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RELIABILITY, STRENGTH, AND WEAR RESISTANCE OF MACHINES AND STRUCTURES

The Stress State in the Boundary Region of a Conical Shell according to a Refined Theory

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Abstract—In this article, on the basis of a refined theory, we study the stress state of a conical shell of the "boundary layer" type. In comparison with the classical theory, the sought displacements of the shell are approximated by polynomials along the coordinate normal to the median surface with a degree two units higher. Based on the equations of the three-dimensional elasticity theory and the Lagrange variational principle, a system of differential equations of equilibrium in displacements with variable coefficients is obtained. The solution of the formulated boundary value problem is carried out by using the methods of finite differences and matrix sweep. The results can be used when assessing the strength and durability of shell structures.

Keywords: conical shell, Lagrange variational principle, refined mathematical model, "boundary layer" stress state, transverse normal stresses

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Currently, conical shells are widely used in aircraft constructions, rocketry, shipbuilding, and the automotive and construction industries. Engineering calculations of conical shells are based on the classical Kirchhoff–Love theory. The hypotheses used in this theory do not permit transverse deformations to be taken into account, which leads to errors in determining the stress state of plates and shells, especially in zones near joints, local loading, and rapidly changing loads.

In the development of an approximate theory of shells, free of Kirchhoff–Love hypotheses, the method of direct asymptotic integration of differential equations of the three-dimensional elasticity theory became widespread. The problem of determining the stress–strain state (SSS) of plates and shells was reduced to the construction of three stress–strain states, corresponding in the first approximation to the internal stress–strain state, determined by the classical theory, and two additional states of the "boundary layer" [2]. There is a bibliography of works on the asymptotic integration of equations of elasticity theory for thin-walled systems [1]. The problem of an additional (with respect to the classical theory) SSS near the clamped edge of plates was solved in [3] using the variational-asymptotic method and a special approximating polynomial function.

An approximate solution to the spatial problem of elasticity theory was obtained by E.M. Zveryaev [6–8] using the modified semi-inverse Saint-Venant method. Two-dimensional resolving equations for determining the basic SSS were obtained, which coincide with the equilibrium equations of the classical theory, as well as additional equations for calculating the SSS of the "boundary layer" type, taking into account shear corrections.

Another approach to the formulation of a refined theory is based on approximation of the unknown displacements of the shell using polynomials along the coordinate normal to the median surface [4]. In [5], within the framework of this approach, a refined theory of calculating the SSS for cylindrical shells was elaborated, according to which there are significant additional local stresses.

In this article, within the framework of the approach presented in [4, 5], the stress–strain state of a conical shell under the action of an axisymmetric load is investigated. The unknown displacements are expanded in polynomials in the coordinate normal to the median plane of the shell. The degree of these polynomials is two units higher than in the classical Kirchhoff–Love–type theory.

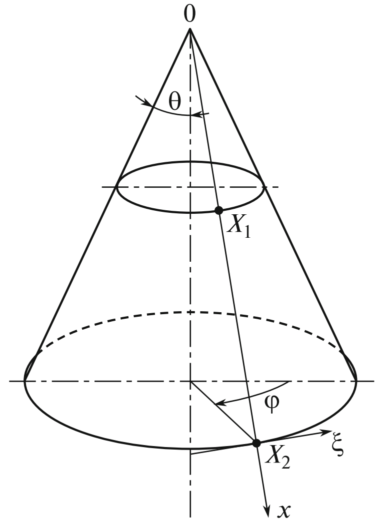

Fig. 1. Conical shell.

Basic equations of a conical shell. We consider a conical shell of constant thickness $2h$ made of an isotropic material, referred to the x, φ, ξ coordinate system (Fig. 1). The Lamé coefficients are determined by the formulas

$$H_i = A_i a_i, \quad H_3 = 1, \quad a_i = 1 + \frac{\xi}{R_i}, \quad i = 1, 2,$$

where A_1, A_2 and R_1, R_2 denote the coefficients of the first quadratic form and principal curvatures of the shell. For the coordinate system used, we have

$$A_1 = 1, \quad A_2 = x\sin\theta, \quad R_1 = \infty, \quad R_2 = \frac{x}{\cot\theta}.$$

We set the following boundary conditions on the lateral and end surfaces of the shell:

$$\sigma_{i3}(\pm h) = q_{i3}^{\pm}, \quad \sigma_{ji} = q_{ji}, \quad i = 1, 2, 3, \quad j = 1, 2.$$

