ISSN 1062-8738, Bulletin of the Russian Academy of Sciences: Physics, 2020, Vol. 84, No. 4, pp. 441–445. © Allerton Press, Inc., 2020.
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Analyzing Cascade Gamma-Decays of Nuclei at Energies of Excitation Below That of Neutron Bonding

A. M. Sukhovoj[a, *], L. V. Mitsyna[a], D. C. Vu[a, b], N. Jovancevic[c], D. Knezevic[c, d], M. Krmar[c], and A. Dragic[d]

[a] *Joint Institute for Nuclear Research, Dubna, Moscow oblast, 141980 Russia*
[b] *Institute of Physics, Vietnam Academy of Sciences and Technology, Hanoi, 100000 Vietnam*
[c] *Department of Physics, Faculty of Science, University of Novi Sad, Novi Sad, 21000 Serbia*
[d] *Belgrade Institute of Physics, Belgrade, 11080 Serbia*
**e-mail: suchovoj@nf.jinr.ru*

Received October 30, 2019; revised November 25, 2019; accepted December 27, 2019

Abstract—Data obtained with the γ-calorimeter of the DANCE spectrometer at the Los Alamos National Laboratory (LANL) are reanalyzed using the empirical model developed at the Joint Institute for Nuclear Research (JINR, Dubna), in which the density and partial radiative widths of nuclear levels are simultaneously derived from the measured intensity of gamma two-stage cascade transitions as a function of primary transition energy E_1.

DOI: 10.3103/S1062873820040322

INTRODUCTION

In modern investigations of the two-stage gamma decays of excited nuclei performed with technologically-advanced detectors, the primary aim is to glean fundamental information on intranuclear processes rather than merely measuring the energy levels, spins, and lifetimes of excited nuclear states.

When probing nucleon interactions in the multistage gamma-cascade decay of a nucleus, the first task is to reconstruct the time ordering of emitted γ-quanta. For an M-stage gamma cascade decay, elementary transitions can be time-ordered a priori in the gamma spectrum (or the decay scheme) in $M!$ different ways. In a two-stage gamma-cascade decay (TGC), elementary transitions can be time-ordered in two different ways, one of which is spurious. At the same time, a five-stage cascade decay allows 120 possible orderings, of which only one is actually observed in nature. The aim of our indirect experiment [1] is to obtain the gamma-ray spectrum for primary transitions of two-stage gamma-cascade decays. In discriminating the primary and secondary transitions, we rely on spectrometric data and consider that the spectra of energetically-allowed and forbidden continuum transitions have different shapes.

Reliably quantifying the parameters of cascade gamma-decays of compound nuclei is of paramount importance in understanding the processes that occur in an excited nucleus. Although the nucleus is usually visualized as a system of noninteracting Fermi particles, we may assume that constituent nucleons form Cooper pairs which break up as soon as the nucleus is excited. The process of Cooper-pair breakup has never been studied experimentally, since no optically-efficient gamma spectrometers with electronvolt-level energy resolution are available. A comparison of neutron bonding energy B_n and constituent-nucleon pairing energy Δ [2] suggests that at least for the investigated nuclei in the $28 \le A \le 200$ mass range, some 3–4 Cooper pairs break up at energies of excitation below B_n.

To understand the mechanisms of intranuclear processes, we must simultaneously measure the density of excited nuclear levels ρ and such cascade gamma-decay parameters as the levels' partial radiative widths Γ or strength functions $k = \Gamma/\left(A^{2/3}E_\gamma^3 D_\lambda\right)$, where A, E_γ, and D_λ denote the nucleus mass number, γ-quantum energy, and mean distance between neighboring compound states, respectively. A way of simultaneously deriving nuclear parameters ρ and Γ from the TGC complete γ-spectrum was proposed and used for the first time at the JINR Laboratory of Neutron Physics in 1984 [3–5]. Two Ge(Li) detectors were initially employed to detect and analyze two sucessively-emitted γ-quanta with net energies of 5–10 MeV, at which a sample of several thousand TGC events with total absorption of the γ-cascade energy was accumulated. Since 2000, TGC measurements have been made with HPGe detectors that are more efficient than their Ge(Li) counterparts. So far, γ-decay

