ISSN 1060-992X, Optical Memory and Neural Networks (Information Optics), 2015, Vol. 24, No. 2, pp. 123–129. © Allerton Press, Inc., 2015.

# Comparison of Learning Methods for Spiking Neural Networks

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Received February 9, 2015; in final form, April 3, 2015

**Abstract**—Investigation of different factor influence on the spike-timing-dependent plasticity learning process was performed. The next factors were analyzed: choice of spike pairing scheme, shapes of postsynaptic currents and the choice of input type signal for learning. Best factors for learning performance were extracted.

*Keywords*: spiking neural network, learning methods, neurosimulators, STDP, spike-pairing scheme

**DOI:** 10.3103/S1060992X15020095

## 1. INTRODUCTION

In recent years there has been growing interest in spiking neural networks (SNN). On the one hand, this was caused by progress in experimental techniques in vitro which allowed to investigate the neuronal culture on the microelectrode array and on the other hand, it was induced by achievements in developing of SNN simulators. However, the learning methods of SNN remain semi-developed, making the study of such techniques actual problem.

Bioinspired models of SNN are considered as the new generation of neural networks relatively to formal models of neural networks with static elements [1]. Neurons and synapses of SNN are dynamic objects, and their properties depend on time. Information richness of biosimilar neurons and synapses with synaptic plasticity potentially increases associative abilities of SNN.

Special interest are the attempts to use spiking neural networks with learning methods to solve practical problems: extraction of temporally correlated features from vision sensors [2], monophonic sound source separation [3], image clustering [4] and other similar problems. However, question which remains open is developing of effective learning algorithms of SNN in complex model including neuron model, synapse model, spike-timing-dependent plasticity model, and assumption of form of input signal. A set of above mentioned models should be formed for answer to this question and parameters of these models should be evaluated.

The two simulators are applied: NEST (NEural Simulation Tool), CSIM(A neural Circuit SIMulator) for investigations of the influence on the learning performance following factors: three types of spike trains signals (*Poisson* (the number of spike in time interval has a Poisson distribution), *normal* (signal has a normal distribution of interspike interval), *uniform* (signal has a uniform distribution of interspike interval)), spike-timing-dependent plasticity (STDP), form of postsynaptic current, spike pairing scheme in STDP. The experiments of learning the SNN with denoted above parameters were performed and most suitable for learning approaches were presented.

## 2. MATERIALS AND METHODS

The three components of SNN may be determined: neuron, synapse, learning approach STDP.

The equation of membrane potential dynamics of leaky integrate-and-fire neuron has view (1):

$$\frac{dv}{dt} = \frac{v_{\text{resting}} - v}{\tau_m} + \frac{I_{\text{syn}}(t)}{C_m} + \frac{I_e}{C_m}, \tag{1}$$

where $v$, $C_m$ are membrane potential, capacitance. When $v$ achieves threshold $v_{\text{th}}$, neuron fires a spike and during the refractory period $\tau_{\text{ref}}$ after spike, value of membrane potential has constant value $v_{\text{reset}}$. Without synaptic currents, the membrane potential relaxes to the resting potential $v_{\text{resting}}$ with specific time $\tau_m$. $I_{\text{syn}}(t)$ is synaptic current. $I_e$ is background constant stimulation current.

**Table 1.** Parameters of neuron model

| Notation | Value |
|---|---|
| $v$ | $v\ (t=0) = v_{\text{resting}}$ |
| $v_{\text{resting}}$ | 0 mV |
| $\tau_m$ | 30 ms |
| $C_m$ | 30 nF |
| $I_e$ | 13.5 nA |
| $v_{\text{th}}$ | See experiments |
| $\tau_{\text{ref}}$ | 3 ms |
| $v_{\text{reset}}$ | 14.2 mV |

**Table 2.** Parameters of short-term plasticity model. Notation $N(\mu;\sigma)$ indicates that the parameter has a normal distribution with mean $\mu$ and standard deviation $\sigma$

| Notation | Value |
|---|---|
| $u_k$ | $u_k\,(t=0) = U$ |
| $R_k$ | $R_k\,(t=0) = 1$ |
| $U$ | $N$(0.5; 0.05) for excitatory<br>$N$(0.25; 0.025) for inhibitory |
| $D$ | $N$(1100; 110) ms for excitatory<br>$N$(700; 70) ms for inhibitory |
| $F$ | $N$(50; 5) ms for excitatory<br>$N$(20; 2) ms for inhibitory |

Values of parameters of neuron model were presented in Table 1.