The required elastic displacements are written in the form

$$\begin{aligned} U_1(x,\varphi,\xi) &= u_0(x,\varphi) + u_1(x,\varphi)\xi + u_2(x,\varphi)\frac{\xi^2}{2!} + u_3(x,\varphi)\frac{\xi^3}{3!}, \\ U_2(x,\varphi,\xi) &= v_0(x,\varphi) + v_1(x,\varphi)\xi + v_2(x,\varphi)\frac{\xi^2}{2!} + v_3(x,\varphi)\frac{\xi^3}{3!}, \\ U_3(x,\varphi,\xi) &= w_0(x,\varphi) + w_1(x,\varphi)\xi + w_2(x,\varphi)\frac{\xi^2}{2!}. \end{aligned} \quad (1)$$

Substituting expansions (1) into the geometric equations of the three-dimensional elasticity theory, we obtain expressions for the shell deformations

$$e_{11} = \sum_{k=0}^{3}\frac{1}{A_1 a_1}\left(\frac{\partial u_k}{\partial x} + \frac{\partial A_1}{\partial\varphi}\frac{v_k}{A_2}\right)\frac{\xi^k}{k!} + \sum_{k=0}^{2}\frac{w_k}{R_1 a_1}\frac{\xi^k}{k!},$$

$$e_{22} = \sum_{k=0}^{3}\frac{1}{A_2 a_2}\left(\frac{\partial v_k}{\partial\varphi} + \frac{\partial A_2}{\partial x}\frac{u_k}{A_1}\right)\frac{\xi^k}{k!} + \sum_{k=0}^{2}\frac{w_k}{R_2 a_2}\frac{\xi^k}{k!}, \quad e_{33} = \sum_{k=1}^{2} w_k \frac{\xi^{k-1}}{(k-1)!},$$

$$e_{12} = \sum_{k=0}^{3}\frac{1}{A_2 a_2}\left(\frac{\partial u_k}{\partial\varphi} - \frac{1}{A_1}\frac{\partial A_2}{\partial x}v_k\right)\frac{\xi^k}{k!} + \sum_{k=0}^{3}\frac{1}{A_1 a_1}\left(\frac{\partial v_k}{\partial x} - \frac{1}{A_2}\frac{\partial A_1}{\partial\varphi}u_k\right)\frac{\xi^k}{k!}, \quad (2)$$

$$e_{13} = \sum_{k=0}^{2}\frac{1}{A_1a_1}\frac{\partial w_k}{\partial x}\frac{\xi^k}{k!} + \sum_{k=1}^{3}u_k\frac{\xi^{k-1}}{(k-1)!} - \sum_{k=0}^{3}\frac{u_k}{R_1a_1}\frac{\xi^k}{k!},$$

$$e_{23} = \sum_{k=0}^{2}\frac{1}{A_2a_2}\frac{\partial w_k}{\partial \varphi}\frac{\xi^k}{k!} + \sum_{k=1}^{3}v_k\frac{\xi^{k-1}}{(k-1)!} - \sum_{k=0}^{3}\frac{v_k}{R_2a_2}\frac{\xi^k}{k!}.$$

We obtain the equilibrium equations for the conical shell from the condition for the minimum of the Lagrange energy functional

$$\begin{aligned}\delta L = \iiint &[(\sigma_{11}\delta e_{11} + \sigma_{22}\delta e_{22} + \sigma_{33}\delta e_{33} + \sigma_{12}\delta e_{12} + \sigma_{13}\delta e_{13} + \sigma_{23}\delta e_{23}) \\ &- (G_1\delta U_1 + G_2\delta U_2 + G_3\delta U_3)]\,A_1A_2a_1a_2\,dx\,d\varphi\,d\xi \\ &- \iint (q_{11}\delta U_1 + q_{12}\delta U_2 + q_{13}\delta U_3)\,A_2a_2\,d\varphi\,d\xi \\ &- \iint (q_{21}\delta U_1 + q_{22}\delta U_2 + q_{23}\delta U_3)\,A_1a_1\,dx\,d\xi \\ &- \iint \Big\{q_{13}^{+}[a_1a_2\delta U_1]_{(\xi=+h)} - q_{13}^{-}[a_1a_2\delta U_1]_{(\xi=-h)} \\ &+ q_{23}^{+}[a_1a_2\delta U_2]_{(\xi=+h)} - q_{23}^{-}[a_1a_2\delta U_2]_{(\xi=-h)} \\ &+ q_{33}^{+}[a_1a_2\delta U_3]_{(\xi=+h)} - q_{33}^{-}[a_1a_2\delta U_3]_{(\xi=-h)}\Big\}\,A_1A_2a_1a_2\,dx\,d\varphi = 0,\end{aligned} \tag{3}$$