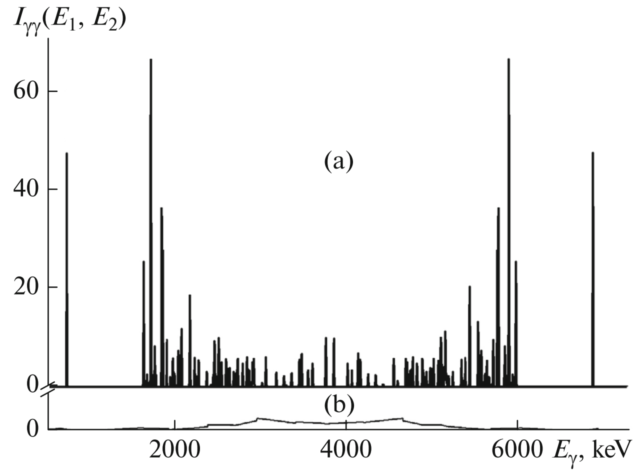

Fig. 1. Intensities of γ-quanta emitted in (a) energetically allowed elementary transitions and (b) the continuum of forbidden ones for the TGC transition to the excited level of a ^{164}Dy nucleus with $E_f = 73$ keV. The former distribution is symmetric with respect to mean energy $E_\gamma = 0.5(E_1 + E_2)$ with the γ-energy regions above and below this value populated by primary and secondary γ-quanta, respectively. The latter distribution includes a small background component.

parameters have been derived from the measured TGC γ-spectra for 44 different nuclei [6] using the evolving techniques pioneered in [3–5].

The experimental gamma-intensity spectrum of a cascade transition contains isolated gamma-lines corresponding to energetically-allowed (intensive) radiative transitions and a continuous distribution due to small-amplitude forbidden transitions with zero mean value [7]. For a TGC transition, the energy distribution of gamma intensity centers on $0.5(E_1 + E_2)$ where E_1 and E_2 are the energies of primary and secondary γ-quanta, respectively. The typical TGC gamma-intensity spectrum shown in Fig. 1 is associated with the TGC transition at a net γ-quanta energy of $E_1 + E_2 =$ 7585 keV to the first excited level of the ^{164}Dy nucleus.

The process of radiative neutron capture by the ^{163}Dy nucleus, which involves TGC transition ^{163}Dy$(n, 2\gamma)$, has been investigated in two experiments using different techniques of analysis:

(1) The experiment performed at the Joint Institute for Nuclear Research (Dubna) employed two Ge(Li) detectors positioned close to each other on both sides of the target, and normal to the direction of the neutron beam. The most probable values of the ρ and Γ parameters at energies of excitation below B_n were obtained from the measured gamma-intensity spectra using the maximum likelihood approach and model parametrizations of nuclear parameters.

(2) The experiment performed at the Los Alamos National Laboratory employed an electromagnetic calorimeter with 4π geometry [8]. The measured spectra of γ-quanta from the decays of ^{164}Dy neutron resonances were analyzed using the DICEBOX algorithm [9], with which multistage gamma cascade transitions with arbitrary multipolarities can be simulated [10].

These two experiments relied on different model assumptions in deriving nuclear parameter values from measured TGC gamma-ray spectra. Let us now discuss and compare the problems that arise in these approaches to data analysis.

