# **Studying the Thermal Neutron Spectrum of the W–Be Photoneutron Source at the Institute for Nuclear Research, Russian Academy of Sciences**

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**Abstract**—The possibility is considered of reconstructing the thermal neutron flux density of a W–Be photoneutron source via neutron activation analysis of Ag, Mg, Fe, Mn, and Ti samples. The use of a simple function in the form *AE*−*B* describing the thermal region of the spectrum is discussed as an example of reconstruction. The dependence of the reaction rates' deviation from the ones calculated on the parameters of the approximating function is investigated. The convergence of the reconstruction results as the number of experimental data grows is obtained. This means of recovery can be used for operational measurements of the thermal neutron flux density of the photoneutron source and to optimize the parameters of the source.

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### INTRODUCTION

In [1], we considered the option of reconstructing the low-energy part of the neutron spectrum of a photoneutron source by means of neutron activation analysis (NAA). The IN-LUE photoneutron W–Be source of neutrons [1, 2] was created on the basis of the linear electron accelerator at the Institute for Nuclear Research, Russian Academy of Sciences. Its main parameters are characteristics of the neutron field that forms during its operation and depend on its design features. These are the neutron flux density inside and outside the source, and the neutron energy distribution (spectrum) inside the source and at the outlet of neutron measuring channels. Simulation of the photoneutron source characteristics reveals the complex neutron spectrum of the source. Epithermal and fast neutrons make contributions in addition to the main contribution from thermal neutrons [3]. According to [1], low-energy segments of the spectrum can be approximated by linear functions in logarithmic coordinates along the abscissa and ordinate axes, or by exponential functions of the form  $\Phi = AE^{-B}$ in the rectangular coordinate system. Here Φ and *Е* are the flux density and the energy of neutrons. In this work, we consider the possibilities of such an approach to studying the thermal neutron spectrum of a photoneutron source.

## ACTIVATION METHOD OF MEASURING NEUTRON FIELD PARAMETERS

The flux density of thermal neutrons inside and outside the source is measured via neutron activation analysis (NAA) [4] using activation detectors made of materials with known activation cross sections of (*n*,γ) reactions. All measurements are based on determining the activity induced by neutrons in the material of the detector.

In the NAA, the area of the gamma-spectrum peak of the *i*-th radionuclide measured by the activated detector is written as

$$
S_i = \frac{m_i g_i N_A p_i \varepsilon}{A_i \lambda_i} K_i J_i, \qquad (1)
$$

where  $S_i$  is the number of counts in the analytical peak of the *i*-th element isotope (without background);  $m<sub>i</sub>$  is the mass of the  $i$ -th element of the detector;  $g_i$  is the content of the *i*-th element analytical isotope in the natural isotope mixture;  $N<sub>A</sub>$  is the Avogadro number;  $p_i$  is gamma-quantum yield per decay of the generated radioactive isotope;  $\varepsilon$  is the efficiency of registering the induced radioactivity;  $A_i$  is the atomic weight of the *i*-th element; and  $\lambda_i$  is the radioactive decay rate of the *i*-th element analytical isotope. Here,  $J_i$  is the integral of convolution (the reaction rate):

$$
J_i = \int_0^\infty \sigma_i(E) \varphi(E) dE, \qquad (2)
$$

where  $\sigma_i(E)$  is the activating nuclear reaction cross section, depending on neutron energy  $E$ ;  $\varphi(E)$  is the spectral density of the neutron flux:

$$
K_i = (1 - e^{-\lambda_i t_a}) e^{-\lambda_i t_s} (1 - e^{-\lambda_i t_m}); \qquad (3)
$$

 $t_a$  is the time of activation;  $t_s$  is the time of holding after irradiation;  $t<sub>m</sub>$  is the measurement time.

The areas of gamma-spectrum peaks of radionuclides are measured during the experiment to obtain the experimental values of convolution integral *J* that correspond to them:

$$
J_i = S_i \Big/ K_i^{\text{exp}} \,, \tag{4}
$$

where  $K_i^{\text{exp}}$  is the coefficient, depending on how we select an isotope and the experimental conditions. The experimental values of *J* obtained for a set of activation detectors are used to reconstruct the neutron spectrum.

To do so, we represent convolution integrals (2) as sum *m* of the products of the values of cross sections σ(*Ej* ) and neutron flux density Φ(*Ej* ) = ϕ(*Ej* )Δ*Ej* averaged over *m* elementary energy bins  $E_j$  with width  $\Delta E_j$ . For the set of *k* detectors, we have a system of *k* convolution equations in unknown values of neutron flux density Φ(*Ej* ):

$$
\begin{cases} \sum_{j=0}^{m} \sigma_{1}(E_{j}) \Phi(E_{j}) = J_{1} \\ \dots \\ \sum_{j=0}^{m} \sigma_{k}(E_{j}) \Phi(E_{j}) = J_{k} \end{cases}
$$
 (5)

We solve system (5) by reducing the number of unknowns in the system and parameterizing spectrum Φ(*Ej* ) with a set of elementary functions [1]. Analysis of the cross sections of different nucleus capture reactions shows that some elements can be used as activation detectors (samples) that are mainly sensitive to thermal neutrons (e.g.,  $^{23}Na$ ,  $^{26}Mg$ ,  $^{27}Al$ ,  $^{37}Cl$ ,  $^{50}Ti$ ,  $55$ Mn,  $63$ Cu, and others). System (5) then contains only a limited sum of  $m_{\text{th}}$  terms corresponding to the low-energy (thermal) section of the spectrum:

$$
\begin{cases} \sum_{j=0}^{m_{\text{th}}} \sigma_{1}(E_{j}) \Phi(E_{j}) = J_{1} \\ \cdots \\ \sum_{j=0}^{m_{\text{th}}} \sigma_{k}(E_{j}) \Phi(E_{j}) = J_{k} \end{cases}
$$
 (6)

