T-Invariance Conditions for Sequential Multistep Statistical Nuclear Reactions

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Abstract—A generalized version of the *R*-matrix theory is used to determine the amplitudes of sequential *n*-step statistical nuclear reactions. *T*-invariance conditions for these amplitudes are analyzed. Within the scope of the unified theory, integral formulas are constructed for the decay amplitudes of the intermediate states of compound nuclei via the matrix elements **QHP** of the system Hamiltonian **H**, where **Q** and **P** operators perform projections onto resonance and energy-continuous states of the system, respectively.

INTRODUCTION

Bohr and Mottelson [1] constructed a formula for amplitude $\mathscr{T}_{bB,aA}$ of a sequential two-step statistical nuclear reaction of the form $a + A \rightarrow r_0 \rightarrow b + B$ in the system of the center of mass where a compound nucleus in resonance state r_0 with total spin J_{r_0}, its projection M_{r_0}, and internal energy E_{r_0} is formed at the first stage of a reaction in which particle a with internal energy E_A, total spin J_a, and relative orbital moment l_a strikes a target nucleus A with spin I_A and internal energy E_A. At the second stage of the reaction, resonance state r_0 decays to emit stable particle b and form stable final nucleus B. Using the formalism of the *R*-matrix theory of nuclear reactions [2], the amplitude of the above reaction $\mathscr{T}_{bB,aA}$ was presented [1] as

$$
\mathscr{T}_{bB,aA} = \sum_{r_0 M_{r_0}} \frac{g\left(I_B J_b l_b \leftarrow J_{r_0} M_{r_0}\right) g\left(J_{r_0} M_{r_0} \leftarrow I_A J_a l_a\right)}{\left(E - E_{r_0} + i/2\,\Gamma_{r_0}\right)}, \quad (1)
$$

where E is the total energy of the reaction, determined by the formula

$$
E = E_a + E_A + T_{aA}, \quad (2)
$$

with T_{aA} being the kinetic energy of relative motion of particle a and target nucleus A, and Γ_{r_0} being the total decay width of state r_0. Quantity $g(J_{r_0} M_{r_0} \leftarrow I_A J_a l_a)$ in Eq. (1) is the amplitude of the formation of resonance state r_0 of the compound nucleus in the aA channel and coincides with amplitude $g(I_A J_a l_a \leftarrow J_{r_0} M_{r_0})$ of the decay of the above state into this channel. This quantity is also related to real amplitude $\sqrt{\Gamma_{aA,r_0}}$ of the

partial decay width of resonance state r_0 and potential scattering phase δ_{aA} in this channel via the formula

$$
g\left(I_A J_a l_a \leftarrow J_{r_0} M_{r_0}\right) = e^{i\delta_{aA}}\sqrt{\Gamma_{aA,r_0}}. \quad (3)
$$

The following relationship was proposed in [1] to determine the value of amplitude $g(I_A J_a l_a \leftarrow J_{r_0} M_{r_0})$:

$$
\begin{aligned}
&g\left(I_A J_a l_a \leftarrow J_{r_0} M_{r_0}\right) \\
&= \sqrt{2\pi}\langle U_{aA} f_{aA}\left(R_{aA}\right) | \mathbf{H'} | \Psi_{J_{r_0} M_{r_0}} \rangle,
\end{aligned} \quad (4)
$$

where $\Psi_{J_{r_0} M_{r_0}}$ is a normalized wave function of resonance state r_0, and U_{aA} is a channel function defined [2] as

$$
U_{aA} = \left\{\left\{\chi_{I_a M_{I_a}} Y_{l_a m_a}\left(\Omega_{\vec{R}_{aA}}\right)\right\}_{J_a M_a} \chi_{I_A M_A}\right\}_{J_{r_0} M_{r_0}}, \quad (5)
$$