IMPORTANCE OF CORRECTLY NORMALIZING THE GAMMA-RAY SPECTRUM

In analyzing the total TGC gamma-ray spectrum for the values of nuclear parameters ρ and Γ (or k), we always rely on some model assumptions. The measured gamma-ray spectrum is always fitted using model functional forms of $\Gamma(E_\gamma)$ and $\rho(E_{\rm ex})$. The procedure for extrapolating uncertainties δS of the experimental spectrum to those on the derived nuclear parameter values merits special attention. Given that the relative error on the experimental spectrum $\delta S/S$ differs from those on the extracted parameters $\delta\rho/\rho$ and $\delta\Gamma/\Gamma$ by a factor of 1.5–2, absolute errors $\delta\rho$ and $\delta\Gamma$ can exceed δS by a factor of 5–10. In the above JINR experiment involving Ge(Li) detectors, the difference between the areas of the experimental and fitted spectra is usually ~1%, once gamma-cascade spectra are averaged over energy intervals of 200–250 keV.

For the cascade decay of a given nucleus, the estimated fraction of TGC net intensity per decay should be the same for all measurements. Correctly normalizing the gamma-intensity spectrum to one decay of the nuclear compound state (neutron resonance) is therefore an important experimental problem for both analyses (those based on gamma-cascade modeling [9] and the maximum likelihood approach).

For all nuclear TGC transitions investigated at the Joint Institute for Nuclear Research [6], the spectra of two-gamma cascades were normalized using the absolute intensities of strong primary transitions obtained in [11, 12]. Branching fractions $b_{\rm r}$ for the intermediate excited levels and the intensities of primary transitions in compound-state decays were thus accurately derived from the experimental data in real time. Formulating the intensity of decay for an individual cascade as $i_{\gamma\gamma} = i_\lambda b_{\rm r}$, where i_λ is the intensity of compound-state decay per decay, the net intensity of all possible gamma-cascade transitions between the top and bottom levels E_λ and E_f through intermediate levels E_i can be obtained as sum $I_{\gamma\gamma} = \sum i_{\gamma\gamma}$. With the latter, we can determine the nuclear-parameter values iteratively by solving a system of nonlinear equations in which the measured TGC gamma-intensity is expressed through parametrized functional forms of $\rho(E_{\rm ex})$ and $\Gamma(E_\gamma)$.

Note, however, that since the above system of equations in nonlinear, the measured total TGC intensity $I_{\gamma\gamma}(E_1, E_2)$ can be reproduced precisely with an infinite variety of very different pairs of correlated $\rho(E_{ex})$ and $\Gamma(E_\gamma)$ functions. On the other hand, using TGC intensities measured only for primary transitions $I_{\gamma\gamma}(E_1)$, nuclear parameters can be probed over smaller regions of energy than with $I_{\gamma\gamma}(E_1, E_2)$.

In the JINR approach based on fitting the spectrum of primary transition intensity $I_{\gamma\gamma}(E_1)$, γ-quanta from TGC transitions are time-ordered using additional spectrometric data, whereupon the latter spectrum can be obtained directly from the full $I_{\gamma\gamma}(E_1, E_2)$ distribution. After numerically improving the time resolution [13], the analysis procedure detailed in [1] allows us to determine primary transition contribution $I_{\gamma\gamma}(E_1)$ to any energy interval with a precision of 10–20% while not distorting the normalization of the TGC spectrum. We thus obtain $I_{\gamma\gamma}(E_1) = 45.9\%$ for the net intensity per ^{164}Dy decay [14].

TGC γ-intensities were not normalized per decay in analyzing the data from the LANL experiment with scintillation detectors [8], and the detector resolution proved to be insufficient for identifying intense elementary transitions. In this experiment, any information on the ρ and Γ values could be derived only from the γ-spectra for cascades with $M = 2$. Time-ordering of emitted γ-quanta was technically impossible in the LANL 4π-experiment, and therefore cannot be used to exclude the spurious ordering of elementary transitions. The intensities of two-stage γ-cascades were also probably underestimated, due to the irreducible cross-talk between the detector crystals caused by annihilation γ-quanta, along with poor detector resolution at low energies. In the DICEBOX-based modeling [9] employed in [8], the strength functions most appropriate for the three forms of the density-level function were selected from those available in [15].

