted line corresponds to $p' = 107.5$ AU ($q' = 50$ AU), solid line corresponds to $p' = 161.25$ AU ($q' = 75$ AU), and dashed line corresponds to $p' = 215$ AU ($q' = 100$ AU).

$$\frac{dv}{dv'} = \frac{\delta^2\mu}{p\sqrt{p}}\frac{r'^2}{\sqrt{\mu\mu' p'}} + \frac{1}{\sqrt{\tilde{h}^2+\tilde{k}^2}\,p\sqrt{p}}\left\{-2a^2\delta^2\sqrt{\tilde{h}^2+\tilde{k}^2}\frac{\partial\tilde{R}}{\partial a} + a\delta^2(1+\delta)\sin v\frac{\partial\tilde{R}}{\partial v}\right.$$
$$\left. - p\left[\frac{\delta^2\tilde{h}}{\sqrt{\tilde{h}^2+\tilde{k}^2}} - \tilde{k}(1+\delta)\sin v\right]\frac{\partial\tilde{R}}{\partial\tilde{h}} - p\left[\frac{\delta^2\tilde{k}}{\sqrt{\tilde{h}^2+\tilde{k}^2}} - \tilde{h}(1+\delta)\sin v\right]\frac{\partial\tilde{R}}{\partial\tilde{k}}\right\}. \quad (21)$$

Here,

$$\delta = 1 + \sqrt{\tilde{h}^2+\tilde{k}^2}\cos v, \quad p = a(1-\tilde{h}^2-\tilde{k}^2).$$

The system of equations (20) for osculating elements is applicable for determining the elements and studying the evolution of Jupiter's orbit, since its inclination and eccentricity are rather small.

4. CHANGE IN THE ORBITAL ELEMENTS OF JUPITER DURING A STELLAR ENCOUNTER WITH THE SOLAR SYSTEM

As an example, we will take a sample star approaching the Solar System with mass m', heliocentric distance q' (in AU), and orbital eccentricity e'. These parameters vary within

$$M_\odot \le m' \le 5M_\odot, \quad 50 \le q' \le 100, \quad 1 < e' \le 5, \quad (22)$$

where $M_\odot$ is the mass of the Sun. Additionally, the angular elements i', Ω', and ω' of the sample star are referred to the coordinate system $Oxyz$ with its origin at the center of the Sun; these elements vary within

$$0° \le i' \le 90°, \quad 0° \le \Omega' \le 180°, \quad 0° \le \omega' \le 180°. \quad (23)$$

The initial values of Jupiter's orbital elements are taken from the Astronomical Yearbook of 1988 and are determined as

$$a_0 = 5.2026032 \text{ (a.e.)}, \quad e_0 = 0.04849485, \quad i_0 = 1°18'11.''77,$$
$$\lambda_0 = 34°21'05.''34, \quad \pi_0 = 14°19'52.''71, \quad \Omega_0 = 100°27'51.''98,$$

Jupiter's angular elements i_0, λ_0, π_0, and Ω_0 are referred to the ecliptic and equinox of the epoch J2000.0, and the gravitational constant equals the Gauss constant: $G = k^2 = 0.000295936$. Additionally, the elements ω_0 and M_0 for Jupiter are determined by the equations $\omega_0 = \pi_0 - \Omega_0$ and $M_0 = \lambda_0 - \pi_0$.

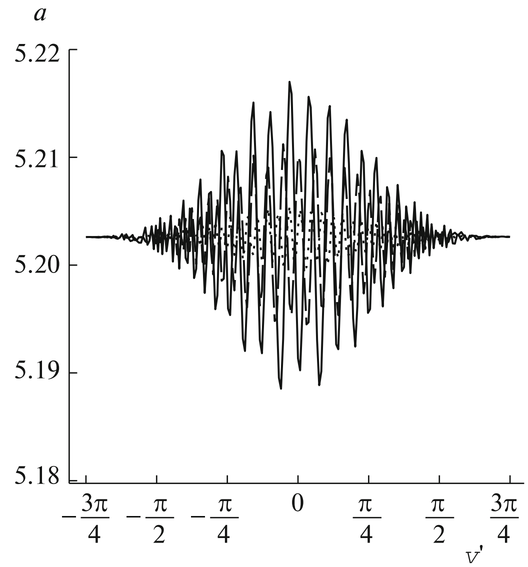

Fig. 2. Variations in the semimajor axis a of Jupiter's orbit at $p' = 161.25$ AU: dotted line corresponds to the mass $m' = M_{\odot}$ of the sample star. dashed line corresponds to $m' = 3M_{\odot}$, and solid line corresponds to $m' = 5M_{\odot}$.

