ISSN 1062-8738, Bulletin of the Russian Academy of Sciences: Physics, 2017, Vol. 81, No. 10, pp. 1192–1195. © Allerton Press, Inc., 2017.
Original Russian Text © E.G. Drukarev, M.G. Ryskin, V.A. Sadovnikova, 2017, published in Izvestiya Rossiiskoi Akademii Nauk, Seriya Fizicheskaya, 2017, Vol. 81, No. 10, pp. 1334–1337.

Three-Body Nuclear Interactions in the QCD Sum Rules[1]

E. G. Drukarev, M. G. Ryskin, and V. A. Sadovnikova*
National Research Center Kurchatov Institute, Konstantinov Nuclear Physics Institute, Gatchina, Leningrad oblast, 188300 Russia
**e-mail: sadovnik@thd.pnpi.spb.ru*

Abstract—The QCD sum rules used to calculate the characteristics of single-particle nucleons are reviewed briefly. The contribution from three-body forces to the nucleon self-energies are calculated.

DOI: 10.3103/S1062873817100094

INTRODUCTION

In this work, we calculate the contribution from three body (3*N*) interactions to the single-particle characteristics of nucleons in nuclear matter. We determine the contributions to vector self-energy Σ_V, and to effective mass m^*. We also obtain the contribution from 3*N* forces to nucleon potential energy *U* and find the dependence of these characteristics on nuclear density ρ. Calculations are performed for symmetrical matter containing equal number of protons and neutrons.

In traditional approaches, the role of 3*N* forces is determined by analyzing nucleon interactions. In the QCD sum rules approach, nucleons of matter interact with a system having baryon (proton) quantum numbers with four-momentum *q* in the rest frame of the matter. The function describing the distribution of this system is often referred to as a polarization operator and is denoted as $\Pi(q)$. At positive values of q^2, function $\Pi(q)$ has singularities associated with real physical states. The lowest pole corresponds to an in-medium proton that we refer to as the probe proton.

Due to the asymptotic freedom of QCD at large and negative q^2, function $\Pi(q)$ can be obtained by means of perturbation theory. A dispersion relation connects the regions of small and large values of q^2. The parameters of the probe proton are thus associated with those of the expansion of function $\Pi(q)$ at large and negative values of q^2.

The QCD sum rules were proposed in [1] to study mesons in a vacuum. The approach was later expanded to calculate the vacuum parameters of baryon [2]. The dispersion relations were analyzed for vacuum polarization operator $\Pi_0(q^2)$. The left-hand sides of the dispersion relations were presented as power series in $1/q_2$. The leading term was a free three-quark loop. The subsequent terms were dictated by quark exchanges between the polarization operator and the vacuum condensates: $\langle 0|\bar{q}q|0\rangle$, $\langle 0|\bar{q}q\bar{q}q|0\rangle$, and so on (see Fig. 1). This is known as the Operator Product Expansion (OPE). Note that in the Feynman diagrams shown in Figs. 1b, 1c, the polarization operator exchanges noninteracting quarks with the vacuum. The inclusion of their lowest order interactions corresponds to radiative corrections to these diagrams on the order of α_s [3].

The vacuum polarization operator can be presented as $\Pi_0(q) = \hat{q}\Pi_0^q(q^2) + I\Pi_0^I(q^2)$, and in the dispersion relations

$$\Pi_0^i(q^2) = \frac{1}{\pi}\int dk^2 \frac{\mathrm{Im}\,\Pi_0^i(k^2)}{k^2 - q^2},\ i = q, I, \qquad (1)$$

the left-hand sides are the lowest order OPE terms. (Below, we see there is no need to worry about subtractions.) The imaginary parts of $\Pi_0^i(k^2)$ are usually approximated using the pole + continuum model, in which the contribution from a pole is treated exactly,

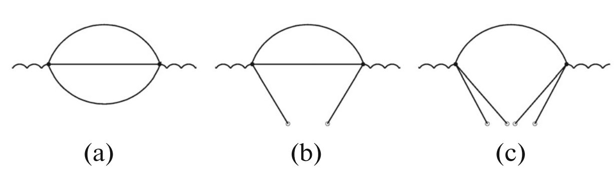

Fig. 1. Main contributions to the polarization operator in a vacuum: (a) from the free quark loop; (b) from the condensate $\langle 0|\bar{q}q|0\rangle$; (c) from the four-quark condensate. The solid lines represent quarks. The wavy lines are for a system with nucleon quantum numbers.