Synapse model expressed by the Eqs. (5), (6) of the synaptic current $I_{\text{syn}}(t)$, which is involved in neuron Eq. (1). The components of model synapse may be determined:

• Model of synaptic plasticity (short-term and long-term) determines the dynamics of synaptic weights relative to activities of the presynaptic and postsynaptic neurons.

• Form of postsynaptic current.

Model of short-term plasticity Maass–Markram [5] was used, which is described below (2):

$$\begin{cases} u_1 = U, \\ R_1 = 1, \\ u_k = U + u_{k-1}(1-U)e^{-\frac{\Delta_{k-1}}{F}}, \\ R_k = 1 + (R_{k-1} - u_{k-1}R_{k-1} - 1)e^{-\frac{\Delta_{k-1}}{D}}, \end{cases} \tag{2}$$

where $R_k$ are fractions of synaptic efficacy available resources; $u_k$ are fractions of available resources, which are activated when a presynaptic spike occurs; $F$ is time constant for recovery from facilitation; $U$ is fraction of synaptic resources that are used for a single spike; $D$ is time constant for recovery from depression; $\Delta_1, \Delta_2, \ldots, \Delta_{k-1}$ are interspike intervals.

Values of parameters of short-term plasticity model were presented in Table 2.

The long-term STDP rule describes synaptic weight changes relative to times of spikes of presynaptic and postsynaptic neurons. Several long-term plasticity rules are used in this article. Consider the additive STDP rule used in simulations with Neural Simulation Tool (NEST) [6].

**Table 3.** Parameters of long-term plasticity model. Notation $N(\mu; \sigma; a; b)$ indicates that the parameter has a normal distribution with mean $\mu$ and standard deviation $\sigma$ with interval constraint [a, b]. In simulators NEST and CSIM values lower than a (greater than b) are replaced by a(b)

| Notation | Value |
|---|---|
| $w_+$ | 0.3 |
| $w_-$ | –0.3105 |
| $\tau_+$ | 20 ms |
| $\tau_-$ | 20 ms |
| $W_{min}$ | 0 |
| $W_{max}$ | $N(54; 10.8; 21.6; 86.4)$ |

On activation of presynaptic neuron weight value decreases, and vice versa, at the activation of the postsynaptic neuron weight value increases. The change of weight described in formula:

$$\Delta w = \begin{cases} w_- e^{-\frac{t_{pre}-t_{post}}{\tau_-}}, & t_{pre} - t_{post} > 0, \\ w_+ e^{-\frac{t_{post}-t_{pre}}{\tau_+}}, & t_{pre} - t_{post} < 0, \end{cases} \tag{3}$$

where $t_{pre}$ is spike time of presynaptic neuron, $t_{post}$ is spike time of postsynaptic neuron, $w_-$ is amplitude parameter of weight decrease, $w_+$ is amplitude parameter of weight increase, $\tau_-$ is time constant for depression, $\tau_+$ is time constant for potentiation. If the weight value is less (greater) than minimum (maximum) allowed $W_{min}(W_{max})$, then $w$ is assigned $W_{min}(W_{max})$.

Analysis based on source code has revealed that simulator CSIM used following STDP model:

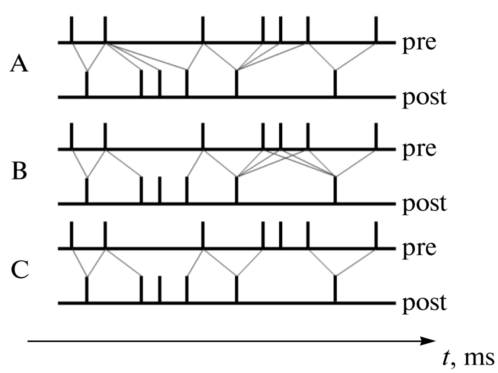
$$\Delta w = \begin{cases} w_- e^{-\frac{t_{pre}-t_{post}}{\tau_-}}, & t_{pre} - t_{post} > 0, \text{ and } (t_{pre} - t_{post}) \in [T_{min}, T_{max}], \\ 0, & t_{pre} - t_{post} > 0, \text{ and } (t_{pre} - t_{post}) \notin [T_{min}, T_{max}], \\ w_+ e^{-\frac{t_{post}-t_{pre}}{\tau_+}}, & t_{pre} - t_{post} < 0, \text{ and } (t_{post} - t_{pre}) \in [T_{min}, T_{max}], \\ 0, & t_{pre} - t_{post} < 0, \text{ and } (t_{post} - t_{pre}) \notin [T_{min}, T_{max}], \end{cases} \tag{4}$$

where $T_{min} = 2$ ms, $T_{max} = 60$ ms.

Values of parameters of long-term plasticity model are presented in Table 3.

It's known, that different schemes of spike pairing [7] exist. These schemes are shown on the figure.

The scheme "A" is used in NEST, the information about scheme used in CSIM wasn't found.




Examples of spike pairing schemes. Pre is presynaptic neuron, post is postsynaptic neuron. Adapted from article [7].

In our work we apply two forms of postsynaptic currents: exponential and $\alpha$-function form. The synapse is used with short-term and long-term plasticity described above. The equation of exponential form of postsynaptic signal:

$$I_{\rm syn}(t) = \sum_{j\in S}\sum_{t_{\rm sp}} w_j(t_{\rm sp})u_j(t_{\rm sp})R_j(t_{\rm sp})\frac{q_j}{\tau_j}e^{-\frac{t-t_{\rm sp}-t_{\rm d}}{\tau_j}}H(t-t_{\rm sp}-t_{\rm d}), \tag{5}$$

where $S$ is set of synapses incoming in neuron; $w_j$ is synaptic weight of $j$-th synapse; $t_{\rm sp}$ is time of emergence of spike on presynaptic neuron; $t_{\rm d}$ is delay of spike propagation (in NEST 0.1 ms, in CSIM—0 ms); $\tau_j$ is spike time decay constant of synaptic current of $j$-th synapse; $q_j$ is charge that is injected into the postsynaptic neuron when presynaptic neuron fires a spike; respectively. $H(t)$ is Heaviside function. $u_j$, $R_j$ defined by the Eq. (2). $q_j$ for excitatory(inhibitory) synapse is 3 pC (6 pC). $\tau_j$ for excitatory(inhibitory) synapse is 3 ms (6 ms).

The equation of postsynaptic $\alpha$-function form signal is:

$$\begin{cases} I_{\rm syn}(t) = \sum\limits_{j\in S}\sum\limits_{t_{\rm sp}} w_j(t_{\rm sp})u_j(t_{\rm sp})R_j(t_{\rm sp})\dfrac{q_j}{\tau_j}\alpha_j(t-t_{\rm sp}-t_{\rm d}), \\ \alpha_j(t) = \dfrac{t}{\tau_j}e^{-\frac{t}{\tau_j}}H(t). \end{cases} \tag{6}$$

The two simulators are employed in this work: NEST [6], CSIM [8].

The NEST is a computer program for simulations of large heterogeneous networks. NEST is more suitable for models that focus on the dynamics, size and structure of networks rather on the detailed individual neuron's properties. NEST uses the opportunities of parallelism based on MPI and OpenMP. The original version of NEST has been modified by authors to be able to use other learning techniques.

CSIM is a tool for simulation of heterogeneous networks with various models of neurons and synapses.

## 3. DESCRIPTION OF EXPERIMENTAL TECHNIQUE

The experimental technique was first described in the paper [9]. The purpose of this experiment was the study of the general convergence of the learning process. The convergence was estimated in two ways: through the convergence to the target weights and through the correlation of input and output signals.