where G_i, $i = \overline{1,3}$ are the bulk forces. Substituting (2) into (3) and making transformations, the following system of equations is obtained:

$$\begin{aligned}&\frac{\partial(A_2M_1^{(k)})}{\partial x} + \frac{\partial(A_1M_{12}^{(k)})}{\partial \varphi} - \frac{\partial A_2}{\partial x}M_2^{(k)} + \frac{\partial A_1}{\partial \varphi}M_{21}^{(k)} + A_1A_2\left(\frac{Q_1^{(k)}}{R_1} - T_1^{(k)} + P_1^{(k)} + X_1^{(k)}\right) = 0, \\ &k = \overline{0,3}; \\ &\frac{\partial(A_1M_2^{(k)})}{\partial \varphi} + \frac{\partial(A_2M_{21}^{(k)})}{\partial x} - \frac{\partial A_1}{\partial \varphi}M_1^{(k)} + \frac{\partial A_2}{\partial x}M_{12}^{(k)} + A_1A_2\left(\frac{Q_2^{(k)}}{R_2} - T_2^{(k)} + P_2^{(k)} + X_2^{(k)}\right) = 0, \\ &k = \overline{0,3}; \\ &\frac{\partial(A_2Q_1^{(k)})}{\partial x} + \frac{\partial(A_1Q_2^{(k)})}{\partial \varphi} - A_1A_2\left(\frac{M_1^{(k)}}{R_1} + \frac{M_2^{(k)}}{R_2} + T_3^{(k)} - P_3^{(k)} - X_3^{(k)}\right) = 0, \quad k = \overline{0,2}.\end{aligned} \tag{4}$$

Here we use the following definitions of the generalized forces:

$$\begin{aligned}&M_1^{(k)} = \int_{-h}^{+h} a_2\sigma_{11}\frac{\xi^k}{k!}d\xi, \quad M_2^{(k)} = \int_{-h}^{+h} a_1\sigma_{22}\frac{\xi^k}{k!}d\xi, \quad M_{12}^{(k)} = \int_{-h}^{+h} a_1\sigma_{12}\frac{\xi^k}{k!}d\xi, \\ &M_{21}^{(k)} = \int_{-h}^{+h} a_2\sigma_{12}\frac{\xi^k}{k!}d\xi, \quad T_1^{(k)} = \int_{-h}^{+h} a_1a_2\sigma_{13}\frac{\xi^{k-1}}{(k-1)!}d\xi, \quad T_2^{(k)} = \int_{-h}^{+h} a_1a_2\sigma_{23}\frac{\xi^{k-1}}{(k-1)!}d\xi, \\ &T_3^{(k)} = \int_{-h}^{+h} a_1a_2\sigma_{33}\frac{\xi^{k-1}}{(k-1)!}d\xi, \quad Q_1^{(k)} = \int_{-h}^{+h} a_2\sigma_{13}\frac{\xi^k}{k!}d\xi, \quad Q_2^{(k)} = \int_{-h}^{+h} a_1\sigma_{23}\frac{\xi^k}{k!}d\xi, \\ &X_1^{(k)} = \int_{-h}^{+h} G_1a_1a_2\frac{\xi^k}{k!}d\xi, \quad X_2^{(k)} = \int_{-h}^{+h} G_2a_1a_2\frac{\xi^k}{k!}d\xi, \quad X_3^{(k)} = \int_{-h}^{+h} G_3a_1a_2\frac{\xi^k}{k!}d\xi, \\ &P_1^{(k)} = q_{13}^{+}\left[a_1a_2\frac{\xi^k}{k!}\right]_{(\xi=+h)} - q_{13}^{-}\left[a_1a_2\frac{\xi^k}{k!}\right]_{(\xi=-h)},\end{aligned} \tag{5}$$

$$P_2^{(k)} = q_{23}^{+}\left[a_1 a_2 \frac{\xi^k}{k!}\right]_{(\xi=+h)} - q_{23}^{-}\left[a_1 a_2 \frac{\xi^k}{k!}\right]_{(\xi=-h)},$$

$$P_3^{(k)} = q_{33}^{+}\left[a_1 a_2 \frac{\xi^k}{k!}\right]_{(\xi=+h)} - q_{33}^{-}\left[a_1 a_2 \frac{\xi^k}{k!}\right]_{(\xi=-h)}.$$