with $\chi_{I_a M_{I_a}}$ and $\chi_{I_A M_A}$ being internal wave functions of particle a and target nucleus A, and $Y_{l_a m_a}\left(\Omega_{\vec{R}_{aA}}\right)$ being a spherical function that describes the relative orbital motion of particle a and target nucleus A. At the same time, the relative radial motion of particle a and target nucleus A was in this case described by a regular radial wave function $f_{aA}(R_{aA})$ that was energy-normalized to the δ-function. Bohr and Mottelson [1] assumed that operator $\mathbf{H'}$ that appears in Eq. (4) and causes the decay of the resonance state r_0 is a small part of the system's full Hamiltonian $\mathbf{H}$. This representation was justified because if the lifetime of a decaying state is long compared to the periods of internal nuclear motions, the decay can be considered as a perturbation described by small operator $\mathbf{H'}$. Taking the self-adjointness of the operator $\mathbf{H'}$ into account and using

Eq. (4), the following T-invariance condition for amplitude $g\left(J_{r_0}M_{r_0} \leftarrow I_A J_a l_a\right)$ was obtained in [1] for a T-invariant quantum system:

$$g\left(J_{r_0}M_{r_0} \leftarrow I_A J_a l_a\right) = g\left(\overline{I_A J_a l_a} \leftarrow \overline{J_{r_0}M_{r_0}}\right), \quad (6)$$

where wave functions $\overline{I_A J_a l_a}$ and $\overline{J_{r_0}M_{r_0}}$ were defined as time-inverse wave functions of states $I_A J_a l_a$ and $J_{r_0}M_{r_0}$, respectively. Bohr and Mottelson [1] used Eq. (6) to derive T-invariance condition $\mathscr{T}_{bB,aA}$ in Eq. (1) for a sequential two-step statistical nuclear reaction:

$$\mathscr{T}_{bB,aA} = \tilde{\mathscr{T}}_{\overline{aA},\overline{bB}} = \sum_{r_0 M_{r_0}} \frac{g\left(\overline{J_{r_0}M_{r_0}} \leftarrow \overline{I_B J_b l_b}\right) g\left(\overline{I_A J_a l_a} \leftarrow \overline{J_{r_0}M_{r_0}}\right)}{\left(E - E_{r_0} + i/2\,\Gamma_{r_0}\right)}, \quad (7)$$

where $\tilde{\mathscr{T}}_{\overline{aA},\overline{bB}}$ is the amplitude of the time-reversed nuclear reaction.

The results in [1] hold for decays associated with weak and electromagnetic interactions, for which the operator $\mathbf{H'}$ is well defined. However, operator $\mathbf{H'}$ in Eq. (4) is not defined for decays related to such nuclear interactions as α-decay, proton decay, cluster decay, and nuclear fission, and its particular form can conflict with the assumption made in [1] about its smallness.

In this work, we plan first of all to prove that the T-invariance condition for a quantum system is generally the T-evenness of its Hamiltonian $\mathbf{H}$; second, to construct a formula for the partial decay width of resonance state r_0 of a compound nucleus under this condition using the unified theory of a nucleus with allowance for strong nuclear interactions; and third, to find the T-invariance condition for the amplitude of a sequential multistep statistical nuclear reaction using the first and second results above.

T-INVARIANCE CONDITIONS FOR QUANTUM SYSTEMS

As understood in classical and quantum mechanics, invariance with respect to time reversal or T-invariance means [1, 3−5] that for every possible state of a system there is a time-inverse state described by the same equations of motion.

To analyze T-invariance conditions for a quantum system, let us consider the Schrödinger equation for wave function $\Psi(\xi,t)$ with a full set of spatial, spin, and other coordinates of system ξ and time-independent Hamiltonian $\mathbf{H}(\xi)$:

$$ih\frac{\partial}{\partial t}\Psi(\xi,t) = \mathbf{H}(\xi)\Psi(\xi,t). \quad (8)$$