The neural network consists of one neuron, to which 100 synapses are connected: 90 excitatory synapses and 10 inhibitory. Excitatory synapses are described by equations of the short-term plasticity (2) and by equations of the long-term plasticity (the Eq. (3) for NEST and Eq. (4) for CSIM). The inhibitory synapses are modeled by Eq. (2). The neuron is modeled by Eq. (1).

Stages of the experiment:

1. *The stage of selecting the maximum values of the weights:*

For excitatory synapses the vector of maximum values of weights $\vec{W}_{\max}$ is generated with normal distribution with average value equal to 54 and standard deviation sd = 10.8. The weights are in the range 54 ± 3sd. The weight's values less than 21.6 (and more than 86.4) are replaced by value 21.6 (86.4) (see Table 3).

2. *The stage of selecting the target weights of the excitatory synapses:*

The one-half of excitatory synapses is randomly selected, and the target weights for this half are set equal to maximum values of the weights, obtained in stage 1. The target weights of the remaining half of excitatory synapses are set to zero. As a result of training the weights of synapses should converge to the vector of target weights. The weights of the inhibitory synapses are set to –54 and they remain constant at each stage of the experiment.

3. *The stage of generation of the input signal:*

The independent signal sequence $S_{in}^{i}(i = 1,2,\dots100)$ is generated for each of 100 inputs. There are three types of signals: Poisson signal (the number of spike in time interval has a Poisson distribution), and the signals with normal and uniform distribution of interspike interval. The one type of signal sequence is used for each experiment. The sequences $\vec{S}_{in}$ are generated with mean frequency equal to 20 Hz during 3000 s. Subsequently the obtained sequences are used in learning process.

**Table 4.** The threshold value of neuron's potential selected on stage 4. Exp, α means the form of postsynaptic current described in Eqs. (5), (6) respectively

| The type of input signal | Simulator | | |
|---|---|---|---|
| | NEST | | CSIM |
| | The form of postsynaptic current | | |
| | exp | α | exp |
| Poisson signal | 15.7 | 15.6 | n/e (no evidence) |
| Normal signal | 15.9 | 15.8 | n/e |
| Uniform signal | 15.7 | 15.7 | 15.1 |

**Table 5.** Value of parameter $\beta(t)$ after learning for different simulators

| Type of input signal | NEST | | CSIM |
|---|---|---|---|
| | exp | α | |
| Poisson signal | 0.1 | 0.2 | n/e |
| Normal signal | 0.3 | Diverge | n/e |
| Uniform signal | 0.1 | 0.2 | 0.03 |

4. *The stage of configuration of the threshold value of the potential:*

For the given input signal $\vec{S}_{in}$ the required value of the threshold potential of the neuron $\nu_{th}$ is selected in order to generate the output signal with mean frequency equal to 25 Hz.

5. *The stage of obtaining the reinforcing signal:*

On this stage for the given input signal $\vec{S}_{in}$ and target of weights $\vec{W}_{\text{target}}$ the output signal is obtained. Subsequently this output signal is used as reinforcing in the learning process. This sequence of reinforcing signals $\vec{S}_{+}$ is used as current impulses of rectangular form with an amplitude equal to 1 μA and duration equal to 0.2 ms. The changing of the weight through the STDP is disabled on this stage.

6. *The learning stage:*

The initial values of the weights of the excitatory synapses are randomly selected on the basis of gamma distribution with average value equal to 9 and standard deviation equal to 6.3. During the learning the input signal $\vec{S}_{in}$ and reinforcing signal $\vec{S}_{+}$ are given. The duration of learning is 3000 s.

In Table 4 the threshold value of neuron's potential is presented. This value was obtained from the condition in which for the input signal with mean frequency of 20 Hz the output signal had mean frequency of 25 Hz.

The parameter $\beta(t)$ was introduced for the estimation of the convergence to the target weights:

$$\beta(t) = \frac{\sum_{i=1}^{90} \left| w_i(t) - w_i^{\text{target}} \right|}{\sum_{i=1}^{90} w_i^{\text{target}}}, \tag{7}$$

where $\vec{W}(t)$ is the vector of excitatory weights in the moment $t$. The summation is made only over weights of excitatory synapses, because only they change during learning process.