ANALYZING EXPERIMENTAL DATA OBTAINED INDIRECTLY

Our analysis relies on the standard maximum likelihood approach to solving the system of equations that express the measured TGC intensities of γ-cascades with unknown numbers n_j of intermediate levels through the sought partial and total widths Γ in the intervals of the energies of primary transitions:

$$I_{\gamma\gamma}(E_1) = \sum_{\lambda,f}\sum_{i}\frac{\Gamma_{\lambda i}}{\Gamma_\lambda}\frac{\Gamma_{if}}{\Gamma_i} = \sum_{\lambda,f}\sum_{j}\frac{\Gamma_{\lambda j}}{\langle\Gamma_{\lambda j}\rangle M_{\lambda j}}n_j\frac{\Gamma_{jf}}{\langle\Gamma_{jf}\rangle m_{jf}}. \qquad (1)$$

Here, $M_{\lambda j}$ is the unknown number of γ-transitions from compound states λ to intermediate levels n_j, and m_{jf} is the number of secondary transitions to bottom levels of cascades f. At each successful iteration, we chart the parameter variation trajectories with minimum χ^2 values and fitted TGC intensities $I_{\gamma\gamma}$. By doing so, we can control the process of reaching absolute minimum

$$\chi^2 = \sum_{n_j}\frac{\left(I_{\gamma\gamma}^{\text{cal}}(E_1) - I_{\gamma\gamma}^{\text{exp}}(E_2)\right)^2}{\sigma^2}, \qquad (2)$$

where $I_{\gamma\gamma}^{\text{exp}}(E_1)$ is the measured TGC intensity, $I_{\gamma\gamma}^{\text{cal}}(E_1)$ is its model parametrization, and σ^2 is the dispersion of the difference between them.

For a given nuclear model with definite parametrizations of functions $\rho(E_{ex})$ and $\Gamma(E_\gamma)$, the system of Eqs. (1) has a single solution. Our state-of-the-art analysis relies on the currently accepted parametrizations of the quasiparticle [16] and vibrational [17] level densities, balancing the entropy and energy variations for quasiparticle states [18], and testable forms [19] of the energy dependences of radiative strength functions.

The uncertainties of our model parametrizations of nuclear parameters result in systematic errors in their fitted values that can be reduced only by refining the corresponding models.

In the analysis of the LANL data [8] which is based on simulating the gamma-ray spectrum, primary transitions are not selected in the TGC spectra, and the radiative widths of the secondary and subsequent transitions remain unknown. We may reasonably assume that in [8], as in [10], the authors analyzed the spectra divided to N energy intervals in terms of the criterion

$$\chi^2 = \sum_{i=1}^{N}\frac{\left(\bar{A}_{\text{exp}}(E_i) - \bar{A}_{\text{sim}}(E_i)\right)^2}{\varepsilon_{\text{exp}}^2(E_i) + \varepsilon_{\text{sim}}^2(E_i)}, \qquad (3)$$

where $\bar{A}_{\text{exp}}(E_i)$ and $\bar{A}_{\text{sim}}(E_i)$ are the numbers of counts in the ith energy interval for the measured and simulated spectra, respectively, and $\varepsilon_{\text{exp}}^2$ and $\varepsilon_{\text{sim}}^2$ are the corresponding dispersions.

In [8], the energy dependence of nuclear-level density was assumed to be smooth and known a priori, while strength functions k matching the measured gamma-ray spectra were derived within existing models. However, different forms of the strength function should be tested simultaneously with model parametrizations of nuclear-level densities.