[1] This article was translated by the authors.

while the higher states are described by continuum. The magnitude of a jump is determined by OPE terms

$$\frac{\mathrm{Im}\Pi^i(k^2)}{\pi} = \xi^i\lambda_{mN}^2\delta(k^2 - m^2) + \frac{\Delta\Pi^{i\mathrm{OPE}}(k^2)}{\pi}\theta(k^2 - W_m^2). \quad (2)$$

Here, $\xi^q = 1$, $\xi^I = m$. The position of proton pole m, residue λ_{mN}^2, and continuum threshold W^2 are unknowns that must be determined from the sum rules. This is how the proton parameters are associated with the vacuum condensates. This approach has proven effective in calculating static hadron characteristics and some features of their dynamics [3].

The QCD sum rules were expanded in [4] to calculate nucleon characteristics in a medium with a finite baryon number density. The structure of the polarization operator in nuclear matter is $\Pi(q) = \hat{q}\Pi^q(q,P) + \hat{P}\Pi^P(q,P) + I\Pi_0^I(q,P)$. Here we introduced four-vector $P = (m,\vec{\mathbf{0}})$ with vacuum nucleon mass m (ignoring small neutron–proton mass splitting). The condensates acquire in-medium values $\langle M|\bar{q}q|M\rangle$, $\langle M|\bar{q}q\bar{q}q|M\rangle$, and so on, where $|M\rangle$ is the nuclear matter's ground state. There are also new condensates that have zero vacuum values. Vector condensate $v_\mu = \langle M|\bar{q}\gamma_\mu q|M\rangle$ is the one most important. The differences between the in-medium and vacuum values determine the changes in single-particle nucleon characteristics.

Functions $\Pi^i(q,P)(i = q,P,I)$ depend on two variables, which can be q^2 and $s = (P+q)^2$. Fixing $s = 4m^2$, we separate the singularities of the polarization operator associated with the in-medium proton from those associated with the excitation of the medium. Employing the pole + continuum model to the spectrum of the polarization operator, we obtain the dispersion relations

$$\Pi^i(q^2,s) = \frac{1}{\pi}\int dk^2\frac{\mathrm{Im}\Pi^i(k^2,s)}{k^2 - q^2}\; i = q,P,I, \quad (3)$$

where the left-hand side is described by several OPE terms, and the imaginary part is

$$\frac{\mathrm{Im}\Pi^i(k^2,s)}{\pi} = \xi^i\lambda_{mN}^2(s)\delta(k^2 - m_m^2(s)) + \frac{\Delta\Pi^{i\mathrm{OPE}}(k^2,s)}{\pi}\theta(k^2 - W_m^2(s)), \quad (4)$$

where $\xi^q = 1$, $\xi^P = -\Sigma_V/m$, $\xi^I = m^*$; Σ_V is the vector self-energy and m^* is the Dirac effective mass. The vector self-energy and the effective mass must be determined from the sum rules, along with parameters λ_m^2 and W_m^2. The position of the proton pole can be expressed in terms of vector and scalar self-energies Σ_V and $\Sigma_s = m^* - m$. In a linear approximation, $m_m = m + \Sigma_s + \Sigma_V$. The proton parameters are thus expressed through the exchange of noninteracting or weakly interacting quarks between the polarization operator and the matter. The latter exchange can be expressed in terms of the in-medium QCD condensate.

The vector and the scalar proton self-energies can be considered a result of meson exchange between a probe proton and the nuclear matter. On the other hand, mesons are strongly correlated quark systems, so the exchange of strongly correlated quarks is expressed in the QCD sum rules through that of weakly correlated quarks.

The OPE of the left-hand sides contains terms $(1/q^2)^n$ with QCD condensates as coefficients. The larger values of n correspond to condensates of higher dimensions. The assumed convergence of OPE thus corresponds to the hypothesis that the condensates of lower dimensions are the ones most important. The left-hand sides of the sum rules can be written as

$$\Pi^{q\mathrm{OPE}}(q^2) = \sum_{n=0} A_n(q^2); \quad \Pi^{I\mathrm{OPE}}(q^2) = \sum_{n=3} B_n(q^2); \quad \Pi^{P\mathrm{OPE}}(q^2) = \sum_{n=3} C_n(q^2), \quad (5)$$

with lower index n denoting the dimension of the condensate ($n = 0$ for a free quark loop).