Wave function $\Psi(\xi,t)$ corresponds to the traditional time description of a system from the past into the future when time changes from $t = -\infty$ to $t = \infty$, and is represented [1] via wave function $\Psi(\xi,0)$ of the system at initial moment $t = 0$ as

$$\Psi(\xi,t) = e^{-i\frac{Ht}{h}}\Psi(\xi,0). \quad (9)$$

Function $\Psi(\xi,0)$ coincides with wave function $\Psi(\xi)$, which is the solution to the Schrödinger steady-state equation

$$\mathbf{H}(\xi)\Psi(\xi) = E\Psi(\xi). \quad (10)$$

Following the concepts of [3−5], we introduce the system's time-inverse wave function $\overline{\Psi}(\xi,t)$ linked to wave function $\Psi(\xi,t)$ via the relation

$$\overline{\Psi}(\xi,t) = \boldsymbol{\tau}\Psi(\xi,-t), \quad (11)$$

where $\boldsymbol{\tau}$ is a time-inverse operator and has [1, 5] the form

$$\boldsymbol{\tau} = \mathbf{OK}, \quad (12)$$

with $\mathbf{K}$ being a complex conjugation operator and $\mathbf{O}$ being a unitary operator that is by definition related to Hermitian conjugate operator $\mathbf{O^+}$:

$$\mathbf{OO^+} = \mathbf{O^+O} = 1.$$

Let us find a condition under which wave function $\overline{\Psi}(\xi,t)$ is a solution to the same Schrödinger equation in Eq. (8) as function $\Psi(\xi,t)$, and may thus be considered time-inverse with respect to original wave function $\Psi(\xi,t)$. We subject Eq. (11) to the action of the operator $ih\frac{\partial}{\partial t}$ to obtain

$$ih\frac{\partial}{\partial t}\overline{\Psi}(\xi,t)$$
$$= ih\frac{\partial}{\partial t}\boldsymbol{\tau}\Psi(\xi,-t) = \mathbf{O}ih\frac{\partial}{\partial t}\Psi^*(\xi,-t). \quad (13)$$

At the same time, when operator $\mathbf{K}$ acts upon Schrödinger equation (8) and we substitute $t \rightarrow -t$, Eq. (8) can be transformed into

$$ih\frac{\partial}{\partial t}\Psi^*(\xi,-t) = \mathbf{H}^*(\xi)\Psi^*(\xi,-t). \quad (14)$$

where $\mathbf{H}^*$ is an operator that is a complex conjugate of operator $\mathbf{H}$.

Substituting (14) into the right-hand side of Eq. (13) and using the definition in Eq. (11), we get the equation

$$ih\frac{\partial}{\partial t}\overline{\Psi}(\xi,t) = \mathbf{O}\mathbf{H}^*(\xi)\mathbf{O}^+\overline{\Psi}(\xi,t).\qquad(15)$$

For wave function $\overline{\Psi}(\xi,t)$, similar to function $\Psi(\xi,t)$ to be a solution to Schrödinger equation (8), the following condition must be fulfilled [5]:

$$\mathbf{H} = \mathbf{O}\mathbf{H}^*\mathbf{O}^+ = \boldsymbol{\tau}\mathbf{H}\boldsymbol{\tau}^{-1}.\qquad(16)$$

We shall consider the meaning of this condition below.

Using the formulas in Eqs. (9) and (12), the time dependence of function $\overline{\Psi}(\xi,t)$ can be presented as

$$\overline{\Psi}(\xi,t) = \mathbf{O}\mathbf{K}e^{i\frac{Ht}{h}}\Psi(\xi)$$
$$= \mathbf{O}e^{-i\frac{H^*t}{h}}\mathbf{K}\Psi(\xi) = e^{-i\frac{Ht}{h}}\overline{\Psi}(\xi),$$

where $\overline{\Psi}(\xi)$ is a time-reversed wave function $\Psi(\xi)$ defined by the formula

$$\overline{\Psi}(\xi) = \boldsymbol{\tau}\Psi(\xi).\qquad(17)$$

The reason why for time-independent function $\Psi(\xi)$ there is function $\overline{\Psi}(\xi)$ which coincides with time-reversed function $\Psi(\xi)$ in Eq. (17) and satisfies the same Schrödinger equation (10) is that functions $\Psi(\xi)$ and $\overline{\Psi}(\xi)$ describe quasi-stationary [1] processes that are characterized through initial and boundary conditions by probability-flow density vectors that depend on the direction of flow of time.