From the physical equations, based on expansions (1), we find the stresses, which we successively substitute into (5) and (4). Transforming the equations obtained with allowance for relations (2), we obtain the following system of differential equations of equilibrium in displacements

$$\sum_{m=0}^{3}\left(Ki_0^{u_m} + Ki_1^{u_m}\frac{\partial}{\partial x} + Ki_{11}^{u_m}\frac{\partial^2}{\partial x^2} + Ki_{22}^{u_m}\frac{\partial^2}{\partial \varphi^2}\right)u_m + \sum_{k=0}^{3}\left(Ki_2^{v_k}\frac{\partial}{\partial \varphi} + Ki_{12}^{v_k}\frac{\partial^2}{\partial x \partial \varphi}\right)v_k$$
$$+ \sum_{n=0}^{2}\left(Ki_0^{w_n} + Ki_1^{w_n}\frac{\partial}{\partial x}\right)w_n = Ki^{q_{13}^{+}}q_{13}^{+} - Ki^{q_{13}^{-}}q_{13}^{-}, \quad i = \overline{1,4};$$
$$\sum_{m=0}^{3}\left(Ki_2^{u_m}\frac{\partial}{\partial \varphi} + Ki_{12}^{u_m}\frac{\partial^2}{\partial x \partial \varphi}\right)u_m + \sum_{k=0}^{3}\left(Ki_0^{v_k} + Ki_1^{v_k}\frac{\partial}{\partial x} + Ki_{11}^{v_k}\frac{\partial^2}{\partial x^2} + Ki_{22}^{v_k}\frac{\partial^2}{\partial \varphi^2}\right)v_k$$
$$+ \sum_{n=0}^{2}Ki_2^{w_n}\frac{\partial w_n}{\partial \varphi} = Ki^{q_{23}^{+}}q_{23}^{+} - Ki^{q_{23}^{-}}q_{23}^{-}, \quad i = \overline{5,8}; \tag{6}$$
$$\sum_{m=0}^{3}\left(Ki_0^{u_m} + Ki_1^{u_m}\frac{\partial}{\partial x}\right)u_m + \sum_{k=0}^{3}Ki_2^{v_k}\frac{\partial v_k}{\partial \varphi}$$
$$+ \sum_{n=0}^{2}\left(Ki_0^{w_n} + Ki_1^{w_n}\frac{\partial}{\partial x} + Ki_{11}^{w_n}\frac{\partial^2}{\partial x^2} + Ki_{22}^{w_n}\frac{\partial^2}{\partial \varphi^2}\right)w_n = Ki^{q_{33}^{+}}q_{33}^{+} - Ki^{q_{33}^{-}}q_{33}^{-},$$
$$i = \overline{9,11}.$$

Here, Ki represent variable coefficients depending on the geometrical parameters, elastic constants of the shell material and coordinate x, and u_m, v_k, w_n are the expansion coefficients of the unknown displacements in expressions (1).

Calculation of a conical shell under an axisymmetric load. In this case, all components of the SSS of the shell are independent of the angle and displacement in the circumferential direction v_k, $k = \overline{0,3}$ (equal to zero). Then the system of differential equations in displacements (6) takes the form

$$\sum_{m=0}^{3}\left(Ki_0^{u_m} + Ki_1^{u_m}\frac{d}{dx} + Ki_{11}^{u_m}\frac{d^2}{dx^2}\right)u_m + \sum_{n=0}^{2}\left(Ki_0^{w_n} + Ki_1^{w_n}\frac{d}{dx}\right)w_n = 0, \quad i = \overline{1,4};$$
$$\sum_{m=0}^{3}\left(Ki_0^{u_m} + Ki_1^{u_m}\frac{d}{dx}\right)u_m + \sum_{n=0}^{2}\left(Ki_0^{w_n} + Ki_1^{w_n}\frac{d}{dx} + Ki_{11}^{w_n}\frac{d^2}{dx^2}\right)w_n = Ki^{q_{33}^{+}}q_{33}^{+} - Ki^{q_{33}^{-}}q_{33}^{-}, \tag{7}$$
$$i = \overline{9,11}.$$