We normally use the Borel transform (an inverse Laplace transform) to improve the convergence of the OPE. This transform converts the functions depending on q^2 to ones of Borel mass M^2. It is important that this transform removes any polynomial in q^2 (which explains why we do not worry about subtractions in Eq. (1)). The Borel transform yields −1, when acts on $1/q^2$. The transformed Borel sum rules are then

$$\mathcal{L}^q(M^2,W_m^2) = \Lambda_m;\; \mathcal{L}^I(M^2,W_m^2) = m^*\Lambda_m; \quad \mathcal{L}^P(M^2,W_m^2) = -\Sigma_P\Lambda_m, \quad (6)$$

with $\Sigma_P = \Sigma_V/m$, $\lambda_m^2 = 32\pi^4\lambda_{mN}^2$; $\Lambda_m = \lambda_m^2 e^{-m_m^2/M^2}$; factor $32\pi^4$ is introduced to deal with values on the

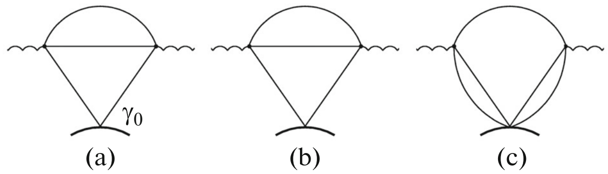

Fig. 2. Main contributions to the polarization operator in nuclear matter that correspond to the inclusion of $2N$ interactions: (a) from the vector condensate; (b) from the scalar condensate; (c) from the four-quark condensate. Bold lines denote the nucleons of the matter.

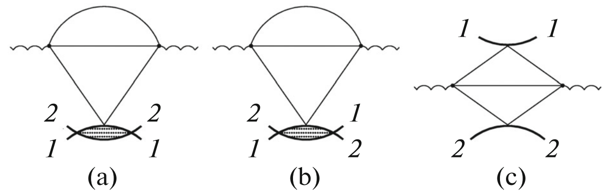

Fig. 3. Main contributions to the polarization operator in nuclear matter that correspond to the inclusion of $3N$ interactions: (a, b) from the nonlinear part of the scalar condensate (the shaded block is for the pion field); (c) from the four-quark condensates.

order of unity. In these equations, we put the continuum contributions on the left-hand sides:

$$\mathcal{L}^q(M^2,W_m^2) = \sum_{n=0} \tilde{A}_n(M^2,W_m^2);$$
$$\mathcal{L}^I(M^2,W_m^2) = \sum_{n=3} \tilde{B}_n(M^2,W_m^2); \quad (7)$$
$$\mathcal{L}^P(M^2,W_m^2) = \sum_{n=3} \tilde{C}_n(M^2,W_m^2),$$

with the tilde denoting the Borel transform.

The lowest dimension condensates with $n = 3$ that contribute to the polarization operator are the vector and scalar condensates:

$$v(\rho) = \langle M | \sum_i \bar{q}^i \gamma_0 q^i | M \rangle;$$
$$\varkappa(\rho) = \langle M | \sum_i \bar{q}^i q^i | M \rangle. \quad (8)$$

They correspond to the exchange of vector and scalar mesons between a probe proton and the matter. The vector condensate is written in the rest frame of the matter. The most important condensates of higher dimensions are four-quark condensates $\langle M | q_\alpha^a \bar{q}_\beta^b q_\gamma^c \bar{q}_\delta^d | M \rangle$, which have dimension $n = 6$.

TWO BODY INTERACTIONS IN QCD SUM RULES

To calculate the contribution from two-body forces to the characteristics of a probe proton, we must include the separate contributions from in-medium nucleons to the polarization operator. In other words, we must calculate the polarization operator in the system of noninteracting nucleons. This means we must include only the linear density dependent terms in the condensates. While the vector condensate is purely linear in density ($v(\rho) = N\rho$, with $N = 3$ being the number of valence quarks in the nucleon), the density dependence of the scalar condensate is somewhat more complicated:

$$\kappa(\rho) = \kappa(0) + \kappa_N \rho + S(\rho);$$
$$\kappa_N = \langle N | \sum_i \bar{q}^i q^i | N \rangle \ , \quad (9)$$

with nonlinear term $S(\rho)$ resulting from the interaction between in-medium nucleons. It was noted above that four-quark condensates are the most important condensates of higher dimensions. In calculating the $2N$ forces, we include only those configurations in which all four-quark operators act on the same in-medium nucleon. The polarization operator corresponding to the inclusion of $2N$ interactions is shown in Fig. 2.