We shall now study matrix element $\langle\Psi_2(\xi)|\mathbf{Q}|\Psi_1(\xi)\rangle$ of arbitrary operator $\mathbf{Q}$, where $\Psi_1(\xi)$ and $\Psi_2(\xi)$ are the eigenfunctions of Hamiltonian $\mathbf{H}$. If we use the relationship in Eq. (17), this matrix element can be written [5] in the form

$$\langle\Psi_2(\xi)|\mathbf{Q}|\Psi_1(\xi)\rangle$$
$$= \langle\boldsymbol{\tau}^{-1}\overline{\Psi}_1(\xi)|\boldsymbol{\tau}^{-1}\boldsymbol{\tau}\mathbf{Q}^+|\boldsymbol{\tau}^{-1}\overline{\Psi}_2(\xi)\rangle^*$$
$$= \langle\mathbf{O}^+\overline{\Psi}_1(\xi)|\mathbf{O}^+\overline{\mathbf{Q}}|\overline{\Psi}_2(\xi)\rangle\qquad(18)$$
$$= \langle\overline{\mathbf{Q}}\overline{\Psi}_2(\xi)|\mathbf{O}|\mathbf{O}^+\overline{\Psi}_1(\xi)\rangle^*$$
$$= \langle\overline{\Psi}_1(\xi)|\overline{\mathbf{Q}}|\overline{\Psi}_2(\xi)\rangle,$$

where $\overline{\mathbf{Q}}$ is time-inverse with respect to operator $\mathbf{Q}$, defined as

$$\overline{\mathbf{Q}} = \boldsymbol{\tau}\mathbf{Q}^+\boldsymbol{\tau}^{-1}.\qquad(19)$$

It then follows that the time-inverse operators of coordinate $\overline{\mathbf{r}}$, momentum $\overline{\mathbf{p}}$, angular momentum $\overline{\mathbf{L}}$, and spin $\overline{\mathbf{s}}$ can be expressed via original operators $\mathbf{r}$, $\mathbf{p}$, $\mathbf{L}$, and $\mathbf{s}$ as

$$\overline{\mathbf{r}} = \mathbf{r};\quad \overline{\mathbf{p}} = -\mathbf{p};\quad \overline{\mathbf{L}} = -\mathbf{L};\quad \overline{\mathbf{s}} = -\mathbf{s},\qquad(20)$$

if we consider their self-adjointness.

We shall assume operator $\mathbf{Q}$ is *T*-invariant if its time-inverse $\overline{\mathbf{Q}}$ coincides with original operator $\mathbf{Q}$ when

$$\overline{\mathbf{Q}} = \mathbf{Q}.\qquad(21)$$

Arbitrary operator $\mathbf{Q}$ can generally be presented as the sum

$$\mathbf{Q} = \mathbf{Q}_T + \mathbf{Q}_{-T},\qquad(22)$$

where $\mathbf{Q}_T(\mathbf{Q}_{-T})$ is a *T*-even (*T*-odd) part of operator $\mathbf{Q}$ that does not (does) change sign when time is reversed:

$$\mathbf{Q}_T = \overline{\mathbf{Q}}_T = \frac{\mathbf{Q}+\overline{\mathbf{Q}}}{2};\quad \mathbf{Q}_{-T} = -\overline{\mathbf{Q}}_{-T} = \frac{\mathbf{Q}-\overline{\mathbf{Q}}}{2}.\qquad(23)$$

Operator $\mathbf{Q}$ is then *T*-invariant in the sense of Eq. (21) if it contains only *T*-even component $\mathbf{Q}_T$ with time reversal.

We are now in a position to consider the physical meaning of the condition in Eq. (16). If we allow for the self-adjointness of operator $\mathbf{H}$, this condition becomes

$$\mathbf{H} = \overline{\mathbf{H}},\qquad(24)$$

where operator $\overline{\mathbf{H}}$ is time-inverse with respect to operator $\mathbf{H}$ defined by the formula in (19) if we replace $\mathbf{Q}$ with $\mathbf{H}$. The condition in Eq. (24) is for the *T*-invariance of Hamiltonian $\mathbf{H}$ under which this Hamiltonian only contains the *T*-even component $\mathbf{H}_T$. It then follows that the *T*-invariance of a quantum system can be violated only if Hamiltonian $\mathbf{H}$ contains *T*-odd members $\mathbf{H}_{-T}$, e.g., the terms like $A[(\mathbf{r},\mathbf{p})+(\mathbf{p},\mathbf{r})]$ that were considered in [6].