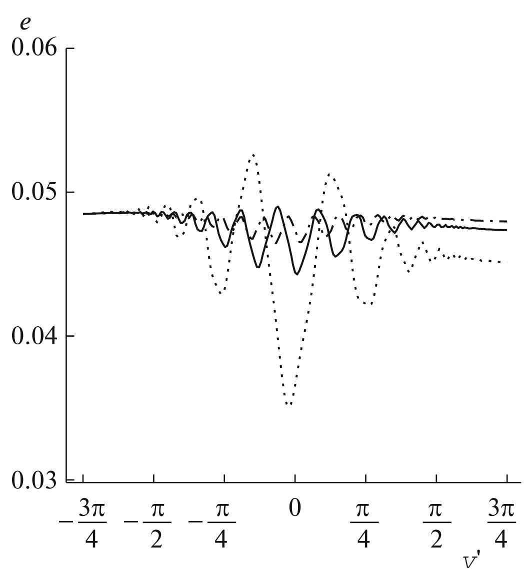

Fig. 3. Variations in the eccentricity e of Jupiter's orbit depending on the true anomaly v' of the sample star moving in a hyperbolic orbit with $e' = 1.15$ and $m' = 5M_{\odot}$: dotted line corresponds to $p' = 107.5$ AU, solid line corresponds to $p' = 161.25$ AU, and dashed line corresponds to $p' = 215$ AU.

Using the above-mentioned initial values for Jupiter's orbital elements, differential equation system (20) is numerically integrated in the Lagrange variables at the initial value of the independent variable $v_0' = -3\pi/4$. The orbital elements of the sample star are taken as $e' = 1.15$, $i' = 5°$, $\Omega' = 0°$, and $\omega' = 40°$. These elements play an important role in the construction of the diagrams and tables.

Figures 1 and 2 show the variations in the semimajor axis a (with the initial value $a_0 = 5.2026032$ AU) of Jupiter's orbit as a function of the true anomaly v' of the star moving in a hyperbolic orbit ($e' > 1$) relative to the Sun at certain values of its mass m' (Fig. 2) and focal parameter of its orbit p' (Fig. 1). The focal

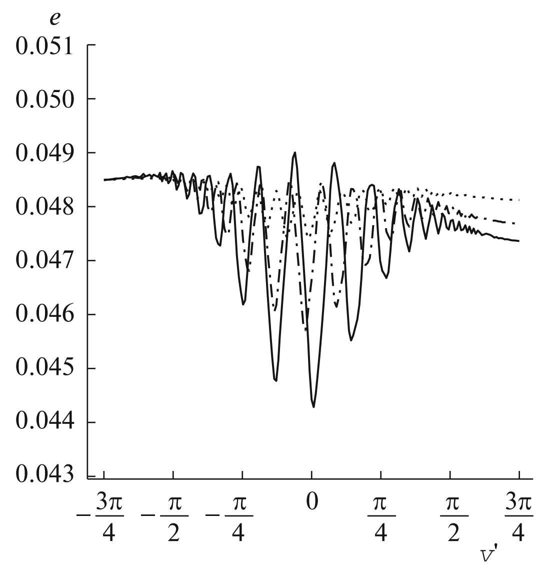

Fig. 4. Variations in the eccentricity e of Jupiter's orbit at $p' = 161.25$ AU: dotted line corresponds to the mass $m' = M_\odot$ of the sample star, dashed line corresponds to $m' = 3M_\odot$, and solid line corresponds to $m' = 5M_\odot$.

parameter p' and perihelion distance q' are related via the equation $p' = q'(1 + e')$ for the hyperbolic orbit of the star.

Figures 3 and 4 illustrate the variations in the eccentricity e (with an initial value $e_0 = 0.04849485$), while Figs. 5 and 6 show the variations in the inclination i (with the initial value $i_0 = 1°.30327$) of Jupiter's orbit with similar changes in the mass and perihelion distance of the star with the orbital eccentricity $e' = 1.15$, depending on the true anomaly v', respectively. As can be seen from the figures, after the star recedes to a large distance from the Sun, Jupiter's orbital elements a, e, and i slightly change and differ from the initial values a_0, e_0, and i_0. However, the maximum changes in the size and shape of Jupiter's orbit occur only in the case when the sample star is at perihelion, and Jupiter is in opposition.

Table 1 lists the changes in the orbital elements of Jupiter Δa, Δe, and Δi depending on the parameter p' for the hyperbolic ($e' = 1.15$) orbit of the star and on its mass m'. As can be seen from the table, the changes in the orbital elements of Jupiter (semimajor axis a and eccentricity e) are minor. The maximum changes in the elements a and e occur where a star with a mass $m' = 5M_\odot$ at a distance $q' = 50$ AU (or $p' = 107.5$ AU) approaches the Solar System in a hyperbolic orbit.

