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Extreme Waves in the Ocean

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Abstract—Issues related to possible mechanisms of the generation of wind-formed extreme waves in the ocean and their forecasting are considered. Data on the generation of solitary waves in a ring wind-and-water tunnel are presented.

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INTRODUCTION

Terms such as killer waves, extreme waves, giant waves, and solitary waves have been used to describe the same sort of natural phenomena in the form of giant waves in the ocean that reach heights of up to 30 m or more. The last term is preferred in respect of shallow water, under both marine and laboratory conditions.

Both theoretical and experimental methods are used to study extreme waves in the ocean. There are several hypotheses regarding the generation of extreme (killer) waves: the focusing of surface waves, their nonlinear interaction, dispersion compression, and modulation instability. Nonlinear Euler equations obtained for ideal, incompressible, and homogeneous fluids are sometimes used in numerical experiments to study such waves. In vector form, these are written

$$\varphi_t + \frac{1}{2}(\nabla\varphi)^2 + gz = \rho, \quad \Delta\varphi = 0, \quad z = \eta(x, y, z), \qquad (1)$$

where φ is the velocity potential of a perfect liquid, η is a free surface shift from the equilibrium horizontal direction, axis z is directed upward, and Δ, ∇ are the Laplacian operator and the gradient along the horizontal coordinates.

This equation is nonlinear, so two types of approximations are sometimes considered for the sake of simplicity:

(a) Waves of infinitesimal amplitude, so that $\lambda \gg a$ and the Euler equation is linear.

(b) Long waves, so that $\lambda \gg H$, where H is the liquid's depth.

As is well known, the dispersion relation for linear waves in deep water has the form

$$\omega^2 = g|\vec{K}|, \qquad (2)$$

where $K = \frac{2\pi}{\lambda}$; $\lambda = \frac{gT^2}{2\pi}$.

In 1847, George Stokes obtained the following approximation based on conformal transformations for deep water waves of finite amplitude:

$$\eta(x,y,t) = \eta(x,z) = a\cos(kx - \omega t) + \frac{1}{2}ka^2\cos 2(kx - \omega t) + \frac{3}{8}k^2a^3\cos 3(kx - \omega t). \qquad (3)$$

Because of the terms containing ka^2, the dispersion relation takes the form

$$\omega = (gk)^{\frac{1}{2}}\left(1 + \frac{1}{2}k^2a^2\right). \qquad (4)$$

In other words, the frequency of the waves depends not only on their wavelength but on their amplitude as well. This is a so-called Stokes wave. Such a wave is symmetrical with respect to the vertical lines passing through its crests and troughs, but not with respect to the plane of the undisturbed level. The crests are located above this plane and the troughs are below it. The troughs have a flatter profile than the crests. With an increase in wave height at a constant length, the wave crests become sharper and approachir the ultimate shape, characterized by the presence of a corner point in their tops. According to Mitchell, the limiting angle at the tops of the Stokes wave is 120 degrees, and the wave steepness is $\frac{h}{\lambda} = 0.142$. The velocity of such a wave's propagation depends on both its length λ and its height h:

$$c = \sqrt{\frac{g\lambda}{2\pi}\left[1 + \left(\frac{2\pi a}{\lambda}\right)^2\right]}. \qquad (5)$$

Due to the open trajectory of particles, the motion of the wave gives rise to a wave current:

$$V = a^2k^2\sqrt{\frac{g\lambda}{2\pi}}e^{ekz}. \qquad (6)$$

It is assumed that in many cases, highly nonlinear physical models based on the Euler equations can be used in the numerical modeling of killer waves, allowing us to describe waves of large amplitude before they decay. Over time, the applicability of the nonlinear Schrödinger equation (NLS) was established for the theory of liquid surface waves in the form of a complex amplitude equation. Zakharov was the first to obtain this for deep-water waves [1]. According to [1], a wave envelope propagates with the group velocity of the carrier wave. The solution to the NLS for a weakly nonlinear wave train describes a solitary wave in the form of a localized wave packet propagating with constant velocity. Zakharov and Shamin [2] proved numerically that a single wave comparable to the New Year's wave observed on the Draupner Platform in the North Sea can emerge in deep-water conditions for a perfect fluid, provided there is long-wave modulation of the Stokes wave train. However, the Draupner platform rests on a shelf at a depth of 80 m; for the New Year's wave, these were conditions of shallow water. Numerical solutions to the nonlinear Schrödinger equation with unsteady disturbances show that the evolution of an unstable wave train leads to a number of cycles of modulation and demodulation, at least when there is no significant viscous dissipation. (Allowing for weak dissipation would show that during the evolution and dissipation of a wave train, the cycles should gradually increase over time). This phenomenon of modulation and demodulation associated with the instability of a nonlinear system is known as the Fermi–Pasta–Ulam recurrence, since it was first discovered by these authors as they studied the anharmonic oscillations of crystal lattices. A clear illustration of this recurrence is the Talbot effect, under which the emergence of pulses was observed every 3000 km in an optical fiber with a total length of 9000 km [3, 4].

According to [5], the propagation of periodic waves of constant shape and finite amplitude can occur in systems as a result of the balance between nonlinear processes and processes of frequency dispersion. It is known that in many cases, these uniform wave trains are unstable with respect to certain small disturbances, and they will decay when propagating over long distances. This effect, known as the Benjamen-Feir instability, occurs with long-wavelength modulation in the wave number of a disturbance less than a certain critical value. It is also known as the instability of side frequencies, since it emerges as the growth of a pair of side components ($\omega_0 \pm w'$, $k_0 \pm k'$) located near the carrier component (ω_0, k_0). If the frequency and wave number are disturbed slightly and take the form $\omega = \omega_0 + \omega'$) and $k = k_0 + k'$, where $\omega' \ll \omega_0$ and $k' \ll k_0$, the disturbances result in modulation of the amplitude and phase of the primary wave: constant amplitude a becomes function $a(x,t)$, and the summand $\theta(x,t)$ is added to the phase. In this case, the disturbances can draw energy from the main wave motion. The elevation of the free water surface can be written as

$$\xi(x,t) = \mathrm{Re}\{A(x,t)\exp[i(k_0x - \omega_0 t)]\}, \tag{7}$$

where

$$A(x,t) = a(x,t)\exp[i\theta i\theta](t) \tag{8}$$

is the complex envelope. The equation for the complex envelope of weakly nonlinear waves in deep water was obtained in [1] and has the form

$$i\left(\frac{\partial A}{\partial t} + \frac{\omega_0}{2k_0}\frac{\partial A}{\partial x}\right) - \frac{\omega_0}{8k_0}\frac{\partial^2 A}{\partial x^2} - \frac{1}{2}\omega_0 k_0^2 |A|^2 A = 0. \tag{9}$$

After the discovery of wave modulation instability, it was considered that the final stage of development fora modulated wave's train could be either its complete decay or the development of a stable wave train of envelope solitons. According to experiments conducted in a laboratory tray, however, it was shown in [6] that unstable modulations grow to a certain maximum height and then collapse. In addition, the have determined the parameters of the waves and their spectral density under conditions of growing instability. They noted that their limited tray length did not allow them to observe more than one cycle of wave evolution.

In order to identify the possible physical mechanisms of extreme wave generation, we performed experiments in a ring wind-and-water tunnel (WWT) [6–9]. The outer and inner diameters of the WWT were 202 and 165 cm, respectively, with a height of 40 cm. Let us note some advantages of a WWT:

(1) A mode of wind wave generation for both capillary and solitary waves. This mode is obtained with the air flow velocity in the WWT initially adjusted for solitary wave generation at a given depth of the liquid.