AMPLITUDE OF A SEQUENTIAL MULTISTEP STATISTICAL NUCLEAR REACTION

Let us now try to describe a sequential *n*-step statistical nuclear reaction of the form $a + A \rightarrow r_0 \rightarrow b_1 + r_1 \rightarrow \ldots \rightarrow b_1 + \ldots + b_{n-1} + r_{n-1} \rightarrow b_1 + \ldots + b_n + B$, in which emitted particles $b_1, b_2, \ldots, b_n$ and final nucleus B are stable but there exist decay resonance states $r_0, r_1, \ldots, r_{n-1}$ of the intermediate nuclei. We can take advantage of the generalization of the standard formalism of the *R*-matrix theory of nuclear reactions that was developed in [2] to describe a case where nucleus *1* that appears as a result of the above two-step statistical nuclear reaction is unstable and can disintegrate from resonance state r_1 with the formation of stable particle b_2 and stable nucleus B. Such a generaliza-

tion allowed Lane and Thomas [2] to describe a sequential three-step statistical nuclear reaction in which three stable particles b_1, b_2, B appear in the final channel. Using the method developed in [2], the amplitude of $\mathcal{T}_{b_1...b_nB,aA}$ of the sequential n-step statistical reaction can then be presented as

$$\mathcal{T}_{b_1...b_nB,aA} = \sum_{r_0M_{r_0}...r_{n-1}M_{r_{n-1}}} \frac{g\left(I_BJ_{b_n}l_{b_n} \leftarrow J_{r_{n-1}}M_{r_{n-1}}\right)...g\left(J_{r_0}M_{r_0} \leftarrow I_AJ_al_a\right)}{\left(E_{r_{n-2}} - E_{r_{n-1}} - E_{b_{n-1}} - T_{b_{n-1},r_{n-1}} + i/2\,\Gamma_{r_{n-1}}\right)...\left(E - E_{r_0} + i/2\,\Gamma_{r_0}\right)}, \tag{25}$$

where total reaction energy E is determined by Eq. (2).

If we use the results in [7–9] of the unified theory of nucleus, the amplitudes in Eq. (25) can be expressed via integral formulas that are exemplified below for amplitude $g(I_AJ_al_a \leftarrow J_{r_0}M_{r_0})$ of the decay of resonance state r_0 of a compound nucleus:

$$\begin{aligned} &g\left(I_AJ_al_a \leftarrow J_{r_0}M_{r_0}\right) \\ &= \sqrt{2\pi}\left\langle U_{aA}f_{aA}\left(R_{aA}\right)\middle|\mathbf{P}(E-\mathbf{H})\mathbf{Q}\middle|\Psi_{J_{r_0}M_{r_0}}\right\rangle, \end{aligned} \tag{26}$$

where operators $\mathbf{P}$ and $\mathbf{Q}$ are projection operators ($\mathbf{P} + \mathbf{Q} = 1$). Operator $\mathbf{P}$ projects wave function $\Psi(\xi)$ (10) of the system for a scattering problem on the internal states of particles a and A that correspond to open decay channels with infinite motion of particles and channel function U_{aA}. Operator $\mathbf{Q}$ in turn projects the above function $\Psi(\xi)$ (10) on the resonance states of a compound nucleus of the r_0 type with wave functions $\Psi_{J_{r_0}M_{r_0}}$ that do not have open decay channels and thus have discrete energies E_{r_0} that correspond to finite movements.

The formula in Eq. (26) is in fact a generalization of Eq. (4) in [1] that allows for different decay channels of resonance state r_0, including decay channels that include such strong nuclear interactions as α-decay, proton decay, cluster decay, and nuclear fission.