## 4. RESULTS

The simulation results in NEST with parameters described in "Materials and Methods" and "Description of experimental technique" sections show that the learning does not occur and weights do not converge to the target weights, parameter $\beta(t)$ tends to 1.

CSIM simulator demonstrates the availability of learning and the convergence parameter $\beta$ tending to 0.03 after training (Table 5).

**Table 6.** Value of parameter β(*t*) after learning for NEST simulator

| Type of input signal | Type of STDP | | | | | |
|---|---|---|---|---|---|---|
| | formula (3) | | | formula (4) | | |
| | spike pairing scheme | | | spike pairing scheme | | |
| | A | B | C | A | B | C |
| Normal signal | 1.0 | 0.25 (0.25) | 0.15 | 1.0 | 0.1 | 0.1 |
| Poisson signal | 1.0 | 0.8 (0.85) | 0.05 | 0.07 | 0.4 (0.4–0.5) | 0.07 |
| Uniform signal | 1.0 | 0.8 (0.8) | 0.05 | 1.0 | 0.4 (0.4–0.6) | 0.07 |

**Table 7.** Value of parameter β(*t*) after learning without neuron

| Type of input signal | Type of STDP | | | | | |
|---|---|---|---|---|---|---|
| | formula (3) | | | formula (4) | | |
| | spike pairing scheme | | | spike pairing scheme | | |
| | A | B | C | A | B | C |
| Normal signal | 1.0 | 0.05 | 0.05 | 1.0 | 0.05 | 0.05 |
| Poisson signal | 0.05 | 0.05 | 0.05 | 0.2 | 0.2 | 0.45 (0.2) |
| Uniform signal | 0.05 | 0.05 | 0.05 | 0.15 | 0.2 | 0.5 (0.2) |

The learning process was registered in the NEST with replacing STDP rule by (4) and replacing spike pairing scheme "A" by "C" (figure) for three types of input signals in case of exponential form of postsynaptic current (Table 5). The learning with normal input signal and the α-function form of synaptic current does not occur and weights diverge.

Furthermore, the convergence evaluation by correlation of target output signal and obtained signal was performed. To do so, new sequences of 100 second duration signals were generated for three types of signals. We transformed spikes by Gaussian filter with standard deviation 5 ms in calculation of correlation of the two signals. The values of correlations for exponential form of postsynaptic current are approximately equal to 0.8 for three types of signals for experiments with NEST.

The experimental results for simulators have showed differences, because the different types of STDP have been used in simulators and it has been necessary to examine STDP parameter sets in these simulators. Further, the NEST is selected as base simulator for comparison and all results performed below are obtained with NEST.

As it was denoted above, the original version of NEST was modified. The experiments for different forms of STDP, spike pairing schemes, type of signals are realized and the results are provided in Table 6. There were cases, when values of parameter β(*t*) did not converge in 3000 s. In such cases the experiment was continued up to 10000 s, and if convergence was observed, the such results β (10000 s) are enclosed by parentheses (for example, case of Poisson signal with STDP formula (3) and spike pairing scheme B provided results: β (3000 s) = 0.8, β (10000 s) = 0.85). If convergence is not observed, then results enclosing in parentheses represent β range in (3000, 10000) s (for example, case of Poisson signal with STDP formula (4) and spike pairing scheme B provided results: β (3000 s) = 0.4, $\beta(t) \in (0.4, 0.5)$ in (3000, 10000) s).

The next step was investigation how STDP functions without neuron, only with input signal $\vec{S}_{in}$ and output signal generated on stage 4 in "Description of experimental technique" without processing it to the reinforcing signal $\vec{S}_{+}$. The results are provided in Table 7.

The obtained results show, that spike pairing scheme "C" is more perspective for developing of learning models.

## 5. CONCLUSIONS

In this work the investigation of STDP learning mechanisms of spiking neural networks in two different simulators was done. The importance of selection of associated set of STDP rules, spike pairing schemes, forms of input signals and forms of the postsynaptic currents for the learning process was shown. On the base of the experimental results combination of these factors (STDP Eq. (3), spike pairing scheme case "C"), suitable for learning process and which can be used for practical applications was selected. This combination is applicable for all types of input signals.

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