(2) A quasisteady mode of wind waves at different stages of their generation through correlating the energy of the air flow in the ring tunnel and the energy lost as a result of friction on the bottom and sides of the tunnel.

(3) A fading mode in the case of a phased or a one-time air flow velocity decrement.

It is important that we study mechanisms of internal wave interaction in all three wind wave modes.

There is one more advantage to a ring tunnel: The generation and development of wind waves is more realistic than in straight tunnels, where waves are generated by wavemakers. It should therefore be noted that the modulation of unstable wind wave trains in a ring tunnel can differ from that in straight tunnels, and can disappear in other wind wave modes.

The disadvantages of a ring tunnel are the effect its size has on the wind wave parameters, vertical circulation caused by the increased water level at the outer wall, and the lateral oscillation of the liquid surface inside the tunnel upon a reduction in airflow velocity.

Earlier, V.V. Shuleikin created a so-called stormy tank in Katseveli, in the Crimea. The outer diameter of

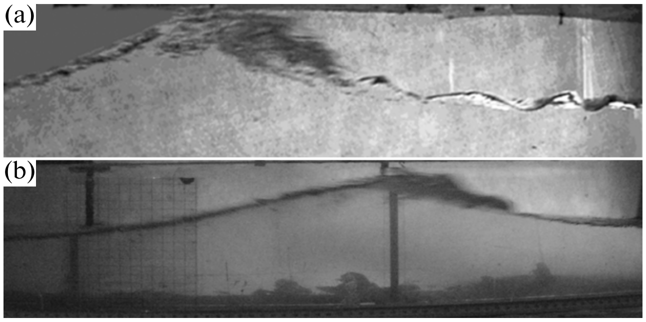

Fig. 1. Solitary wave in a ring wind-and-water tunnel: (a) in pure water; (b) with surfactants.

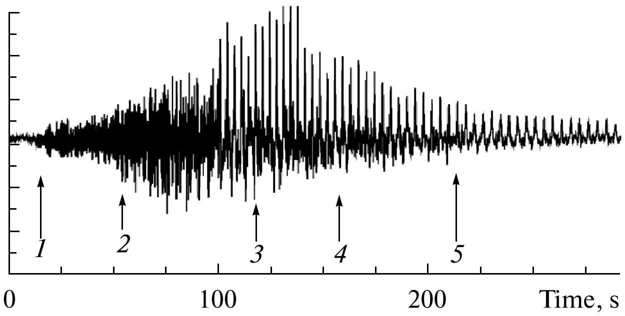

Fig. 2. Wind wave train (in compressed form) in a ring tunnel, from generation to decay. The numbers indicate nominal sections of the train: (*1*) stage of wind wave development; (*2*) wave train development; (*3*) generation of two solitary waves; (*4*) generation of one solitary wave (stage of decay); (*5*) decay.

the tank was 40 m; its inner diameter was 38 m, and its height was 5.6 m. Twenty powerful fans were installed on the roof, creating an air flow of up to 19 m/s. The air was boosted through special tubes at an angle of 30 degrees, and the tank was filled with seawater to a depth of 2 m. It was shown that wave trains formed in the course of wind wave development. A tenth wave was observed in each wave train.

The generation of solitary waves in our ring wind-and-water tunnel occurred in the following sequence: After the fan was turned on, gravity waves similar to harmonic waves formed, height h and length λ of which gradually increased. Long waves then formed that lined up in proportion to their values of h and λ so that the longest of them were ahead of the others. With time, they overtook the smaller waves and passed through them. As a result, the total number of long waves in the tunnel fell. Four solitary waves then formed.

One extreme wind wave later formed as a result of the pairwise nonlinear interaction between the two last solitary waves in the tunnel (Fig. 1).

As can be seen from Fig. 1a, the highest extreme wave was generated in pure water, with visible fluid oscillations in the leading edge caused by the wind in the interval between two consecutive passages of the wave in front of the camera. Figure 1b shows a solitary wave in the presence of surfactants.