The boundary conditions at the rigidly clamped edges of the shell are written in the form

$$u_m = 0, \quad (m = \overline{0,3}); \quad w_n = 0, \quad (n = \overline{0,2}) \quad \text{for} \quad x = x_1, \quad x = x_2. \tag{8}$$

System of equations (7) is solved by the finite-difference method. The derivatives of the first and second orders are approximated by the central differences of the second order of accuracy

$$\frac{dy_j}{dx} = \frac{y_{j+1} - y_{j-1}}{2s}; \quad \frac{d^2 y_j}{dx^2} = \frac{y_{j+1} - 2y_j + y_{j-1}}{s^2}.$$

Based on system (7), taking into account boundary conditions (8), we obtain the following finite-difference system

$$\sum_{m=0}^{3}\left(\left(\frac{Ki_{11}^{u_m}}{s^2} - \frac{Ki_1^{u_m}}{2s}\right)u_m^{j-1} + \left(\frac{-2Ki_{11}^{u_m}}{s^2} + Ki_0^{u_m}\right)u_m^j + \left(\frac{Ki_{11}^{u_m}}{s^2} + \frac{Ki_1^{u_m}}{2s}\right)u_m^{j+1}\right)$$

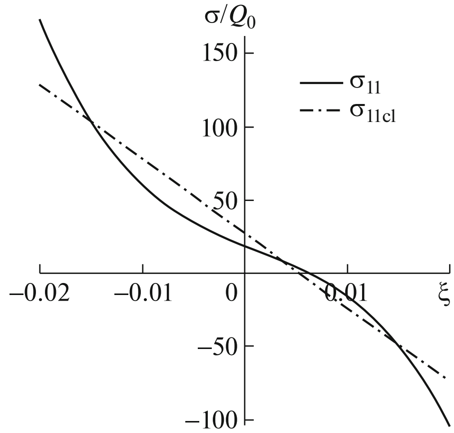

Fig. 2. σ_{11} variation over thickness at edge $x = x_2$.

$$+\sum_{n=0}^{2}\left(\frac{-Ki_1^{w_n}}{2s}w_n^{j-1}+Ki_0^{w_n}w_n^j+\frac{Ki_1^{w_n}}{2s}w_n^{j+1}\right)=0,$$

$$i=\overline{1,4};$$

$$\sum_{m=0}^{3}\left(\frac{-Ki_1^{u_m}}{2s}u_m^{j-1}+Ki_0^{u_m}u_m^j+\frac{Ki_1^{u_m}}{2s}u_m^{j+1}\right)$$
$$+\sum_{n=0}^{2}\left(\left(\frac{Ki_{11}^{w_n}}{s^2}-\frac{Ki_1^{w_n}}{2s}\right)w_n^{j-1}+\left(\frac{-2Ki_{11}^{w_n}}{s^2}+Ki_0^{w_n}\right)w_n^j+\left(\frac{Ki_{11}^{w_n}}{s^2}+\frac{Ki_1^{w_n}}{2s}\right)w_n^{j+1}\right) \qquad (9)$$
$$=Ki^{q_{33}^+}q_{33}^+-Ki^{q_{33}^-}q_{33}^-;$$
$$i=\overline{9,11};\quad j=\overline{1,N-1};$$
$$u_m^0=u_m^N=w_n^0=w_n^N=0;\quad m=\overline{0,3};\quad n=\overline{0,2},$$

where s and $(N + 1)$ are, correspondingly, the pitch of the finite-difference scheme and the number of nodes.

System (9) is a system of linear algebraic equations and is solved by the matrix sweep method using a computer program. Having determined the displacements, the tangential stresses are found by Hooke's law. Transverse stresses are obtained by direct integration of the equations of equilibrium of the three-dimensional elasticity theory

$$\sigma_{13}=-\frac{1}{a_1^2a_2}\int_{-h}^{\xi}\left[\frac{a_1a_2}{A_1}\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial A_2}{\partial x}\frac{1}{A_1A_2}a_1^2(\sigma_{11}-\sigma_{22})\right]d\xi,$$
$$\sigma_{33}=-\frac{1}{a_1a_2}\int_{-h}^{\xi}\left[\frac{a_2}{A_1}\frac{\partial\sigma_{13}}{\partial x}-\frac{a_2}{R_1}\sigma_{11}-\frac{a_1}{R_2}\sigma_{22}+\frac{\partial A_2}{\partial x}\frac{1}{A_1A_2}a_1\sigma_{13}\right]d\xi+\frac{(a_1a_2)_{\xi=-h}}{a_1a_2}q_{33}^-.$$