RELATIONSHIP BETWEEN DIRECT AND TIME-REVERSED AMPLITUDES OF A SEQUENTIAL MULTISTEP STATISTICAL NUCLEAR REACTION

As defined above, a necessary condition for the T-invariance of a quantum system is the T-evenness of the Hamiltonian of the system in Eq. (24). Let us study the above amplitude $\mathcal{T}_{b_1...b_nB,aA}$ for a sequential multistep statistical nuclear reaction under the effects of a time-reversal operation with allowance for this condition.

We shall first analyze the T-invariance condition for the decay amplitudes of resonance nuclear states of the form $g(I_AJ_al_a \leftarrow J_{r_0}M_{r_0})$ (26) contained in the formula in Eq. (25). We use the formula in (18) and transform the matrix element that defines amplitude (26) as

$$\begin{aligned} &\left\langle U_{aA}f_{aA}\left(R_{aA}\right)\middle|\mathbf{P}(E-\mathbf{H})\mathbf{Q}\middle|\Psi_{J_{r_0}M_{r_0}}\right\rangle \\ &= \left\langle\overline{\Psi}_{J_{r_0}M_{r_0}}\middle|\overline{\mathbf{P}(E-\mathbf{H})\mathbf{Q}}\middle|\overline{U}_{aA}\overline{f}_{aA}\left(R_{aA}\right)\right\rangle, \end{aligned} \tag{27}$$

where $\overline{\Psi}_{J_{r_0}M_{r_0}}$, $\overline{f}_{aA}(R_{aA})$, and $\overline{U}_{aA}$ are time-inverse with respect to wave function $\Psi_{J_{r_0}M_{r_0}}$ of resonance state r_0 of the compound nucleus, the regular radial wave function of relative motion of particle a and target nucleus A, and channel function U_{aA}, respectively, while $\overline{\mathbf{P}(E-\mathbf{H})\mathbf{Q}}$ is a time-reversed operator $\mathbf{P}(E-\mathbf{H})\mathbf{Q}$ defined by Eq. (19). If we use this formula and allow for condition (24) of the T-invariance of Hamiltonian $\mathbf{H}$, operator $\overline{\mathbf{P}(E-\mathbf{H})\mathbf{Q}}$ can be presented as

$$\begin{aligned} \overline{\mathbf{P}(E-\mathbf{H})\mathbf{Q}} &= \boldsymbol{\tau}[\mathbf{P}(E-\mathbf{H})\mathbf{Q}]^+\boldsymbol{\tau}^{-1} \\ &= \mathbf{OKQ}^*(E-\mathbf{H})\mathbf{P}^*\mathbf{KO}^+ \\ &= \mathbf{OQ}(E-\mathbf{H}^*)\mathbf{PO}^+ = \mathbf{Q}(E-\mathbf{H})\mathbf{P}. \end{aligned} \tag{28}$$

In light of transformation (28), formula (27) can be written as

$$\begin{aligned} &\left\langle U_{aA}f_{aA}\left(R_{aA}\right)\middle|\mathbf{P}(E-\mathbf{H})\mathbf{Q}\middle|\Psi_{J_{r_0}M_{r_0}}\right\rangle \\ &= \left\langle\overline{\Psi}_{J_{r_0}M_{r_0}}\middle|\mathbf{Q}(E-\mathbf{H})\mathbf{P}\middle|\overline{U}_{aA}\overline{f}_{aA}\left(R_{aA}\right)\right\rangle, \end{aligned} \tag{29}$$

which leads to a T-invariance condition for g-amplitudes of the form

$$g\left(I_AJ_al_a \leftarrow J_{r_0}M_{r_0}\right) = \tilde{g}\left(\overline{J_{r_0}M_{r_0}} \leftarrow \overline{I_AJ_al_a}\right), \tag{30}$$

from which it follows that time-reversed amplitude $\tilde{g}(\overline{J_{r_0}M_{r_0}} \leftarrow \overline{I_AJ_al_a})$ can be constructed by moving from operator $\mathbf{P}(E-\mathbf{H})\mathbf{Q}$ the noncoincident operator $\mathbf{Q}(E-\mathbf{H})\mathbf{P}$. This corresponds to moving from the decay of resonance state r_0 into the aA channel to the inverse process of the formation of the resonance state from the aA channel.