Our studies in shallow water showed that the extreme wave magnitude in the ring tunnel varied, depending on the liquid's depth and the air flow velocity. In other words, a series of multiple solitary waves whose magnitude grows along with depth and wind velocity can be generated in the ring tunnel.

We may assume that an extreme wave in shallow water (on a shelf) is a solitary wind wave that extends along the water column from surface to bottom with a swirling leading edge. When the wind slows, an extreme wave decays rapidly. In view of the wind effect and bottom friction, the parameters of such waves can be determined using the Korteweg–de Vries equation. The rapid generation of an extreme wave can occur upon reciprocal interaction between two or more huge waves.

It is well known that a large water surface area can be coated with oil films as a result of accidents with platforms and tankers. Surfactant films both weaken the existing surface waves and hinder their generation. Small wind waves do not form when surfactants are present, due to the reduced membrane frictional stress in the water sheet. This means that the main source of wave generation is a normal component of pressure that is inherent to a perfect fluid.

We also conducted a series of flotation tests on a liquid surface. It was found that increased flotation reduces the amplitude of a solitary wave, increasing its length and time of formation. At a certain critical mass, wind waves were generated only within 1–2 h of blower operation. At the same time, long waves formed as in the case with surfactants, and one solitary wave was generated. Upon a further increase in flotation, no solitons formed at all. It should therefore be noted that it takes more time to generate extreme wind waves in northern seas coated with broken ice.

Some authors set the initial wave profiles arbitrarily in their numerical and laboratory experiments. The question then arises: How do the time of the generation of extreme waves (solitons) and their parameters depend on the initial wave height? To answer this question, we measured wave parameters, giving special attention to the formation of the maximum wave heights at different stages of wave generation and a constant wind velocity.

We found that extreme wave parameters depended on the time of generation at a constant wind velocity. In addition, there is gradual growth of an extreme wave as the air flow rate and liquid's depth increase.

A wave train is shown in Fig. 2 to provide a qualitative representation of wind wave development and decay in the ring tunnel. Numbers 1–5 indicate the

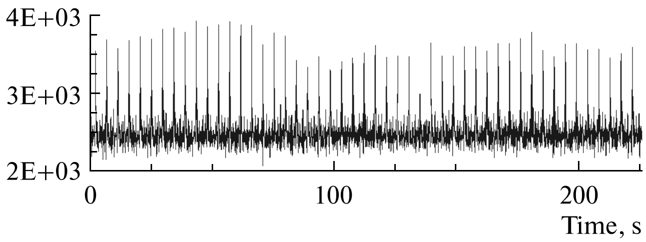

Fig. 3. The 200-second wave record of the same solitary wave in the ring tunnel. The interval between adjacent peaks is 4 s.

locations of the selected areas of wave formation in the real wave scale. Figure 2 shows wave generation in sector 1; the formation of wave trains in sector 2; the generation of two solitary waves in sector 3; the generation of a solitary wave in sector 4; and the decay of a wave, accompanied by a reduction in the wave's height and an increase in its length, in sector 5.

Figure 3 shows the 200-second wave record of the same solitary wave in the ring tunnel.

CONCLUSIONS

The discovery of solitary wave generation in shallow water in our ring wind-and-water tunnel under the effect of air flow was an important step in conceptualizing the mechanisms of extreme wave generation in the ocean as a result of interaction between it and the atmosphere.

It was shown that the decay of solitary wind waves produced in the ring tunnel took longer than their development.

When solitary waves decayed in the ring tunnel, their height was reduced while their length and period increased. The decay of a solitary wave in a shelf zone can be observed even when there is no wind, but with a smaller solitary wave.

It was found that flotation slows the generation of a solitary wind wave, reducing its height and increasing its length.

We believe that the most important factor of wave generation in a ring wind-and-water tunnel is the air flow rate, and the internal wave interaction leads to concentration of the wave energy in certain areas of the spectrum.

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