Table 1. Changes in the orbital elements of Jupiter Δa, Δe, and Δi depending on the focal parameter p' for the hyperbolic orbit ($e' = 1.15$) of the star and on its mass m'

p' (AU)	m'	Δa (AU)	Δe	Δi (deg.)
107.5	$M_\odot$	−0.009631	0.000214	0.075275
	$3M_\odot$	0.025592	−0.003532	0.169424
	$5M_\odot$	−0.030154	−0.012621	0.238626
161.25	$M_\odot$	0.002507	−0.001059	0.039625
	$3M_\odot$	0.007816	−0.002124	0.087279
	$5M_\odot$	−0.012734	−0.004218	0.121404
215	$M_\odot$	0.001077	−0.000491	0.025249
	$3M_\odot$	0.003302	−0.001224	0.055089
	$5M_\odot$	0.005928	−0.001482	0.076197

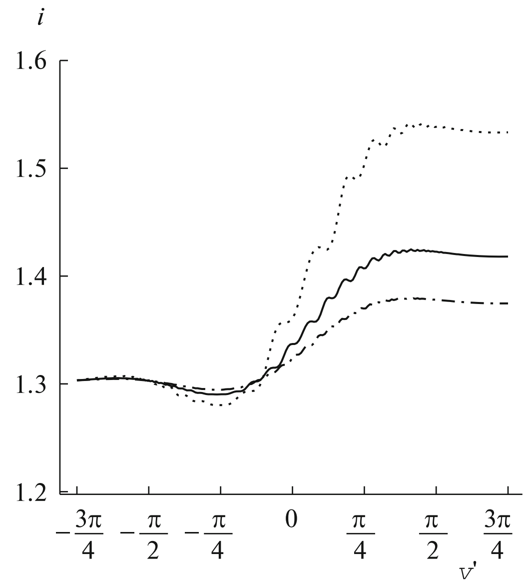

Fig. 5. Variations in the inclination i of Jupiter's orbit depending on true anomaly v' of the sample star moving in a hyperbolic orbit with $e' = 1.15$ and $m' = 5M_\odot$: dotted line corresponds to $p' = 107.5$ AU, solid line corresponds to $p' = 161.25$ AU, and dashed line corresponds to $p' = 215$ AU.

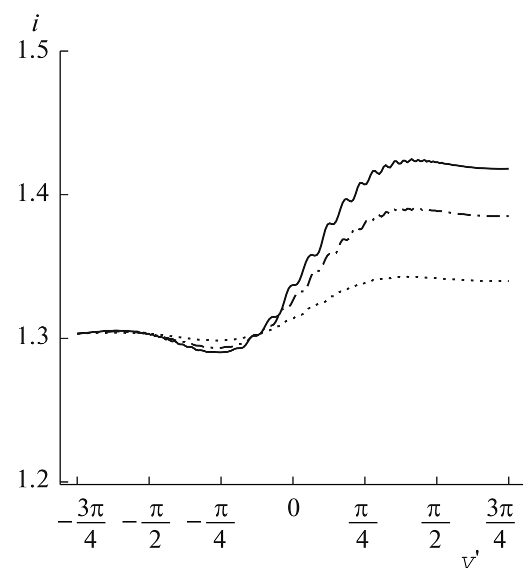

Fig. 6. Variations in the inclination i of Jupiter's orbit at $p' = 161.25$ AU: dotted line corresponds to the mass $m' = M_\odot$ of the sample star, dashed line corresponds to $m' = 3M_\odot$, and solid line corresponds to $m' = 5M_\odot$.

5. CONCLUSIONS

The problem of the evolution of Jupiter's orbit during stellar encounters with the Solar System has been considered within the restricted hyperbolic three-body problem. The influence of a disturbing body (a star) as it approaches the center body (the Sun) in a hyperbolic orbit on the orbit of a passively gravitating body (Jupiter) has been studied. The exact expression of the force function has been used without expansion in series.

Variations in the orbital elements of Jupiter depending on the true anomaly of the star moving in a hyperbolic orbit relative to the central body have been determined. Also, the variations in the orbital elements of Jupiter depending on the perihelion distance of the star (or the focal parameter of its orbit) and on its mass have been found. The results are presented in figures and tables.

It has been shown that a star with a mass of one to five solar masses approaching the Solar System in a hyperbolic orbit at a minimum distance between 50 and 100 AU from the Sun significantly influences the shape and dimensions of Jupiter's orbit only in the case when the sample star is at the perihelion, and Jupiter is in opposition.

REFERENCES

1. A. G. Mamedov, "Averaged parabolic restricted three-body problem," Astron. J. **66**, 377–384 (1989).
2. A. G. Mamedov, "About secular perturbations of elements in the restricted parabolic three-body problem," Astron. J. **68**, 1323–1327 (1991).
3. A. G. Mammadli, "The limiting case of the double-averaged parabolic restricted three-body problem," Astron. Vestn. **41**, 186–189 (2007).
4. K. V. Kholshevnikov and Yu. F. Mishchuk, "The effect of stellar encounters on planetary orbits," Vestn. Leningr. Univ., No. 7, 72–81 (1983).
5. G. N. Duboshin, *Celestial Mechanics. The Main Problems and Methods* (Nauka, Moscow, 1968) [in Russian].
6. E. P. Aksenov, *Theory of the Motion of Artificial Earth's Satellites* (Nauka, Moscow, 1977) [in Russian].
7. M. F. Subbotin, *An Introduction into Theoretical Astronomy* (Nauka, Moscow, 1968) [in Russian].

Translated by M. Chubarova