It should be noted that T-invariance condition (30) is different from its counterpart in Eq. (6) that was derived in [1], in the sense that operator $\mathbf{H}'$ in (6) is replaced with the nondiagonal matrix element of the full system Hamiltonian $\mathbf{P}(E-\mathbf{H})\mathbf{Q}$, which changes in the time-reversal operation and is generally not small.

Since the denominator contained in the expression in Eq. (25) and associated with the product of propagators of the form $(E_{r_{k-1}} - E_{r_k} - E_{b_k} - T_{b_k,r_k} + i/2\Gamma_{r_k})^{-1}$

does not change in the time-reversal operation, we can use the above formula in (30) and present the *T*-invari-

ance condition for the amplitude of sequential multi-step statistical nuclear reaction $\mathscr{T}_{b_1...b_nB,aA}$ as

$$\mathscr{T}_{b_1...b_nB,aA} = \tilde{\mathscr{T}}_{\overline{aA},\overline{b_1...b_nB}} = \sum_{r_0M_{r_0}...r_{n-1}M_{r_{n-1}}} \frac{\tilde{g}\left(\overline{I_AI_al_a} \leftarrow \overline{I_{r_0}M_{r_0}}\right)...\tilde{g}\left(\overline{I_{r_{n-1}}M_{r_{n-1}}} \leftarrow \overline{I_BI_{b_n}l_{b_n}}\right)}{\left(E - E_{r_0} + i/2\Gamma_{r_0}\right)...\left(E_{r_{n-2}} - E_{r_{n-1}} - E_{b_{n-1}} - T_{b_{n-1},r_{n-1}} + i/2\Gamma_{r_{n-1}}\right)}. \quad (31)$$

It follows from Eq. (31) that three operations must be performed in order to move from the matrix element of original reaction $\mathscr{T}_{b_1...b_nB,aA}$ to the matrix element of time-reversed reaction $\tilde{\mathscr{T}}_{\overline{aA},\overline{b_1...b_nB}}$: the initial and final states must be swapped in amplitude; these states must be changed to their time-reversed states; and the sequence of amplitudes in reaction $a + A \rightarrow r_0 \rightarrow b_1 + r_1 \rightarrow ... \rightarrow b_1 + ... + b_{n-1} + r_{n-1} \rightarrow b_1 + ... + b_n + B$ must be reversed to the inverse sequence $b_1 + ... + b_n + B \rightarrow b_1 + ... + b_{n-1} + r_{n-1} \rightarrow ... \rightarrow b_1 + r_1 \rightarrow r_0 \rightarrow a + A$, which requires moving from the matrix of original reaction $\mathscr{T}$ to the matrix of inverse reaction $\tilde{\mathscr{T}}$.

CONCLUSIONS

It was shown that the *T*-invariance condition for a quantum system is the *T*-evenness of its Hamiltonian.

Using the unified theory of a nucleus [7, 9] and projection operators **Q** and **P**, the integral formula in Eq. (26) was constructed for amplitude $g(I_AJ_al_a \leftarrow J_{r_0}M_{r_0})$ of the decay of resonance state r_0. This formula includes a nondiagonal matrix element of the system Hamiltonian with respect to the operators **Q** and **P** that relates the wave functions of the resonance states with those of the open decay channels. Using a generalized version of the *R*-matrix theory of nuclear reactions [2] that allows for the appearance of unstable nuclei in the decay of resonance states, the amplitude of the sequential *n*-step statistical nuclear reaction was constructed via the decay amplitudes of the resonance states of intermediate nuclei.

It was shown that the time-reversed amplitude of the sequential multistep statistical reaction coincides with the original amplitude if the following three operations are performed:

The wave functions of the initial and final states are swapped;

These functions are replaced with their time-reversed counterparts;

Inverse reaction matrix $\tilde{\mathscr{T}}$ is used instead of the matrix $\mathscr{T}$ of the direct reaction.

Results were obtained that can be used in describing the characteristics of ternary and quaternary nuclear fission reactions, given the sequential mechanisms of these processes.

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