RESULTS FROM OUR REANALYSIS OF THE LANL EXPERIMENTAL DATA IN [8]

Below, we report the results from our reanalysis of the measurements of TGC γ-intensities for the ^{164}Dy nucleus made with the 4π-calorimeter [8]. In this reanalysis of the data using the maximum likelihood approach, we simultaneously derive the values of nuclear parameters ρ and Γ while allowing for the anticorrelation between them.

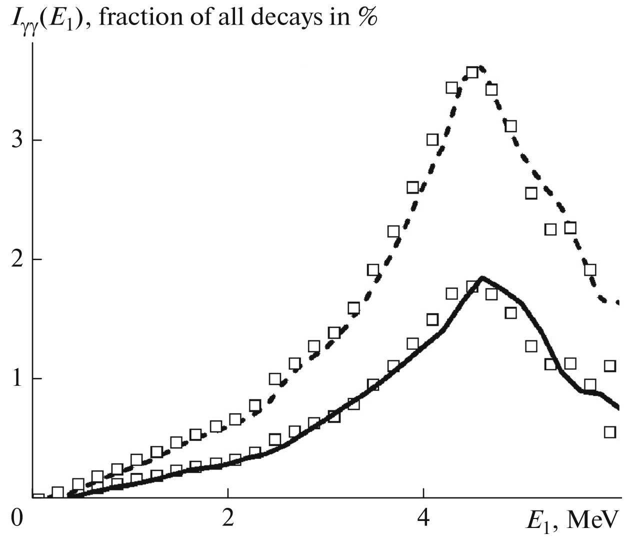

Fig. 2. Primary transition TGC intensity $I_{\gamma\gamma}(E_1)$ for the ^{164}Dy nucleus, obtained from the LANL data in [8], assuming that $I_{\gamma\gamma} = 45$ and 22%, for the net intensity per decay (top and bottom squares fitted by the dashed and solid curves, respectively). The γ-intensities falling within each 200-keV bin have been summed.

To obtain the $I_{\gamma\gamma}(E_1)$ distribution from the LANL data [8] stored in the database in [14], we consider the component of intensity that deals mostly with primary transitions while maintaining the condition $I_{\gamma\gamma}(E_1) = I_{\gamma\gamma}(E_2)$. In the calculus, we substitute $I_{\gamma\gamma} = 45\%$ for the net TGC intensity per decay as experimentally estimated in [14] for the same nucleus and, for comparison, the lower value of $I_{\gamma\gamma} = 22\%$. The resulting $I_{\gamma\gamma}(E_1)$ distributions are shown in Fig. 2.

The results of our reanalysis of the LANL experimental data in [8] are compared to those obtained in the JINR experiment [20] and shown in Figs. 3 and 4. The original analysis of the 4π-calorimeter data [8] yielded 3.05(6) and 5.0(1) MeV for the thresholds of the breakup of the second and third Cooper nucleon pairs, respectively. In the JINR experiment [20] in which the process of thermal-neutron capture was used, the same energy thresholds were found to be 2.57(1) and 5.48(5), respectively. The data in Figs. 3 and 4 show that varying the normalization of TGC intensity strongly affects the derived values of nuclear parameters (note that ρ is affected to a lesser extent).

The theory of internuclear processes in excited nuclei [15, 22] suggests that the structure of the wavefunctions of excited nuclear states varies as the energy of excitation rises. The energy dependences of the nuclear-level density and radiative strength functions are thereforenot expected to be smooth. Since the γ-cascade intensity spectrum is largely affected by the functional form of $\rho(E_{ex})$, the dependence of nuclear parameters on the structure of excited state wave functions cannot be probed if the anticorrelation between