Matrix element κ_N can be expressed through the pion–nucleon σ-term: $\kappa_N = 2\sigma_N/(m_u + m_d)$, where $m_{u,d}$ denotes the light quark masses. The experimental and theoretical values of sigma-term σ_N lie in the interval between 35 [5] and 70 MeV [6]. The conventional value is $\sigma_N = 45$ MeV. This is how the characteristics of a probe nucleon are expressed in terms of observables.

Calculations for four-quark condensates require model assumptions on the quark structure of the nucleon. We employed the relativistic model formulated in [7] with some modifications proposed in [4]. The results in [4] agree with those obtained by means of traditional nuclear physics.

THREE-BODY INTERACTIONS IN THE SUM RULES

The above analysis shows we must include the scalar condensate in which we consider the $2N$ forces between in-medium nucleons. In other words, we must calculate function $S(\rho)$ on the right-hand side of Eq. (9), limiting ourselves to $2N$ forces. We must include also the configurations of the four-quark condensates in which two pairs of quark operators act on two different in-medium nucleons (see Fig. 3).

It was shown in [4] that the main contribution to function $S(\rho)$ comes from the pion field created by nucleons. The contributions from one- and two-pion exchanges to function $S(\rho)$ were determined in [8] using the Chiral perturbation theory. Separating the contribution from the $2N$ in-medium interactions in the equations found in [8], we find the required contribution to the scalar condensate.

The scalar–scalar and scalar–vector condensates appear to be the most important ones among the four-quark condensates, and can be calculated independently of the model. We used a model in which $2N$ interactions are calculated to obtain the other four-quark condensates.

Employing Eq. (7) with these condensates, we find that in symmetric nuclear matter at saturation value of density $\rho_0 = 0.16\,\mathrm{fm}^{-3}$, $3N$ interactions lower the value of the effective mass by 26 MeV. They reduce the vector self-energy by 37 MeV. Our approach allows us to find the density dependence of the $3N$ forces. Introducing $x = \rho/\rho_0$, we can approximate the contributions from these forces to the nucleon parameters as

$$\delta m^* \simeq 26x^2; \; \delta\Sigma_V \simeq -37x^2. \tag{10}$$

Note that the inclusion of $3N$ interactions between in-medium nucleons strongly raises the value of $S(\rho)$ [8], resulting in a large contribution from four body ($4N$) interactions to the nucleon effective mass (around + 100 MeV). The $4N$ forces do not noticeably change the nucleon vector self-energy. Detailed analysis of the role of $4N$ interactions will be the subject of a future work.

REFERENCES

1. Shifman, M.A., Vainshtein, A.I., and Zakharov, V.I., *Nucl. Phys. B*, 1979, vol. 147, p. 385; Shifman, M.A., Vainshtein, A.I., and Zakharov, V.I., *Nucl. Phys. B*, 1979, vol. 147, p. 448; Shifman, M.A., Vainshtein, A.I., and Zakharov, V.I., *Nucl. Phys. B*, 1979, vol. 147, p. 519.
2. Ioffe, B.L., *Nucl. Phys. B*, 1981, vol. 188, p. 317.
3. Ioffe, B.L., Lipatov, L.N., and Fadin, V.S., *Quantum Chromodynamics*, Cambridge Univ. Press, 2010.
4. Drukarev, E.G., Ryskin, M.G., and Sadovnikova, V.A., *Phys. At. Nucl.*, 2012, vol. 75, p. 334.
5. Stahov, J., Clement, H., and Wagner, G.J., *Phys. Lett. B*, 2013, vol. 726, p. 685.
6. Hite, G.E., Kaufmann, W.B., and Jacob, R.J., *Phys. Rev. C*, 2005, vol. 71, p. 065201.
7. Gutsche, Th. and Robson, D., *Phys. Lett. B*, 1989, vol. 229, p. 333.
8. Kaiser, N., de Homont, P., and Weise, W., *Phys. Rev. C*, 2008, vol. 77, p. 025204.