Calculation example. As an example of a calculation, a conical shell is considered, rigidly clamped at two edges, with the following parameters: conical angle $\theta = \frac{\pi}{4}$, the front and rear end of the shell along the x axis are $x_1 = 1$ m, $x_2 = 3$ m, the shell half-thickness is $h = 2$ cm, Poisson's coefficient $\mu = 0.3$, and Young's modulus $E = 2\times10^5$ MPa. The shell is under the action of a load uniformly distributed on the inner surface. Figures 2–5 illustrate the changes in the normal and shear stresses in the edge region of the shell. Note that the subscript "cl" corresponds to the calculation results according to the classical theory.

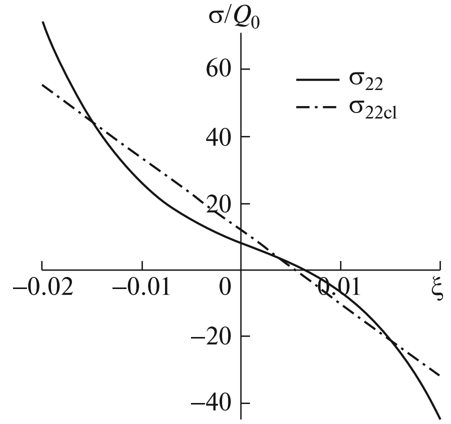

Fig. 3. σ_{22} variation over thickness at edge $x = x_2$.

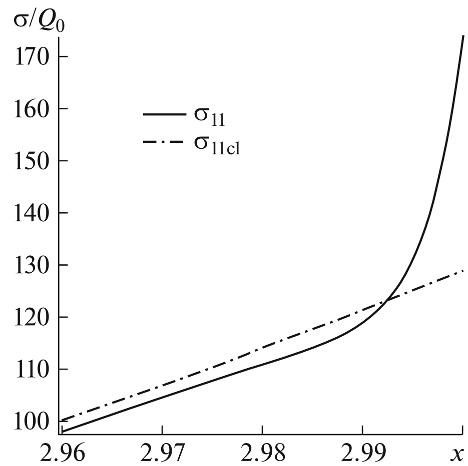

Fig. 4. σ_{11} variation in the boundary region.

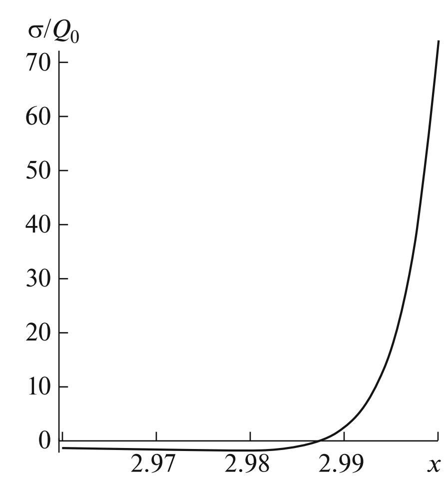

Fig. 5. σ_{33} variation in the boundary region.

Analyzing the graphs in Figs. 2–5, one can determine the stresses in the zone of the rigidly clamped edge significantly more precisely: the maximum normal stress σ_{11} by 35% (Figs. 2, 4) and σ_{22} by 36% (Fig. 3). The maximum transverse normal stress σ_{33} is 42.5% of the main flexural stress σ_{11} (Fig. 5).

CONCLUSIONS

Based on the three-dimensional equations of elasticity theory, using the Lagrange variational principle, differential equations of the equilibrium of conical shells in displacements are obtained, which make it possible to take into account the transverse shear of the shell.

The calculation results of the stress state of the "boundary layer" type according to the refined theory of a conical shell under the action of an axisymmetric load are given and compared with the data of the classical theory. The normal tangential stresses in the region of the rigidly clamped edge of the shell were determined much more precisely.

In the edge zone, the transverse normal stresses, which are neglected in the classical theory, are of the same order of magnitude as the maximum stress values corresponding to the classical theory. Such high additional stresses must be taken into account when assessing the strength and durability of structural elements in mechanical engineering constructions, including aviation and rocket technology.

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CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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