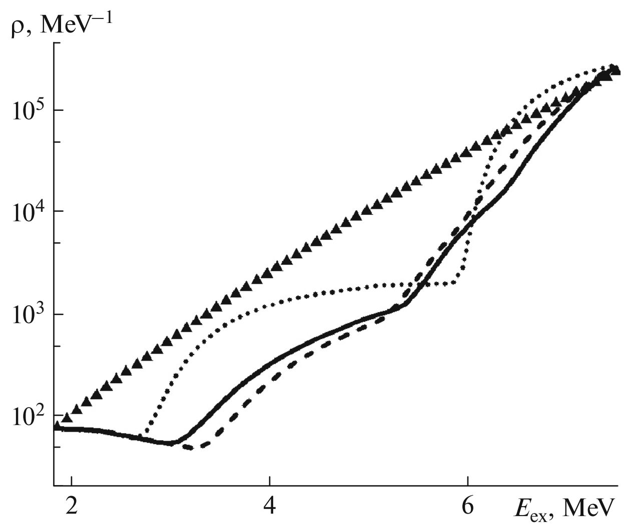

Fig. 3. Nuclear-level density ρ as a function of excitation energy E_{ex} for the ^{164}Dy nucleus. The functional forms obtained in our reanalysis of the LANL data in [8], assuming that $I_{\gamma\gamma} = 45$ and 22%, are depicted by the dashed and solid curves, respectively. The dotted curve is the ρ form obtained by fitting the TGC intensities measured in the JINR experiment [20], assuming that $I_{\gamma\gamma} = 45\%$. The calculated form based on the Fermi gas model with reverse displacement [21] is shown by triangles.

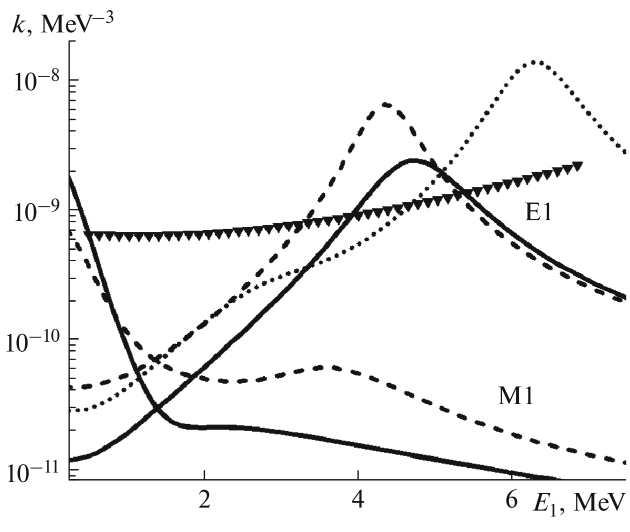

Fig. 4. Radiative strength functions of E1 and M1 transitions in gamma-cascades with $M = 2$ in ^{164}Dy. Those obtained in our reanalysis of TGC intensities measured in [8], assuming that $I_{\gamma\gamma} = 45$ and 22% are depicted by dashed an solid curves, respectively. The dotted curve is the E1 radiative strength function as obtained in [20] by analyzing the JINR data. The form of the latter strength function derived within the theoretical scheme in [19], assuming that k(M1) = const, is shown by the triangles.

these parameters is ignored as in the original LANL analysis [8].

CONCLUSIONS

Using the analysis procedure developed at the Joint Institute for Nuclear Research, experimental TGC γ-intensity spectra can be reproduced adequately by parametrizing nuclear parameter dependences $\rho(E_{ex})$ and $\Gamma(E_1)$ with the functional forms in [16–19]. This allows us to derive the values of both nuclear parameters from the experimental data simultaneously. So far, the measured γ-spectra are best reproduced with the n-quasiparticle level density as modeled in [16].

The fitted form of $\rho(E_{ex})$ shows a step function–like structure with an energy pitch of nearly 2Δ, where Δ is the pairing energy of the last constituent nucleon. These steps can be interpreted as the breakup points of Cooper nucleon pairs in the nucleus.

Since elementary primary transitions cannot be identified in the TGC γ-spectra obtained in the LANL 4π-experiment, these data cannot be used either to probe the intranuclear processes in excited nuclei or test existing models of nuclear parameters.

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Translated by A. Asratyan