Approximating this thermal region of the neutron spectrum with an exponential function in the form  $\Phi_j = A E_j^{-B}$ , we can solve system (6) for two unknowns *A* and *B*. Here, the value of  $\Phi_{\text{th}} = AE_{\text{th}}^{-B}$  at thermal energy of neutrons  $E_{\text{th}} = 0.025$  eV corresponds to the value of the thermal neutron flux density of the photoneutron source.  $AE_{\rm th}^{-B}$ 



**Fig. 1.** Contour diagram *B*–*A* of the functional in the lefthand side of formula (7). Solid lines represent those of levels (isolines) and the corresponding values of the functional. Line 0 is the set of solutions to Eq. (7) for  $k = 2$ :  $J_{\text{Mn}}$  and  $J_{\text{Ti}}$ .

## RECONSTRUCTING THE THERMAL NEUTRON SPECTRUM

To solve the system of equations of convolution (6), we shall use the results from experimental measurements of reaction rate *J* using five activation detectors based on <sup>107</sup>Ag (nuclide <sup>108</sup>Ag, line 633 keV), <sup>58</sup>Fe  $(59Fe, 1292 keV),$   $^{26}Mg$   $(27Mg, 844 keV),$   $^{55}Mn$   $(56Mn, 1292 keV),$ 847 keV), and <sup>50</sup>Ti (<sup>51</sup>Ti, 320 keV) [1]. Cross sections in the sums of (6) are taken in the interval of 0.01–0.3 eV.

We solve system (6) via simple substitution  $\Phi_j =$  $AE_j^{-B}$  and search for *A* and *B*, which are the roots of Eq. (7):

$$
\sum_{i=1}^{k} \left( J_i - \sum_{j=0}^{m_{\text{th}}} \sigma_i(E_j) \Phi(E_j) \right) = 0.
$$
 (7)

For two experimental values of *J*, we obtain the multiple solutions presented in Fig. 1 in the contour diagram of the *B*–*A* functional in the left-hand side of formula (7) in the form of narrow continuous strip 0 corresponding to when Eq. (7) is equal to zero.

The experimental value of *J* is measured with error Δ*J*. We therefore search for the solution to system (6) of the equations of convolution by minimizing functional  $\chi^2$ :

$$
\chi^2 = \sum_{i=1}^k \chi_i^2,\tag{8}
$$



**Fig. 2.** Contour diagram  $B-A$  of functional  $\chi^2$  (8) for samples of Ag and Mn. The main denotations are the same as in Fig. 1.



**Fig. 3.** Mean values of thermal neutron density  $\Phi_{\text{th}}$  for different numbers of experimental points (Ag, Fe, Mg, Mn, and Ti samples). Standard errors of the mean are specified as errors for 1–4 sample sets.

where

$$
\chi_i^2 = \left(\frac{\sum_{j=0}^{m_{\text{th}}} \sigma_i(E_j) \Phi(E_j) - J_i}{\Delta J_i}\right)^2.
$$
 (9)

Contour diagram  $B-A$  of functional  $\chi^2$  is characterized by local minima with different depths (Fig. 2). The solution is the global minimum (i.e., the minimum with the lowest value of  $\chi^2$ ).

Figure 2 shows contour diagram *B*–*A* of functional  $\chi^2$  (8) for two samples of Ag and Mn. Functions of  $\chi^2$ for Fe, Mn, and Ti samples irradiated by the source are obtained in a similar manner. This form of diagram *B*–*A* is characteristic of any single sample and any combination of five samples. Upon an increase in the number of samples (the number of equations in system (6) and terms in functional  $\chi^2$  (8)), there are no changes in either the numbers or positions of the minima. However, they do become narrower, and the accuracy of reconstructing coefficients *A* and *B* is improved.

The obtained optimal  $A_{opt}$  and  $B_{opt}$  for each set of experimental *J* were used to calculate the values of thermal neutron flux densities  $\Phi_{\text{th}} = A_{\text{opt}} E_{\text{th}}^{-B_{\text{opt}}}$  at  $E_{\text{th}} =$ 0.025 eV. Figure 3 presents the obtained mean densities of thermal neutron flux  $\Phi_{th}$  for different numbers of experimental points (samples). Standard errors are specified as errors of the mean for sets of 1–4 samples. It is shown that the error in determining  $\Phi_{th}$  falls as the number of data grows. The convergence of the results from reconstructing the thermal neutron flux density is observed upon raising the number of experimental data to  $\Phi_{\text{th}} \sim 10^7$  neutron cm<sup>-2</sup> s<sup>-1</sup>.  $E_{\rm th}^{-B_{\rm opt}}$ 

#### **CONCLUSIONS**

The possibilities of reconstructing the thermal neutron flux density of a photoneutron source using data from neutron activation analysis were considered. The use of a simple function describing the thermal region of the spectrum was examined as an option for reconstruction. The optimum parameters of this function were found by minimizing functional  $\chi^2$  using experimental data obtained upon the simultaneous activation of five activation detectors based on Ag, Mg, Fe, Mn, and Ti in the photoneutron source. The behavior of the  $\chi^2$  dependence on the parameters of the approximating function was studied. Convergence of the results from reconstruction was obtained upon increasing the number of experimental data to  $\Phi_{\text{th}} \sim$ 107 neutron cm−2 s−1. This reconstruction procedure can be used for real-time measurements of the thermal neutron flux density of a photoneutron source, and in optimizing its parameters.

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